

Calculus Semester 1 Final Exam Review Packet (FERP)

Exam: Multiple Choice & Free Response Problems

Use: Notecard 3x5 (Handwritten, Front/Back, Name on it)

Topics: Definitions: **Limit**
Continuous & Discontinuous
Differentiable

Domain/Range (interval notation)

Limits: Graph (left, right, $+\infty$, $-\infty$, $f(x)$)
Direct Substitution
Simplify first (factor)
End-Behavior ($x \rightarrow \pm\infty$)
Trig Limits
Graphically (can always check)

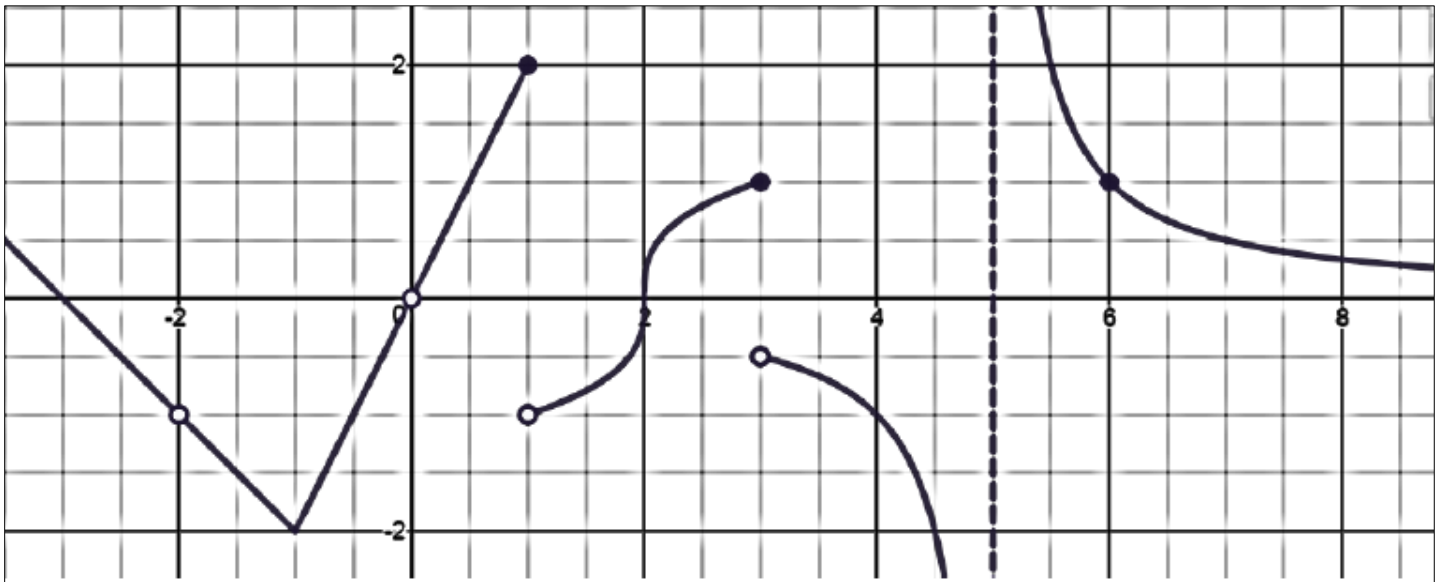
Derivatives: Definition of Derivative
Shortcuts (x^n , product rule, quotient rule)
Chain Rule
Trig Derivatives
Exponential/Logs ($\log_b u$, $\ln u$, e^u , a^u)
Implicit Differentiation
2nd Derivative (y'' , or implicit)
Derivative at a point
Find the equation of the tangent or normal line at a point
Graph the derivative

Points and Types of Discontinuity and/or Differentiability

Particle/Projectile Motion

Displacement
Average Velocity
Instantaneous Velocity
Acceleration
Change Direction

1. Determine the following from the graph of $f(x)$ below.



a. $\lim_{x \rightarrow 3^-} f(x)$ _____

b. $\lim_{x \rightarrow 3^+} f(x)$ _____

c. $\lim_{x \rightarrow 3} f(x)$ _____

d. $f(3)$ _____

e. $\lim_{x \rightarrow -\infty} f(x)$ _____

f. $\lim_{x \rightarrow \infty} f(x)$ _____

g. $\lim_{x \rightarrow -1} f(x)$ _____

h. $f(2)$ _____

i. $\lim_{x \rightarrow 6} f(x)$ _____

j. $\lim_{x \rightarrow 5^+} f(x)$ _____

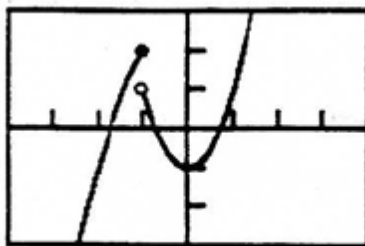
k. $\lim_{x \rightarrow 0} f(x)$ _____

l. $\lim_{x \rightarrow 1^-} f(x)$ _____

2. Use the graph above to Name the x-coordinate(s) of the point(s) of discontinuity and the type of discontinuity.

Locations (@x =)	Type of discontinuity

3. Use the graph to estimate the limits and value of the function.



$[-4, 4]$ by $[-3, 3]$

a. $\lim_{x \rightarrow \infty} f(x)$ _____

b. $\lim_{x \rightarrow -1^+} f(x)$ _____

c. $\lim_{x \rightarrow -1^-} f(x)$ _____

d. $f'(0)$ _____

4. Find $\lim_{x \rightarrow 3^+} \frac{x^2 - 4}{x^2 + x - 2}$ if it exists.

5. Find $\lim_{x \rightarrow -2} \begin{cases} 2 - x, & x < -2 \\ x^2 - 2x, & x > -2 \end{cases}$ if it exists.

6. Find $\lim_{x \rightarrow 3^-} e$ if it exists.

7. Find $\lim_{x \rightarrow \infty} \frac{6x - 3x^3 - 3}{5x^3 + 9}$ if it exists.

8. Find $\lim_{x \rightarrow 16} \frac{x - 16}{\sqrt{x} - 4}$ if it exists.

9. Find $\lim_{x \rightarrow -3^-} \frac{1}{x + 3}$ if it exists.

10. Find $\lim_{x \rightarrow 0} \frac{7 \sin x}{6x}$ if it exists.

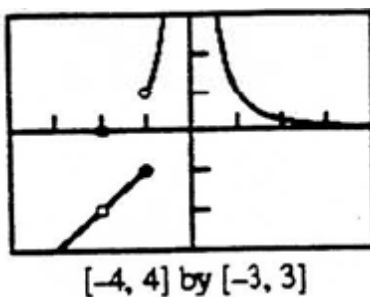
11. Find $\lim_{x \rightarrow 0} \frac{\tan 4x}{\tan 3x}$ if it exists.

12. Find $\lim_{x \rightarrow 2} \csc\left(\frac{\pi}{6}x\right)$

13. Find $\lim_{x \rightarrow -2} \frac{2x^2 - 9x + 9}{x^2 - 9}$ if it exists.

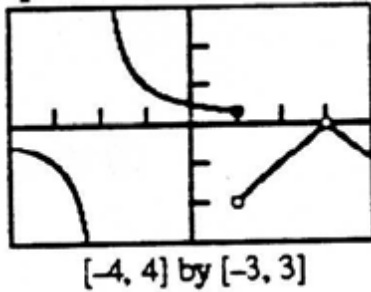
14. Find the points of discontinuity of the function whose graph is shown here.

For each discontinuity, identify the type of discontinuity (removable, jump, infinite, or oscillating)



15. Find the points of discontinuity of the function whose graph is shown here.

For each discontinuity, identify the type of discontinuity (removable, jump, infinite, or oscillating)



16. Find the points of discontinuity of the function: $y = \frac{x^2 + 9x - 36}{x^2 - 9}$

- | | |
|--|---|
| a. Oscillating discontinuity at $x = -3$
Removable discontinuity at $x = 3$ | b. Removable discontinuity at $x = 3$
Infinite discontinuity at $x = -3$ |
| c. Removable discontinuity at $x = -12$
Removable discontinuity at $x = 12$ | d. Removable discontinuity at $x = 3$
Infinite discontinuity at $x = 12$ |

17. Find the points of discontinuity of the function: $y = \frac{x^2 + 13x + 36}{x^2 - 16}$

- | | |
|--|--|
| a. Removable discontinuity at $x = -4$
Infinite discontinuity at $x = 4$ | b. Removable discontinuity at $x = -4$
Infinite discontinuity at $x = 9$ |
| c. Removable discontinuity at $x = -9$
Removable discontinuity at $x = 9$ | d. Oscillating discontinuity at $x = 4$
Removable discontinuity at $x = -4$ |

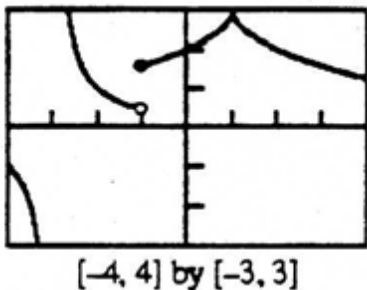
18. Find the derivative of $x^5 - 17xy + y^3 = -23$ at $(1, 2)$.

19. Find the slope, m , of the curve $y = 5x^3 + 7$ at $x = -3$.

20. Find derivative of $y = x \cdot \cos x$ at $x = \frac{\pi}{2}$

21. Use the definition of a derivative to find the derivative of $f(x) = 6x^2 + 7x - 21$. Set up the equation only.
Hint: use the long way, no power rule.

22. The graph of $y = f(x)$ is shown below. At what values of x does $f(x)$ appear to be nondifferentiable?



23. Find the derivative of $f(x) = \frac{2+x}{5-x}$ for $x = 4$.

24. Find the derivative of $f(x) = \frac{8+x}{7-x}$ for $x = 8$.

25. Find y' if $y = \frac{\cos x}{5 + \cos x}$.

26. Find $\frac{dy}{dx}$ if $y = (5x + 7)^6$.

27. Find $\frac{dy}{dx}$ if $y = (4x + 13)^5$.

28. Find $\frac{dy}{dx}$ if $y = -\cos(3x^2 + 6)$.

29. Given $f(x) = 2e^{6x}$, find $f''(x)$.

30. Find $\frac{dy}{dx}$ if $y = 6^{-6x}$.

31. Find $\frac{dy}{dx}$ if $y = 4x^5 \ln(6x^2)$.

32. Find y' if $y = \left(\frac{x-1}{x+3}\right)^4$

33. A body's position is given by $s(t) = t^3 - 7t^2 + 15t + 6$, where t is in seconds and s is in meters.

- Find the displacement after the first 4 seconds.
- Find the average velocity of the particle for the first 4 seconds.
- Find the instantaneous velocity when $t = 4$ seconds.
- Find the acceleration of the boy when $t = 4$ seconds.
- At what value(s) of t does the body change direction? Justify your answer.
- When does the body move to the left? When does the body move to the right?
- Find the position when the velocity is zero.

34. Find the domain and range (in interval notation) of the functions. Show work.

a. $y = \frac{x^2 - 3x + 2}{x^2 - 5x + 6}$

Domain: _____

Range: _____

b. $f(x) = 3x^2 - 5x + 7$

Domain: _____

Range: _____

c. $y = \sqrt{16 - x^2}$

Domain: _____

Range: _____

d. $f(x) = \sqrt{x^2 - 4x}$

Domain: _____

Range: _____

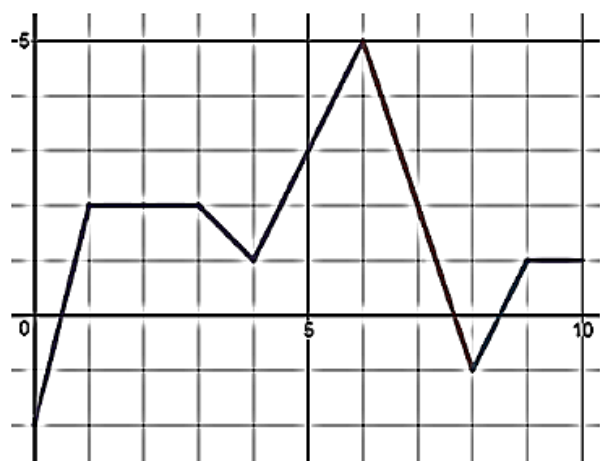
35. Find the slope of the tangent line at $(2, 6)$ for $2x^2 - 3xy + y^2 = 4x$.

36. Find an equation for the line tangent to the graph $y = \cos x + 1$ at $x = \frac{\pi}{2}$.

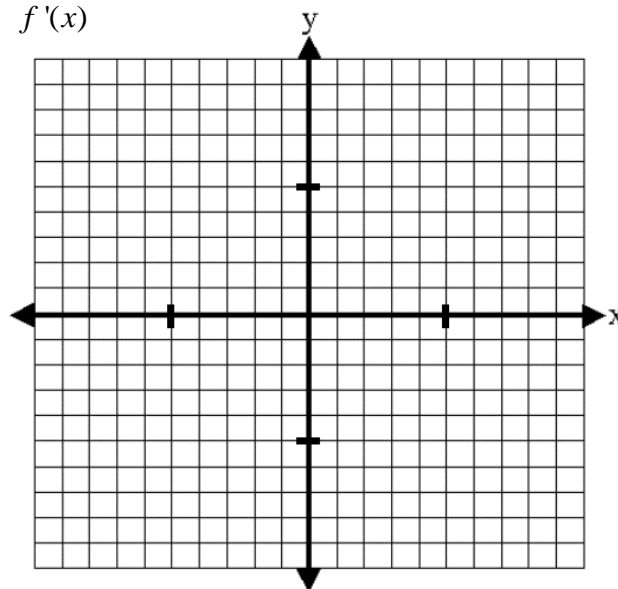
37. Find the equation of the tangent line to the curve $f(x) = 3x^2$ at the point $(1, 3)$.

38. The graphs of $f(x)$ is given. Sketch the graph of $f'(x)$.

$f(x)$



$f'(x)$



39. Find $\frac{d^2y}{dx^2}$ of $y = \frac{x+2}{x-3}$

40. Find $\frac{dy}{dx}$ of $y = 2\sec x - \tan x$

43. Use the table to symbolically evaluate the following derivatives.

a) $h(x) = f(x) + 2g(x)$ at $x = 1$

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	2	1	1	2
2	3	0	3	0
3	2	-1	1	-2

b) $h(x) = f(x) \cdot g(x)$ at $x = 2$

c) $h(x) = f(g(x))$ at $x = 3$

d) $h(x) = \frac{f(x)}{g(x)}$ at $x = 1$

44. The following is a graph of a particle's velocity (meters) over time (sec):

a. When is the particle moving to the:

left:

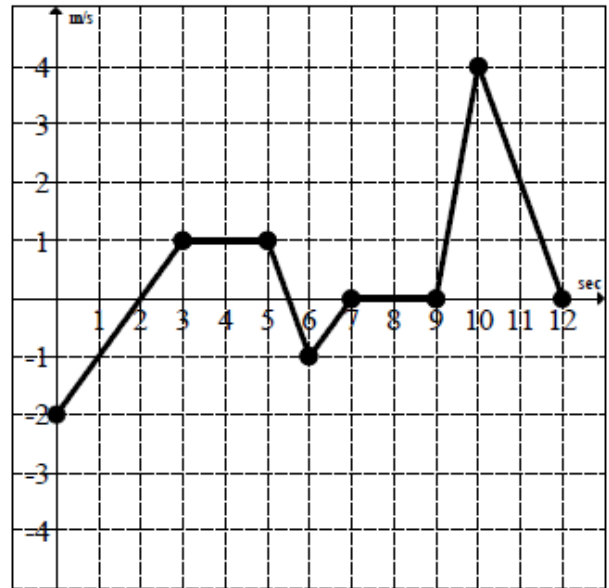
right:

b. When is the particle:

speeding up:

slowing down:

standing still :



c. What is the particle's greatest velocity?