Calculus Name Key

HW 14 : Unit 1 & 2 Test Review

Find the requested information and give your answer in interval notation. Show all work.

1. $f(x)=\sqrt{36-x^2}$

Domain: $[-6,6]$

2. $f(x)=\dfrac{x+3}{x^2-3x-18}$

$\dfrac{\cancel{x+3}}{(x-6)\cancel{(x+3)}}=\dfrac{1}{x-6}$

$x\neq 6,\ x\neq -3$

Domain: $(-\infty,-3)\cup(-3,6)\cup(6,\infty)$

Range: $(-\infty,0)\cup(0,\infty)$

3. $f(x)=\dfrac{5x}{\sqrt{x+7}}$

Domain: $(-7,\infty)$

4. $f(x)=\dfrac{1}{\sqrt{x^2-49}}$

Domain: $(-\infty,-7)\cup(7,\infty)$

5. $f(x)=\dfrac{1}{x^2+13}$

Domain: $\mathbb{R}$

6. $f(x)=\dfrac{1}{\sqrt{x^2+6x+9}}$

$\dfrac{1}{\sqrt{(x+3)^2}}$

Domain: $(-\infty,-3)\cup(-3,\infty)$

Find each limit. Show all work. If a limit does not exist, explain why.

7. $\lim\limits_{x\to 0}\dfrac{\dfrac{1}{x+2}-\dfrac{1}{2}}{x}=\boxed{-1/4}$

$\lim\limits_{x\to 0}\dfrac{\dfrac{2}{2}\cdot\dfrac{1}{x+2}-\dfrac{1}{2}\cdot\dfrac{(x+2)}{(x+2)}}{x}$

$\lim\limits_{x\to 0}\dfrac{\dfrac{2-x-2}{2(x+2)}}{\dfrac{x}{1}}=\dfrac{-\cancel{x}}{2(x+2)}\cdot\dfrac{1}{\cancel{x}}$

$=-\dfrac{1}{2(x+2)}\Big|_{x=0}=\boxed{-\dfrac{1}{4}}$

8. $\lim\limits_{x\to 0}\dfrac{\tan x}{\sin(2x)}=\boxed{1/2}$

$\lim\limits_{x\to 0}\left(\dfrac{x}{1}\right)\cancel{\dfrac{\tan x}{x}}^{1}\left(\cancel{\dfrac{2x}{\sin 2x}}^{1}\right)\dfrac{1}{2x}=\lim\limits_{x\to 0}\dfrac{\cancel{x}}{2\cancel{x}}=\dfrac{1}{2}$

9. $\lim_{x\to\infty} \frac{\sqrt{3x^2+4}}{2x+5} =$ $\frac{\sqrt{3}}{2}$

End behavior

$\frac{\sqrt{3}\sqrt{x^2}}{2x} = \frac{\sqrt{3}|\cancel{x}|}{2\cancel{x}} = \frac{\sqrt{3}}{2}$

10. $\lim_{x\to 5} \frac{\sqrt{x}-\sqrt{5}}{x-5} =$ $\frac{1}{2\sqrt{5}}$

$\frac{(\sqrt{x}-\sqrt{5})(\sqrt{x}+\sqrt{5})}{(x-5)(\sqrt{x}+\sqrt{5})} = \frac{\cancel{x-5}}{\cancel{(x-5)}(\sqrt{x}+\sqrt{5})}$

$= \frac{1}{\sqrt{x}+\sqrt{5}}\Big|_{x=5} = \frac{1}{2\sqrt{5}}$

11. $\lim_{x\to 0} \frac{\sin(4x)}{\tan(4x)} =$ 1

$\lim_{x\to 0}$

$\frac{\sin 4x}{\cancel{4x}} \cdot \frac{4x}{1} \cdot \cancel{\left(\frac{4x}{\tan 4x}\right)} \cdot \frac{1}{4x} = \frac{4\cancel{x}}{4\cancel{x}} = 1$

12. $\lim_{x\to 0} \frac{\tan(2x)}{\sin(3x)} =$ $2/3$

$\frac{\tan 2x}{\cancel{2x}} \cdot \frac{2x}{1} \cdot \frac{3x}{\sin 3x} \cdot \frac{1}{3x} = \frac{2\cancel{x}}{3\cancel{x}} = 2/3$

13. $\lim_{x\to 0} \frac{x^2-16}{x-4} =$ 4

$\frac{(x+4)\cancel{(x-4)}}{\cancel{x-4}} = x+4\Big|_{x=0} = 4$

14. $f(x) = \begin{cases} x^3, & x < -1 \\ x, & -1 < x < 1 \\ 1-x, & x \geq 1 \end{cases}$

14a) $\lim_{x\to 1^-} f(x) =$ 1

$x = (1) = 1$

14b) $\lim_{x\to 1^+} f(x) =$ 0

$1-1=0$

14c) $\lim_{x\to -1} f(x) =$ -1

$(-1)^3 = -1$

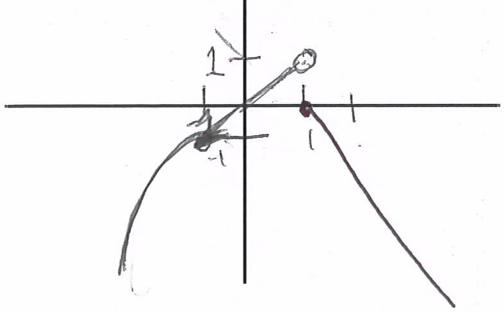
14d) $f(1) =$ 0

$1-1=0$

15. Graph the function. State the points of discontinuity and the type (removable, jump, infinite, oscillate).

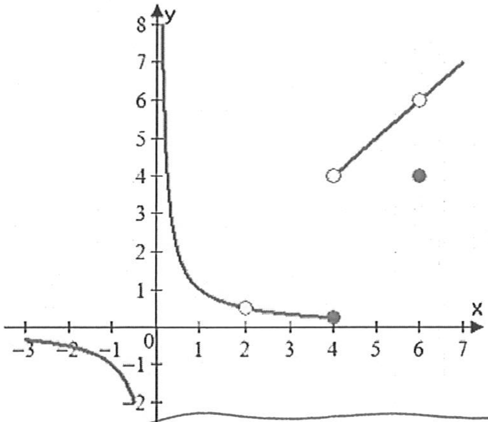
$f(x) = \begin{cases} x^3, & x < -1 \\ x, & -1 < x < 1 \\ 1-x, & x \geq 1 \end{cases}$

$-x+1$



Location	Type
$x=1$	Gap

16. State the points of discontinuity and the type (removable, jump, infinite, oscillate).



X=0 I.D. or VASY
x = 2 hole or R.D.
x=4 gap or jump
x=6 hole or R.D.

17. Use the sandwich (squeeze) theorem to evaluate the limit.

$$\lim_{x\to\infty} \frac{1-\cos(3x)}{2x^2} = \_\_\_\_\_\_$$

WAIT

18. Use the Intermediate Value Theorem to prove that the equation $x^5 - 4x^3 - 3x = -1$ has at least one solution between $x = 2$ and $x = 3$.

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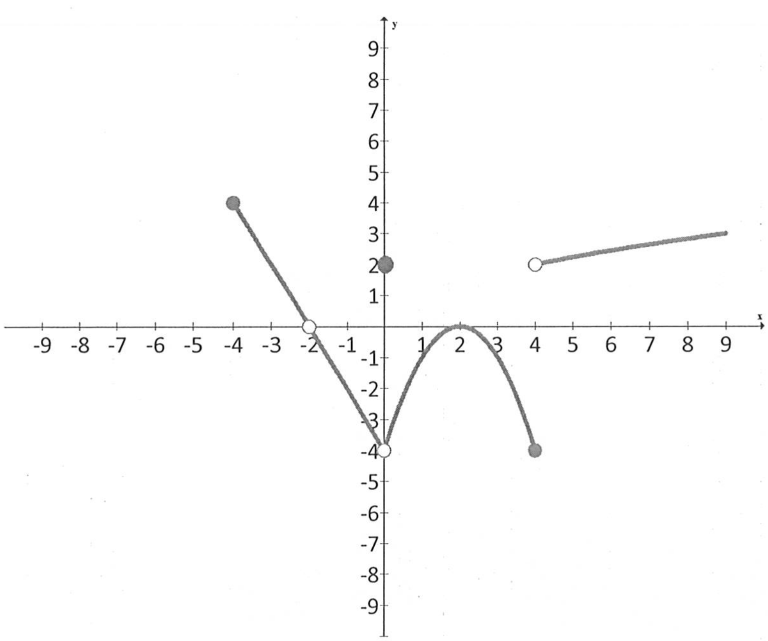
19. Find the requested information from the graph.



a. $\lim_{x\to 4^-} f(x) =$

b. $\lim_{x\to 4^+} f(x) =$

c. $\lim_{x\to 4} f(x) =$

d. $f(4) =$

e. $\lim_{x\to 0^-} f(x) =$

f. $\lim_{x\to 0^+} f(x) =$

g. $\lim_{x\to 0} f(x) =$

h. $f(0) =$