HW#38 (Key)

(1) $4y^2+2=3x^2$

$8y\frac{dy}{dx} = 6x$

$\boxed{\frac{dy}{dx}} = \frac{6x}{8y} = \boxed{\frac{3x}{4y}}$

let $u=3x$ $u'=3$
$v=4y$ $v'=4\frac{dy}{dx}$

(*) $\frac{d^2y}{dx^2} = \frac{4y(3)-3x(4\frac{dy}{dx})}{(4y)^2} = \frac{12y-12x\frac{dy}{dx}}{16y^2} = \frac{12y-12x\left(\frac{3x}{4y}\right)}{16y^2}$

$= \frac{12y-\frac{36x^2}{4y}}{16y^2} = \left(12y-9\frac{x^2}{y}\right)\frac{1}{16y^2}$

$= \frac{y}{y}\left(\frac{12y}{16y^2}-\frac{9x^2}{16y^3}\right)$ Comm den.

$= \frac{12y^2-9x^2}{16y^3} = \frac{d^2x}{dy^2}$

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(2) $5=4x^2+5y^2$

$0 = 8x+10y\frac{dy}{dx}$

$\boxed{\frac{dy}{dx}} = \frac{-8x}{10y} = \boxed{\frac{-4x}{5y}}$

let $u=-4x$ $u'=-4$
$v=5y$ $v'=5\frac{dy}{dx}$

$\frac{d^2y}{dx^2} = \frac{-20y+20x\frac{dy}{dx}}{(5y)^2} = \frac{-20y+20x\left(\frac{-4x}{5y}\right)}{25y^2} = \left(-20y-\frac{80x^2}{5y}\right)\frac{1}{25y^2}$

$= \frac{5y}{5y}\left(\frac{-20y}{25y^2}-\frac{80x^2}{125y^3}\right)$ Get Comm. denominator

$\boxed{\frac{-100y^2-80x^2}{125y^3}} = \boxed{\frac{d^2y}{dx^2}} = \frac{-20y^2-16x^2}{25y^3}$

③ $x^2+y^2=1$

$2x+2y\frac{dy}{dx}=0$

$2y\frac{dy}{dx}=-2x$

$\frac{dy}{dx}=\frac{-2x}{2y}=\frac{-x}{y}$

Let $u=-x$ $u'=-1$

$v=y$ $v'=\frac{dy}{dx}$

$$\frac{d^2y}{dx^2}=\frac{-y+x\frac{dy}{dx}}{y^2}=\frac{-y+x\left(\frac{-x}{y}\right)}{y^2}=\frac{-y-\frac{x^2}{y}}{y^2}$$

$$=\left(-y-\frac{x^2}{y}\right)\left(\frac{1}{y^2}\right)=\frac{\frac{-y}{(\frac{y}{y})}}{y^2}-\frac{x^2}{y^3}$$ Get comm. den.

$$=\frac{-y^2-x^2}{y^3}=\frac{d^2y}{dx^2}$$

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④ $y^2=x^2+2x$

$2y\frac{dy}{dx}=2x+2$

$\frac{dy}{dx}=\frac{2x+2}{2y}=\frac{x+1}{y}$

Let $u=x+1$ $u'=1$

$v=y$ $v'=\frac{dy}{dx}$

$$\frac{d^2y}{dx^2}=\frac{y(1)-(x+1)\frac{dy}{dx}}{y^2}=\frac{y-(x+1)\left(\frac{x+1}{y}\right)}{y^2}=\left[y-\frac{(x^2+2x+1)}{y}\right]\frac{1}{y^2}$$

$$=\frac{\frac{y^2}{y^2}}{y}-\frac{x^2+2x+1}{y^3}$$ Get comm den.

$$\frac{y^2-x^2-2x-1}{y^3}=\frac{d^2y}{dx^2}$$

⑤ $y^2+2y=2x+1$

$u=1 \quad u'=0$

$v=y+1 \quad v'=\frac{dy}{dx}$

$2y\frac{dy}{dx}+2\frac{dy}{dx}=2$

$\frac{dy}{dx}(2y+2)=2$

$\frac{dy}{dx}=\frac{2}{2y+2}=\frac{1}{y+1}$ (let $u=2 \quad u'=0$, $v=2y+2 \quad v'=2\frac{dy}{dx}$)

$\frac{d^2y}{dx^2}=\frac{(2y+2)(0)-2(2\frac{dy}{dx})}{(2y+2)^2}=\frac{-4\frac{dy}{dx}}{(2y+2)^2}=\frac{-4\left(\frac{2}{2y+2}\right)}{(2y+2)^2}$

$=\frac{-8}{2y+2}\cdot\frac{1}{(2y+2)^2}$

$\frac{d^2y}{dx^2}=\frac{-8}{(2y+2)^3}=\frac{-1}{(y+1)^3}$

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⑥ $x^2+xy-y^2=1$ at $(2,3)$

$\Rightarrow y-3=m(x-2)$

$2x+x\frac{dy}{dx}+y-2y\frac{dy}{dx}=0$

$\frac{dy}{dx}(x-2y)=-2x-y$

$\frac{dy}{dx}=\frac{-2x-y}{x-2y}\Big|_{(2,3)}=\frac{-2(2)-3}{2-2(3)}=\frac{-4-3}{2-6}=\frac{-7}{-4}=\frac{7}{4}$

$\Rightarrow y-3=\frac{7}{4}(x-2)$ (tangent)

$y-3=-\frac{4}{7}(x-2)$ (normal)

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⑦ $x\sin 2y=y\cos 2x \quad (\pi/4,\pi/2)$ $\quad y-\pi/2=m(x-\pi/4)$

$x\cos 2y(2\frac{dy}{dx})+\sin 2y=-2y\sin 2x+\cos 2x\frac{dy}{dx}$

$\frac{dy}{dx}(2x\cos 2y-\cos 2x)=-2y\sin 2x-\sin 2y$

$\frac{dy}{dx}=\frac{-2y\sin 2x-\sin 2y}{2x\cos 2y-\cos 2x}\Big|_{(\pi/4,\pi/2)}=\frac{-\pi\sin\pi/2-\sin\pi}{\pi/2\cos\pi-\cos\pi/2}=\frac{-\pi}{-\frac{\pi}{2}}=2$

$y-\pi/2=2(x-\pi/4)$

$y-\pi/2=-\frac{1}{2}(x-\frac{\pi}{2})$