Calculus HW 37 Name Answers

Unit 3.8 Implicit differentiation- 2nd derivatives

For each problem, use implicit differentiation to find $\frac{d^2y}{dx^2}$ in terms of x and y.

1) $x^3 = 2y^2 + 5$

$3x^2 = 4yy' \Rightarrow y' = \frac{3x^2}{4y}$

$y'' = \frac{4y \cdot 6x - 3x^2 \cdot 4y'}{16y^2} = \frac{24xy - 12x^2 \cdot \frac{3x^2}{4y}}{16y^2}$

$= \frac{96xy^2 - 36x^4}{64y^3} = \frac{24xy - 9x^4}{16y^3}$

2) $5x + 3y^2 = 1$

$5 + 6yy' = 0 \Rightarrow y' = \frac{-5}{6y}$

$y'' = \frac{6y(0) + 5 \cdot 6y'}{36y^2} = \frac{30 \cdot \frac{-5}{6y}}{36y^2}$

$= \frac{-150}{6y \cdot 36y^2} = \frac{-150}{216y^3} = \frac{-25}{36y^3}$

3) $5x^2 = 5y^2 + 4$

$10x = 10yy' \Rightarrow y' = \frac{x}{y}$

$y'' = \frac{y(1) - xy'}{y^2} = \frac{y - \frac{x^2}{y}}{y^2}$

$y'' = \frac{y^2 - x^2}{y^3}$

4) $x^3 + 4y^2 = 1$

$3x^2 + 8yy' = 0 \Rightarrow y' = \frac{-3x^2}{8y}$

$y'' = \frac{8y(-6x) - (-3x^2)(8y')}{64y^2}$

$= \frac{-48xy + 24x^2\left(\frac{-3x^2}{8y}\right)}{64y^3} = \frac{-48xy^2 - 9x^4}{64y^3}$

5) $3x^2 + y^2 = 2$

$6x + 2yy' = 0 \Rightarrow y' = \frac{-6x}{2y} = \frac{-3x}{y}$

$y'' = \frac{y(-3) + 3x(y')}{y^2} = \frac{-3y + 3x \cdot \left(\frac{-3x}{y}\right)}{y^2}$

$= \frac{-3y^2 - 9x^2}{y^3}$

6) $5 = 4x^3 - 4y^2$

$0 = 12x^2 - 8yy' \Rightarrow y' = \frac{12x^2}{8y} = \frac{3x^2}{2y}$

$y'' = \frac{2y(6x) - 3x^2 \cdot 2y'}{4y^2} = \frac{12xy - 6x^2\left(\frac{3x^2}{2y}\right)}{4y^2}$

$= \frac{24xy^2 - 18x^4}{8y^3} = \frac{12xy^2 - 9x^4}{4y^3}$

7) $2 = 2x^2 - 4y^2$

$0 = 4x - 8yy' \Rightarrow y' = \frac{4x}{8y} = \frac{x}{2y}$

$y'' = \frac{2y(1) - x \cdot 2y'}{4y^2} = \frac{2y - 2x\left(\frac{x}{2y}\right)}{4y^2}$

$= \frac{4y^2 - 2x^2}{8y^3} = \frac{2y^2 - x^2}{4y^3}$

8) $x^2 + 4y^2 = 5$

$2x + 8yy' = 0 \Rightarrow y' = \frac{-2x}{8y} = \frac{-x}{4y}$

$y'' = \frac{4y(-1) + 4xy'}{16y^2} = \frac{-4y + 4x\left(\frac{-x}{4y}\right)}{16y^2}$

$= \frac{-16y^2 - 4x^2}{64y^3} = \frac{-4y^2 - x^2}{16y^3}$

9) $2x - 5y^2 = 3$

$2 - 10yy' = 0 \Rightarrow y' = \frac{2}{10y} = \frac{1}{5y}$

$y'' = \frac{5y(0) - 1 \cdot 5y'}{25y^2} = \frac{-5\left(\frac{1}{5y}\right)}{25y^2}$

$= \frac{-5}{125y^3} = \frac{-1}{25y^3}$

10) $5x^3 = -4y^2 + 4$

$15x^2 = -8yy' \Rightarrow y' = \frac{15x^2}{-8y}$

$y'' = \frac{-8y(30x) - 15x^2 \cdot (-8y')}{64y^2}$

$= \frac{-240xy + 120x^2\left(\frac{15x^2}{-8y}\right)}{64y^2} = \frac{-240xy^2 - 225x^4}{64y^3}$