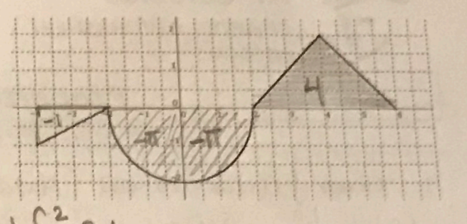
5. The graph of $f(x)$ is shown. Evaluate each integral by using geometric formulas.



a) $\int_0^2 f(x)\,dx = \frac{1}{4}\pi(2)^2 = \boxed{\pi u^2}$

b) $\int_2^6 f(x)\,dx = \boxed{4u^2}$

c) $\int_{-4}^2 f(x)\,dx = \boxed{2\pi - 1}\,u^2$

d) $\int_{-4}^6 f(x)\,dx = -1 - 2\pi + 4 = \boxed{3 - 2\pi}\,u^2$

e) $\int_{-4}^2 |f(x)|\,dx$ Total Area $|-1| + |-\pi - \pi| = \boxed{1 + 2\pi}\,u^2$

f) $\int_{-4}^2 [f(x) + 2]\,dx = \int_{-4}^2 f(x)\,dx + \int_{-4}^2 2\,dx = -1 - 2\pi + 2x\big|_{-4}^{2} = -1 - 2\pi + [4 + 8] = -1 - 2\pi + 12 = \boxed{11 - 2\pi}\,u^2$

6. Consider the function f that is continuous in the interval $[-5, 5]$ and for which $\int_0^5 f(x)\,dx = 4$.

Evaluate each integral.

a) $\int_0^5 [f(x) + 3]\,dx = \int_0^5 f(x)\,dx + \int_0^5 3\,dx = 4 + 3x\big|_0^5 = 4 + 15 = \boxed{19u^2}$

b) $\int_{-2}^3 f(x+2)\,dx$ (*Hint:* assume the graph for $f(x)$ is known, and sketch the graph of $f(x+2)$) → moved to the left 2 units, but the distance (a,b) is the same. $= \boxed{4u^2}$

c) $\int_{-5}^5 f(x)\,dx$ (f is even.) $= \int_{-5}^0 f(x)\,dx + \int_0^5 f(x)\,dx = 4 + 4 = \boxed{8u^2}$

d) $\int_{-5}^5 f(x)\,dx$ (f is odd.) $\int_{-5}^0 f(x)\,dx + \int_0^5 f(x)\,dx = -4 + 4 = \boxed{0u^2}$

In 7–10, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.

7. $\int_a^b [f(x) + g(x)]\,dx = \left[\int_a^b f(x)\,dx\right] + \left[\int_a^b g(x)\,dx\right]$ True

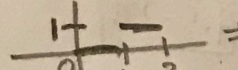
8. $\int_a^b [f(x) \cdot g(x)]\,dx = \left[\int_a^b f(x)\,dx\right] \cdot \left[\int_a^b g(x)\,dx\right]$ False, b/c think "polynomial"?

9. The value of $\int_a^b f(x)\,dx$ must be positive. No b/c of "net" area (below the x-axis)

10. If $\int_a^b f(x)\,dx > 0$, then f is nonnegative for all x in $[a, b]$. No b/c of "net" area is an accumulation of what is above and below the axis over (a,b).

11. Evaluate, if possible, the integral $\int_0^2 \lfloor x \rfloor\,dx$ (*Hint:* sketch the graph of $y = \lfloor x \rfloor$ for $0 \le x \le 2$ first.

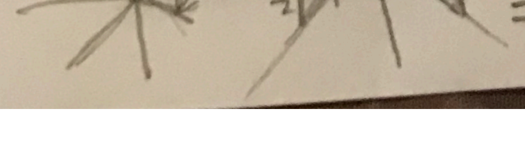
Remember that $y = \lfloor x \rfloor$ is the greatest integer function and it always rounds down to the nearest integer value.)



$\Rightarrow \int_0^2 \text{int}(x)\,dx = 0 + 1 = \boxed{1u^2}$

12. Sketch the region whose area is given by the definite integral. Then use geometric formulas to evaluate the integral.

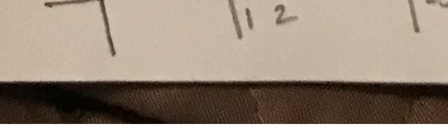
a) $\int_{-2}^2 (1 - |x|)\,dx$ P.F: $y = |x|$ Transformations: reflected over x-axis and moved up 1 unit.



$\Rightarrow$ ③ Area $= -\frac{1}{2} + 1 - \frac{1}{2} = \boxed{0u^2}$

b) $\int_0^3 |3x - 6|\,dx = \int_0^3 3|x-2|\,dx = 3\int_0^3 |x-2|\,dx$

P.F: $y = x$ moved ® 2 units and mult by 3.



$= \boxed{7.5u^2}$