Answers to Student Edition Exercises

Chapter 1
Prerequisites for Calculus

1.1 Lines (pp. 1–9)

Quick Review 1.1

1. -2 **2.** -1

3. -1 **4.** $\frac{5}{4}$

5. (a) Yes **(b)** No

6. (a) Yes **(b)** No

7. $\sqrt{2}$ **8.** $\frac{5}{3}$

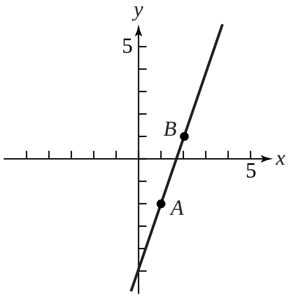
9. $y = \frac{4}{3}x - \frac{7}{3}$ **10.** $y = \frac{2}{5}x - \frac{3}{5}$

Section 1.1 Exercises

1. $\Delta x = -2, \Delta y = -3$ **2.** $\Delta x = 2, \Delta y = -4$

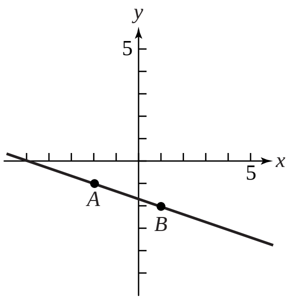
3. $\Delta x = -5, \Delta y = 0$ **4.** $\Delta x = 0, \Delta y = -6$

5. (a) and **(c)**



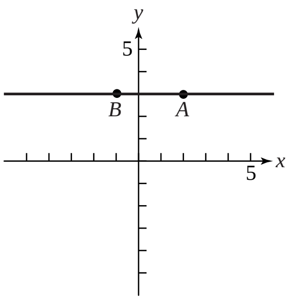
(b) 3

6. (a) and **(c)**



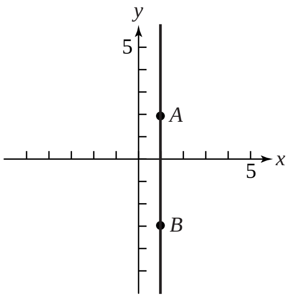
(b) $-\frac{1}{3}$

7. (a) and **(c)**



(b) 0

8. (a) and **(c)**



(b) Has no slope (undefined)

9. (a) $x = 2$ **(b)** $y = 3$

10. (a) $x = -1$ **(b)** $y = \frac{4}{3}$

11. (a) $x = 0$ **(b)** $y = -\sqrt{2}$

12. (a) $x = -\pi$ **(b)** $y = 0$

13. $y = 1(x - 1) + 1$ **14.** $y = -1(x + 1) + 1$

15. $y = 2(x - 0) + 3$ **16.** $y = -2(x + 4) + 0$

17. $3x - 2y = 0$ **18.** $y = 1$

19. $x = -2$ **20.** $3x + 4y = -2$

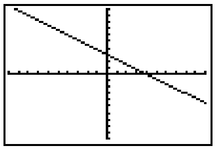
21. $y = 3x - 2$ **22.** $y = -x + 2$

23. $y = -\frac{1}{2}x - 3$ **24.** $y = \frac{1}{3}x - 1$

25. $y = \frac{5}{2}x$ **26.** $y = \frac{2}{5}x$

27. (a) $-\frac{3}{4}$ **(b)** 3

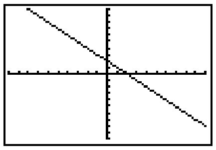
(c)



$[-10, 10]$ by $[-10, 10]$

28. (a) -1 **(b)** 2

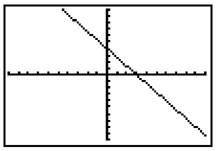
(c)



$[-10, 10]$ by $[-10, 10]$

29. (a) $-\frac{4}{3}$ **(b)** 4

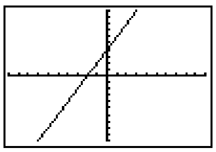
(c)



$[-10, 10]$ by $[-10, 10]$

30. (a) 2 **(b)** 4

(c)



$[-10, 10]$ by $[-10, 10]$

31. (a) $y = -x$ **(b)** $y = x$

32. (a) $y = -2x - 2$ **(b)** $y = \frac{1}{2}x + 3$

33. (a) $x = -2$ **(b)** $y = 4$

34. (a) $y = \frac{1}{2}$ **(b)** $x = -1$

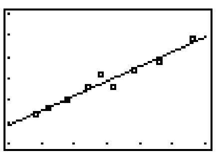
35. $m = \frac{7}{2}, b = -\frac{3}{2}$ **36.** $m = -\frac{3}{2}, b = 2$

37. $y = -1$ **38.** $x = -6$

39. (a) $y = 0.680x + 9.013$

(b) The slope is 0.68. It represents the approximate average weight gain in pounds per month.

(c)



[15, 45] by [15, 45]

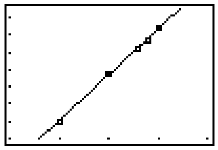
39. continued

(d) 29 pounds

40. (a) $y = 1{,}060.4233x - 2{,}077{,}548.669$

(b) The slope is 1,060.4233. It represents the approximate increase in earnings in dollars per year.

(c)



[1975, 1995] by [20,000, 35,000]

(d) Approximately $43,298

41. $y = 1(x - 3) + 4$

$y = x - 3 + 4$

$y = x + 1$, which is the same equation.

42. (a) When $y = 0$, $x = c$; when $x = 0$, $y = d$.

(b) The x-intercept is $2c$ and the y-intercept is $2d$.

43. (a) $k = 2$ **(b)** $k = -2$

44. (a) -3.75 degrees/inch **(b)** -16.1 degrees/inch

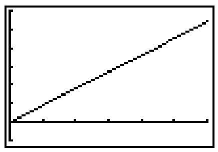
(c) -7.1 degrees/inch

(d) Best: fiberglass; poorest: gypsum wallboard The best insulator will have the largest temperature change per inch, because that will allow larger temperature differences on opposite sides of thinner layers.

45. 5.97 atmospheres ($k = 0.0994$)

46. (a) $d(t) = 45t$

(b)



$[0, 6]$ by $[-50, 300]$

(c) Slope is 45, which is the speed in miles per hour.

(d) Suppose the car has been traveling 45 mph for several hours when it is first observed at point P at time $t = 0$.

(e) The car starts at time $t = 0$ at a point 30 miles past P.

47. (a) $y = 5632x - 11{,}080{,}280$

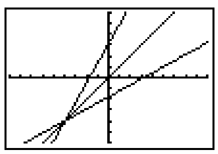
(b) The rate at which the median price is increasing in dollars per year

(c) $y = 2732x - 5{,}362{,}360$

(d) In the Northeast

48. (a) Yes, -40 degrees

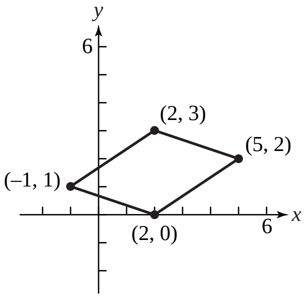
(b)

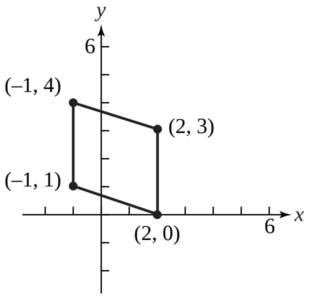


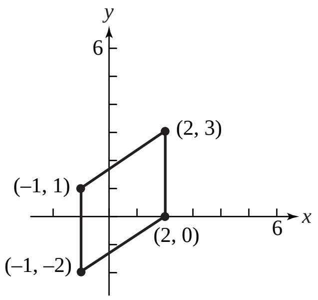
$[-90, 90]$ by $[-60, 60]$

It's related because all three lines pass through the point $(-40, -40)$ where the Fahrenheit and Celsius temperatures are the same.

49. The coordinates of the three missing vertices are (5, 2), (−1, 4) and (−1, −2).







50. Suppose that the vertices of the original quadrilateral are (a, b), (c, d), (e, f), and (g, h). When the midpoints are connected, the pairs of opposite sides of the resulting figure have slopes $\frac{f-b}{e-a}$ or $\frac{h-d}{g-c}$, and opposite sides are parallel.

51. $y = -\frac{3}{4}(x - 3) + 4$ or $y = -\frac{3}{4}x + \frac{25}{4}$

52. **(a)** $y = \frac{B}{A}(x - a) + b$

(b) The coordinates are $\left(\frac{B^2a + AC - ABb}{A^2 + B^2}, \frac{A^2b + BC - ABa}{A^2 + B^2}\right)$.

(c) Distance $= \frac{|Aa + Bb - C|}{\sqrt{A^2 + B^2}}$

1.2 Functions and Graphs (pp. 9–19)

Quick Review 1.2

1. $[-2, \infty)$ **2.** $(-\infty, 0) \cup (2, \infty)$

3. $[-1, 7]$ **4.** $(-\infty, -3] \cup [7, \infty)$

5. $(-4, 4)$ **6.** $[-3, 3]$

7. Translate the graph of f 2 units left and 3 units downward.

8. Translate the graph of f 5 units right and 2 units upward.

9. **(a)** $x = -3, 3$ **(b)** No real solution

10. **(a)** $x = -\frac{1}{5}$ **(b)** No solution

11. **(a)** $x = 9$ **(b)** $x = -6$

12. **(a)** $x = -7$ **(b)** $x = 28$

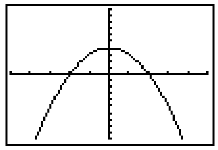
Section 1.2 Exercises

1. $A = \frac{\pi d^2}{4}$ **2.** $h = \frac{\sqrt{3}}{2}s$

3. $S = 6e^2$ **4.** $V = \frac{4}{3}\pi r^3$

5. **(a)** All reals **(b)** $(-\infty, 4]$

(c)

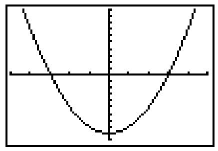


$[-5, 5]$ by $[-10, 10]$

(d) Symmetric about y-axis

6. **(a)** All reals **(b)** $[-9, \infty)$

(c)

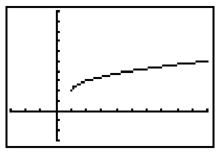


$[-5, 5]$ by $[-10, 10]$

(d) Symmetric about y-axis

7. **(a)** $[1, \infty)$ **(b)** $[2, \infty)$

(c)

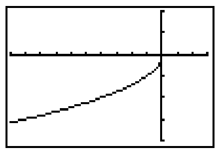


$[-3, 10]$ by $[-3, 10]$

(d) None

8. **(a)** $(-\infty, 0]$ **(b)** $(-\infty, 0]$

(c)

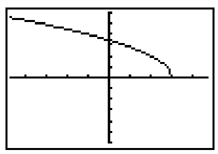


$[-10, 3]$ by $[-4, 2]$

(d) None

9. **(a)** $(-\infty, 3]$ **(b)** $[0, \infty)$

(c)



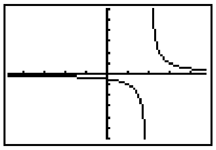
$[-4.7, 4.7]$ by $[-6, 6]$

(d) None

10. **(a)** $(-\infty, 2) \cup (2, \infty)$ **(b)** $(-\infty, 0) \cup (0, \infty)$

10. continued

(c)

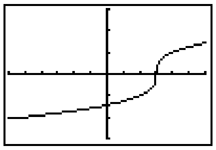


[−4.7, 4.7] by [−6, 6]

(d) None

11. (a) All reals **(b)** All reals

(c)

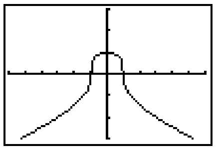


[−6, 6] by [−3, 3]

(d) None

12. (a) All reals **(b)** $(-\infty, 1]$

(c)

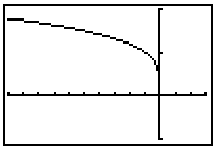


[−6, 6] by [−3, 3]

(d) Symmetric about *y*-axis

13. (a) $(-\infty, 0]$ **(b)** $[0, \infty)$

(c)

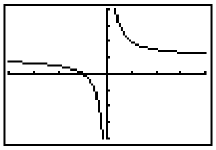


[−10, 3] by [−1, 2]

(d) None

14. (a) $(-\infty, 0) \cup (0, \infty)$ **(b)** $(-\infty, 1) \cup (1, \infty)$

(c)

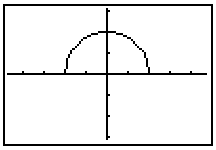


[−4, 4] by [−4, 4]

(d) None

15. (a) $[-2, 2]$ **(b)** $[0, 2]$

(c)

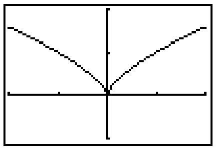


[−4.7, 4.7] by [−3.1, 3.1]

(d) Symmetric about *y*-axis

16. (a) All reals **(b)** $[0, \infty)$

(c)

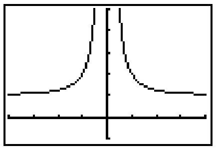


[−2, 2] by [−1, 2]

(d) Symmetric about *y*-axis

17. (a) $(-\infty, 0) \cup (0, \infty)$ **(b)** $(1, \infty)$

(c)

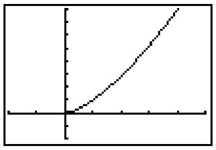


[−4, 4] by [−1, 5]

(d) Symmetric about *y*-axis

18. (a) $[0, \infty)$ **(b)** $[0, \infty)$

(c)



[−2, 5] by [−2, 8]

(d) None

19. Even **20.** Neither

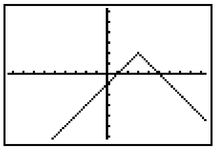
21. Neither **22.** Even

23. Even **24.** Odd

25. Odd **26.** Neither

27. Neither **28.** Even

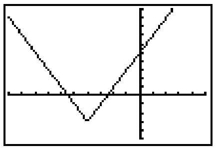
29. (a)



[−9.4, 9.4] by [−6.2, 6.2]

(b) All reals **(c)** $(-\infty, 2]$

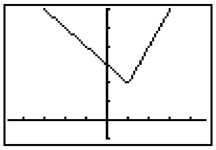
30. (a)



[−10, 5] by [−5, 10]

(b) All reals **(c)** $[-3, \infty)$

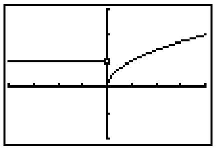
31. (a)



[−4.7, 4.7] by [−1, 6]

(b) All reals **(c)** $[2, \infty)$

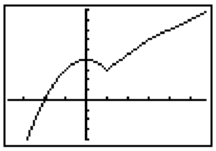
32. (a)



[−4, 4] by [−2, 3]

(b) All reals **(c)** $[0, \infty)$

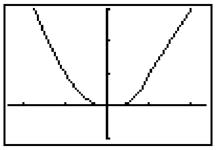
33. (a)



[−3.7, 5.7] by [−4, 9]

(b) All reals **(c)** All reals

34. (a)



[−2.35, 2.35] by [−1, 3]

(b) All reals **(c)** $[0, \infty)$

35. Because if the vertical line test holds, then for each x-coordinate, there is at most one y-coordinate giving a point on the curve. This y-coordinate would correspond to the value assigned to the x-coordinate. Since there's only one y-coordinate, the assignment would be unique.

36. If the curve is not $y = 0$, there must be a point (x, y) on the curve where $y \neq 0$. That would mean that (x, y) and $(x, -y)$ are two different points on the curve and it is not the graph of a function since it fails the vertical line test.

37. No **38.** Yes

39. Yes **40.** No

41. $f(x) = \begin{cases} x, & 0 \le x \le 1 \\ 2 - x, & 1 < x \le 2 \end{cases}$

42. $f(x) = \begin{cases} 2, & 0 \le x < 1 \\ 0, & 1 \le x < 2 \\ 2, & 2 \le x < 3 \\ 0, & 3 \le x \le 4 \end{cases}$

43. $f(x) = \begin{cases} 2 - x, & 0 < x \le 2 \\ \frac{5}{3} - \frac{x}{3}, & 2 < x \le 5 \end{cases}$

44. $f(x) = \begin{cases} -3x - 3, & -1 < x \le 0 \\ 2x + 3, & 0 < x \le 2 \end{cases}$

45. $f(x) = \begin{cases} -x, & -1 \le x < 0 \\ 1, & 0 < x \le 1 \\ \frac{3}{2} - \frac{x}{2}, & 1 < x < 3 \end{cases}$

46. $f(x) = \begin{cases} \frac{x}{2}, & -2 \le x \le 0 \\ 2x + 2, & 0 < x \le 1 \\ -1, & 1 < x \le 3 \end{cases}$

47. $f(x) = \begin{cases} 0, & 0 \le x \le \frac{T}{2} \\ \frac{2}{T}x - 1, & \frac{T}{2} < x \le T \end{cases}$

48. $f(x) = \begin{cases} A, & 0 \le x < \frac{T}{2} \\ -A, & \frac{T}{2} \le x < T \\ A, & T \le x < \frac{3T}{2} \\ -A, & \frac{3T}{2} \le x \le 2T \end{cases}$

49. (a) $x^2 + 2$ **(b)** $x^2 + 10x + 22$

(c) 2 **(d)** 22

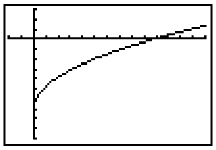
(e) -2 **(f)** $x + 10$

50. (a) x **(b)** x

(c) 0 **(d)** 0

(e) -4 **(f)** $x + 2$

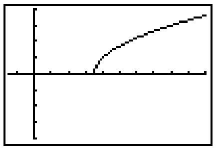
51. (a) For $f \circ g$:



[−10, 70] by [−10, 3]

Domain: $[0, \infty)$; Range: $[-7, \infty)$

For $g \circ f$:

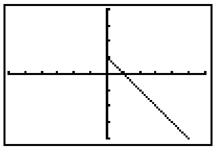


[−3, 20] by [−4, 4]

Domain: $[7, \infty)$; Range: $[0, \infty)$

(b) $(f \circ g)(x) = \sqrt{x} - 7$; $(g \circ f)(x) = \sqrt{x - 7}$

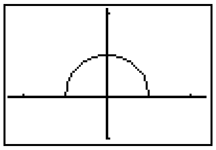
52. (a) For $f \circ g$:



[−6, 6] by [−4, 4]

Domain: $[0, \infty)$; Range: $(-\infty, 1]$

For $g \circ f$:



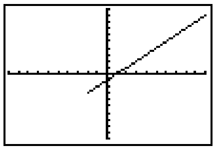
[−2.35, 2.35] by [−1, 2.1]

Domain: $[-1, 1]$; Range: $[0, 1]$

(b) $(f \circ g)(x) = 1 - (\sqrt{x})^2 = 1 - x, x \ge 0$

$(g \circ f)(x) = \sqrt{1 - x^2}$

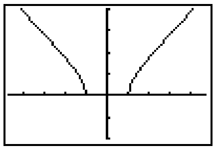
53. (a) For $f \circ g$:



$[-10, 10]$ by $[-10, 10]$

Domain: $[-2, \infty)$; Range: $[-3, \infty)$

For $g \circ f$:



$[-4.7, 4.7]$ by $[-2, 4]$

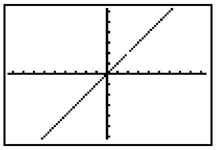
Domain: $(-\infty, -1] \cup [1, \infty)$; Range: $[0, \infty)$

(b) $(f \circ g)(x) = (\sqrt{x + 2})^2 - 3$

$$= x - 1, x \geq -2$$

$(g \circ f)(x) = \sqrt{x^2 - 1}$

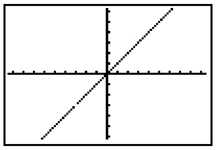
54. (a) For $f \circ g$:



$[-9.4, 9.4]$ by $[-6.2, 6.2]$

Domain: $(-\infty, 2) \cup (2, \infty)$;
Range: $(-\infty, 2) \cup (2, \infty)$

For $g \circ f$:



$[-9.4, 9.4]$ by $[-6.2, 6.2]$

Domain: $(-\infty, -3) \cup (-3, \infty)$
Range: $(-\infty, -3) \cup (-3, \infty)$

(b) $(f \circ g)(x) = x, x \neq 2$
$(g \circ f)(x) = x, x \neq -3$

55. Domain: $(-\infty, -2) \cup (2, \infty)$; Range: $(0, \infty)$

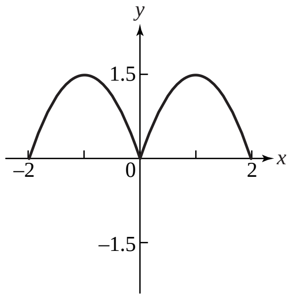
56. Domain: $(-3, 3)$; Range: $\left[\frac{2}{\sqrt{3}}, \infty\right)$

57. Domain: $(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$

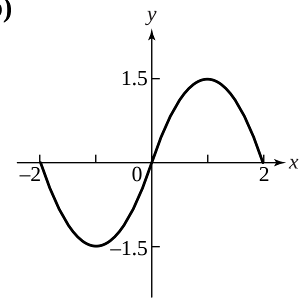
Range: $(-\infty, 0) \cup \left[\frac{2}{\sqrt[3]{9}}, \infty\right)$

58. Domain: $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$
Range: $(-\infty, -1] \cup (0, \infty)$

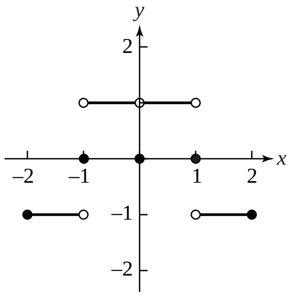
59. (a)



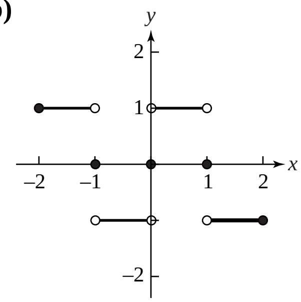
(b)



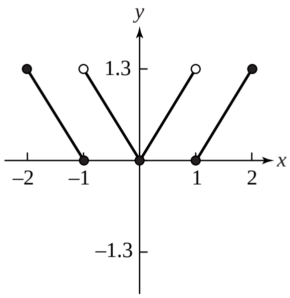
60. (a)



(b)

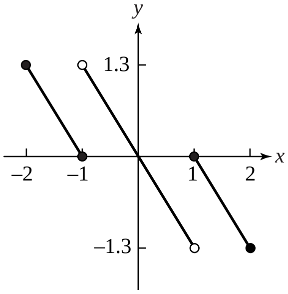


61. (a)

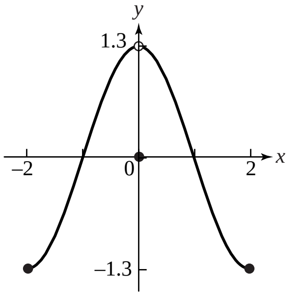


61. continued

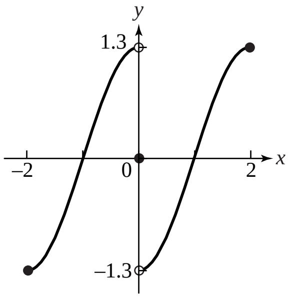
(b)



62. (a)



(b)



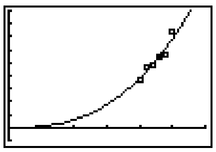
63. (a) $g(x) = x^2$ **(b)** $g(x) = \dfrac{1}{x-1}$

(c) $f(x) = \dfrac{1}{x}$ **(d)** $f(x) = x^2$

(Note that the domain of the composite is $[0, \infty)$).

64. (a) $y = 27.1094x^{2.651044}$

(b)



[0, 30] by [−20,000, 180,000]

(c) Approximately $223,374

(d) Approximately $200,064

65. (a) Because the circumference of the original circle was 8π and a piece of length x was removed.

(b) $r = \dfrac{8\pi - x}{2\pi} = 4 - \dfrac{x}{2\pi}$

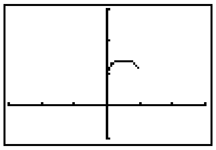
(c) $h = \sqrt{16 - r^2} = \dfrac{\sqrt{16\pi x - x^2}}{2\pi}$

(d) $V = \dfrac{1}{3}\pi r^2 h = \dfrac{(8\pi - x)^2\sqrt{16\pi x - x^2}}{24\pi^2}$

66. (a) $C(x) = 180\sqrt{800^2 + x^2} + 100(10{,}560 - x)$

(b) $C(0) = \$1{,}200{,}000$
$C(500) \approx \$1{,}175{,}812$
$C(1000) \approx \$1{,}186{,}512$
$C(1500) = \$1{,}212{,}000$
$C(2000) \approx \$1{,}243{,}732$
$C(2500) \approx \$1{,}278{,}479$
$C(3000) \approx \$1{,}314{,}870$
Values beyond this are all larger. It would appear that the least expensive location is less than 2000 feet from point *P*.

67. (a)



[−3, 3] by [−1, 3]

(b) Domain of y_1: $[0, \infty)$
Domain of y_2: $(-\infty, 1]$
Domain of y_3: $[0, 1]$

(c) The results for $y_1 - y_2$, $y_2 - y_1$, and $y_1 \cdot y_2$ are the same as for $y_1 + y_2$ above.

Domain of $\dfrac{y_1}{y_2}$: $[0, 1)$

Domain of $\dfrac{y_2}{y_1}$: $(0, 1]$

(d) The domain of a sum, difference, or product of two functions is the intersection of their domains.
The domain of a quotient of two functions is the intersection of their domains with any zeros of the denominator removed.

68. (a) Yes. Since

$$\begin{aligned}(f \cdot g)(-x) &= f(-x) \cdot g(-x)\\ &= f(x) \cdot g(x)\\ &= (f \cdot g)(x),\end{aligned}$$

the function $(f \cdot g)$ will also be even.

(b) The product will be even, since

$$\begin{aligned}(f \cdot g)(-x) &= f(-x) \cdot g(-x)\\ &= (-f(x)) \cdot (-g(x))\\ &= f(x) \cdot g(x)\\ &= (f \cdot g)(x).\end{aligned}$$

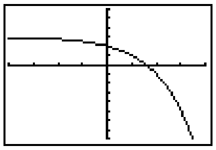
1.3 Exponential Functions (pp. 20–26)

Quick Review 1.3

1. 2.924 **2.** 4.729
3. 0.192 **4.** 2.5713
5. 1.8882 **6.** ±1.0383
7. $630.58 **8.** $1201.16
9. $x^{-18}y^{-5} = \dfrac{1}{x^{18}y^5}$ **10.** $a^2b^{-1}c^{-6} = \dfrac{a^2}{bc^6}$

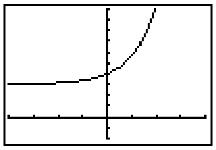
Section 1.3 Exercises

1. (a)
2. (d)
3. (e)
4. (c)
5. (b)
6. (f)

7.


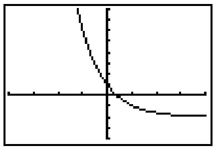
[−4, 4] by [−8, 6]

Domain: All reals
Range: $(-\infty, 3)$
x-intercept: ≈ 1.585
y-intercept: 2

8.


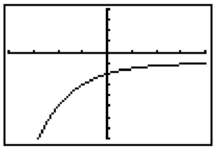
[−4, 4] by [−2, 10]

Domain: All reals
Range: $(3, \infty)$
x-intercept: None
y-intercept: 4

9.


[−4, 4] by [−4, 8]

Domain: All reals
Range: $(-2, \infty)$
x-intercept: ≈ 0.405
y-intercept: 1

10.


[−4, 4] by [−8, 4]

Domain: All reals
Range: $(-\infty, -1)$
x-intercept: None
y-intercept: −2

11. 3^{4x}
12. 2^{12x}
13. 2^{-6x}
14. 3^{-3x}
15. $x \approx 2.3219$
16. $x \approx 1.3863$
17. $x \approx -0.6309$
18. $x \approx -1.5850$

19.

x	y	Δy
1	−1	2
2	1	2
3	3	2
4	5	

20.

x	y	Δy
1	1	−3
2	−2	−3
3	−5	−3
4	−8	

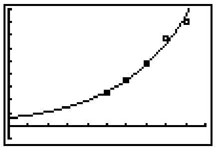
21.

x	y	Δy
1	1	3
2	4	5
3	9	7
4	16	

22.

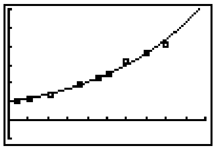
x	y	Δy
1	8.155	2.718
2	22.167	2.718
3	60.257	2.718
4	163.794	

23. After 19 years
24. **(a)** 1915: 12,315
1940: 24,265
(b) 1967 [76.651 years after 1890]
25. **(a)** $A(t) = 6.6\left(\frac{1}{2}\right)^{t/14}$
(b) About 38.1145 days later
26. ≈10.129 years
27. ≈11.433 years
28. ≈11.119 years
29. ≈11.090 years
30. ≈19.650 years
31. ≈19.108 years
32. ≈19.106 years
33. $2^{48} \approx 2.815 \times 10^{14}$
34. **(a)** ≈10.319 years **(b)** ≈41.275 years
35. Since $\Delta x = 1$, the corresponding value of Δy is equal to the slope of the line. If the changes in x are constant for a linear function, then the corresponding changes in y are constant as well.
36. **(a)** 100 **(b)** 6394
(c) After about 1 hour, which is the doubling time
37. **(a)** Regression equation:
$P(x) = 6.033(1.030)^x$, where $x = 0$ represents 1900



[0, 100] by [−10, 90]

(b) Approximately 6.03 million, which is not very close to the actual population
(c) The annual rate of growth is approximately 3%.
38. **(a)** Regression equation:
$P(x) = 4.831(1.019)^x$



[0, 100] by [−5, 30]

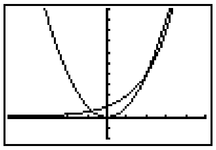
(b) 26.3 million

38. continued

(c) The annual rate of growth is approximately 1.9%.

39. 7609.7 million

40. (a)



$[-5, 5]$ by $[-2, 10]$

In this window, it appears they cross twice, although a third crossing off-screen appears likely.

(b)

x	change in $Y1$	change in $Y2$
1		
	3	2
2		
	5	4
3		
	7	8
4		

(c) $x = -0.7667, x = 2, x = 4$

(d) $(-0.7667, 2) \cup (4, \infty)$

41. $a = 3, k = 1.5$ **42.** $a = 0.5, k = 3$

1.4 Parametric Equations (pp. 26–31)

Quick Review 1.4

1. $y = -\frac{5}{3}x + \frac{29}{3}$ **2.** $y = -4$

3. $x = 2$

4. x-intercepts: $x = -3$ and $x = 3$
y-intercepts: $y = -4$ and $y = 4$

5. x-intercepts: $x = -4$ and $x = 4$
y-intercepts: None

6. x-intercept: $x = -1$
y-intercepts: $y = -\frac{1}{\sqrt{2}}$ and $y = \frac{1}{\sqrt{2}}$

7. (a) Yes **(b)** No
(c) Yes

8. (a) Yes **(b)** Yes
(c) No

9. (a) $t = \frac{-2x - 5}{3}$ **(b)** $t = \frac{3y + 1}{2}$

10. (a) $a \geq 0$ **(b)** All reals
(c) All reals

Section 1.4 Exercises

1. Graph (c).
Window: $[-4, 4]$ by $[-3, 3]$, $0 \leq t \leq 2\pi$

2. Graph (a).
Window: $[-2, 2]$ by $[-2, 2]$, $0 \leq t \leq 2\pi$

3. Graph (d).
Window: $[-10, 10]$ by $[-10, 10]$, $0 \leq t \leq 2\pi$

4. Graph (b).
Window: $[-15, 15]$ by $[-15, 15]$, $0 \leq t \leq 2\pi$

5. (a) The resulting graph appears to be the right half of a hyperbola in the first and fourth quadrants. The parameter a determines the x-intercept. The parameter b determines the shape of the hyperbola. If b is smaller, the graph has less steep slopes and appears "sharper." If b is larger, the slopes are steeper and the graph appears more "blunt."

(b) This appears to be the left half of the same hyperbola.

(c) Because both sec t and tan t are discontinuous at these points. This might cause the grapher to include extraneous lines (the asymptotes to the hyperbola) in its graph.

(d) $\left(\frac{x}{a}\right)^2 - \left(\frac{y}{b}\right)^2 = (\sec t)^2 - (\tan t)^2 = 1$
by a standard trigonometric identity.

(e) This changes the orientation of the hyperbola. In this case, b determines the y-intercept of the hyperbola, and a determines the shape.

The parameter interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ gives the upper half of the hyperbola. The parameter interval $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$ gives the lower half.

The same values of t cause discontinuities and may add extraneous lines to the graph.

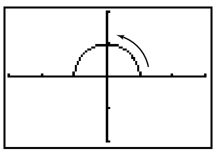
6. (a) h determines the x-coordinate of the center of the circle. It causes a horizontal shift of the graph.

(b) k determines the y-coordinate of the center of the circle. It causes a vertical shift of the graph.

(c) $x = 5 \cos t + 2, y = 5 \sin t - 3, 0 \leq t \leq 2\pi$

(d) $x = 5 \cos t - 3, y = 2 \sin t + 4, 0 \leq t \leq 2\pi$

7. (a)

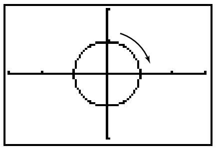


$[-3, 3]$ by $[-2, 2]$

Initial point: $(1, 0)$
Terminal point: $(-1, 0)$

(b) $x^2 + y^2 = 1$; upper half (or $y = \sqrt{1 - x^2}$; all)

8. (a)

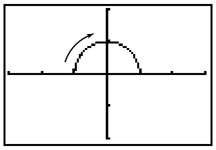


$[-3, 3]$ by $[-2, 2]$

Initial and terminal point: $(0, 1)$

(b) $x^2 + y^2 = 1$; all

9. (a)



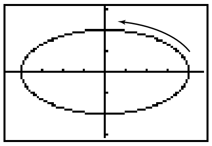
$[-3, 3]$ by $[-2, 2]$

Initial point: $(1, 0)$

Terminal point: $(0, 1)$

(b) $x^2 + y^2 = 1$; upper half (or $y = \sqrt{1 - x^2}$; all)

10. (a)

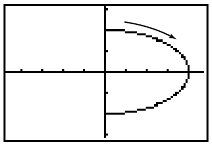


$[-4.7, 4.7]$ by $[-3.1, 3.1]$

Initial and terminal point: $(4, 0)$

(b) $\left(\frac{x}{4}\right)^2 + \left(\frac{y}{2}\right)^2 = 1$; all

11. (a)



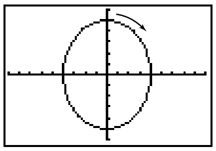
$[-4.7, 4.7]$ by $[-3.1, 3.1]$

Initial point: $(0, 2)$

Terminal point: $(0, -2)$

(b) $\left(\frac{x}{4}\right)^2 + \left(\frac{y}{2}\right)^2 = 1$;
right half (or $x = 2\sqrt{4 - y^2}$; all)

12. (a)

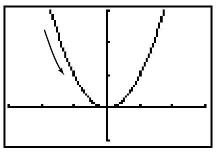


$[-9, 9]$ by $[-6, 6]$

Initial and terminal point: $(0, 5)$

(b) $\left(\frac{x}{4}\right)^2 + \left(\frac{y}{5}\right)^2 = 1$; all

13. (a)

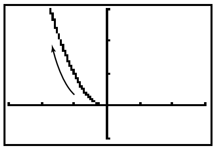


$[-3, 3]$ by $[-1, 3]$

No initial or terminal point

(b) $y = x^2$; all

14. (a)



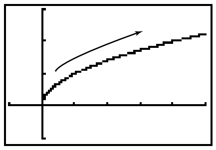
$[-3, 3]$ by $[-1, 3]$

Initial point: $(0, 0)$

Terminal point: None

(b) $y = x^2$; left half (or $x = -\sqrt{y}$; all)

15. (a)



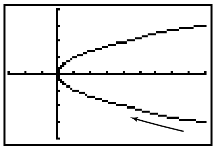
$[-1, 5]$ by $[-1, 3]$

Initial point: $(0, 0)$

Terminal point: None

(b) $y = \sqrt{x}$; all (or $x = y^2$; upper half)

16. (a)

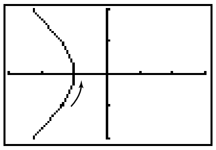


$[-3, 9]$ by $[-4, 4]$

No initial or terminal point

(b) $x = y^2$; all

17. (a)

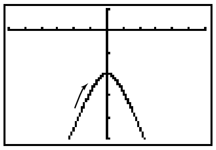


$[-3, 3]$ by $[-2, 2]$

No initial or terminal point

(b) $x^2 - y^2 = 1$; left branch (or $x = -\sqrt{y^2 + 1}$; all)

18. (a)

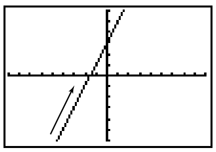


$[-6, 6]$ by $[-5, 1]$

No initial or terminal point

(b) $\left(\frac{y}{2}\right)^2 - x^2 = 1$; lower branch
(or $y = -2\sqrt{x^2 + 1}$; all)

19. (a)

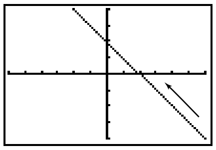


$[-9, 9]$ by $[-6, 6]$

No initial or terminal point

(b) $y = 2x + 3$; all

20. (a)

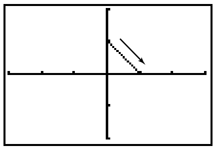


[−6, 6] by [−4, 4]

No initial or terminal point

(b) $y = -x + 2$; all

21. (a)

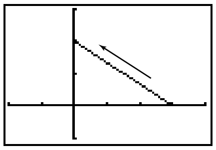


[−3, 3] by [−2, 2]

Initial point: (0, 1)
Terminal point: (1, 0)

(b) $y = -x + 1$; (0, 1) to (1, 0)

22. (a)

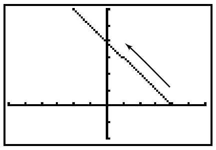


[−2, 4] by [−1, 3]

Initial point: (3, 0)
Terminal point: (0, 2)

(b) $y = -\frac{2}{3}x + 2$; (3, 0) to (0, 2)

23. (a)

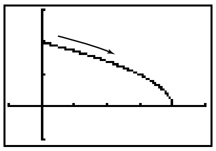


[−6, 6] by [−2, 6]

Initial point: (4, 0)
Terminal point: None

(b) $y = -x + 4$; $x \le 4$

24. (a)

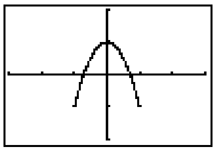


[−1, 5] by [−1, 3]

Initial point: (0, 2)
Terminal point: (4, 0)

(b) $y = \sqrt{4 - x}$; $x \ge 0$

25. (a)

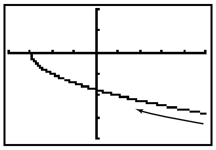


[−3, 3] by [−2, 2]

The curve is traced and retraced in both directions, and there is no initial or terminal point.

(b) $y = -2x^2 + 1$; $-1 \le x \le 1$

26. (a)



[−4, 5] by [−4, 2]

Initial point: None
Terminal point: (−3, 0)

(b) $x = y^2 - 3$; lower half (or $y = -\sqrt{x + 3}$; all)

27. Possible answer:
$x = -1 + 5t,\ y = -3 + 4t,\ 0 \le t \le 1$

28. Possible answer:
$x = -1 + 4t,\ y = 3 - 5t,\ 0 \le t \le 1$

29. Possible answer:
$x = t^2 + 1,\ y = t,\ t \le 0$

30. Possible answer:
$x = t,\ y = t^2 + 2t,\ t \le -1$

31. Possible answer:
$x = 2 - 3t,\ y = 3 - 4t,\ t \ge 0$

32. Possible answer:
$x = -1 + t,\ y = 2 - 2t,\ t \ge 0$

33. $1 < t < 3$

34. $3 < t \le 5$

35. $-5 \le t < -3$

36. $-3 < t < 1$

37. Possible answer: $x = t,\ y = t^2 + 2t + 2,\ t > 0$

38. Possible answer: $x = t,\ y = \sqrt{t + 3},\ t > 0$

39. Possible answers:
(a) $x = a \cos t,\ y = -a \sin t,\ 0 \le t \le 2\pi$
(b) $x = a \cos t,\ y = a \sin t,\ 0 \le t \le 2\pi$
(c) $x = a \cos t,\ y = -a \sin t,\ 0 \le t \le 4\pi$
(d) $x = a \cos t,\ y = a \sin t,\ 0 \le t \le 4\pi$

40. Possible answers:
(a) $x = -a \cos t,\ y = b \sin t,\ 0 \le t \le 2\pi$
(b) $x = -a \cos t,\ y = -b \sin t,\ 0 \le t \le 2\pi$
(c) $x = -a \cos t,\ y = b \sin t,\ 0 \le t \le 4\pi$
(d) $x = -a \cos t,\ y = -b \sin t,\ 0 \le t \le 4\pi$

41. $x = 2 \cot t,\ y = 2 \sin^2 t,\ 0 < t < \pi$

42. (a) If $x_2 = x_1$ then the line is a vertical line and the first parametric equation gives $x = x_1$, while the second will give all real values for y since it cannot be the case that $y_2 = y_1$ as well. Otherwise, solving the first equation for t gives

$$t = \frac{x - x_1}{x_2 - x_1}$$

Substituting that into the second equation for t gives

$$y = y_1 + \left[\frac{y_2 - y_1}{x_2 - x_1}\right](x - x_1)$$

which is the point-slope form of the equation for the line through (x_1, y_1) and (x_2, y_2). Note that the first equation will cause x to take on all real values, because $x_2 - x_1$ is not zero. Therefore, all of the points on the line will be traced out.

(b) Use the equations for x and y given in part (a) with $0 \le t \le 1$.

1.5 Functions and Logarithms (pp. 32–40)

Quick Review 1.5

1. 1 **2.** 5

3. $x^{2/3}$ **4.** $(x - 1)^{2/3} + 1$

5. Possible answer: $x = t, y = \dfrac{1}{t-1}, t \geq 2$

6. Possible answer: $x = t, y = t, t < -3$

7. (4, 5) **8.** $\left(\dfrac{8}{3}, -3\right) \approx (2.67, -3)$

9. (a) (1.58, 3) **(b)** No intersection

10. (a) (−1.39, 4) **(b)** No intersection

Section 1.5 Exercises

1. No **2.** Yes

3. Yes **4.** No

5. Yes **6.** No

7. Yes **8.** No

9. No **10.** Yes

11. No **12.** Yes

13. $f^{-1}(x) = \dfrac{x-3}{2}$ **14.** $f^{-1}(x) = \dfrac{5-x}{4}$

15. $f^{-1}(x) = (x+1)^{1/3}$ or $\sqrt[3]{x+1}$

16. $f^{-1}(x) = (x-1)^{1/2}$ or $\sqrt{x-1}$

17. $f^{-1}(x) = -x^{1/2}$ or $-\sqrt{x}$

18. $f^{-1}(x) = x^{3/2}$

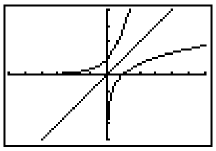
19. $f^{-1}(x) = 2 - (-x)^{1/2}$ or $2 - \sqrt{-x}$

20. $f^{-1}(x) = x^{1/2} - 1$ or $\sqrt{x} - 1$

21. $f^{-1}(x) = \dfrac{1}{x^{1/2}}$ or $\dfrac{1}{\sqrt{x}}$ **22.** $f^{-1}(x) = \dfrac{1}{x^{1/3}}$ or $\dfrac{1}{\sqrt[3]{x}}$

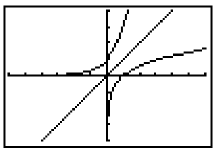
23. $f^{-1}(x) = \dfrac{1-3x}{x-2}$ **24.** $f^{-1}(x) = \dfrac{2x+3}{x-1}$

25.



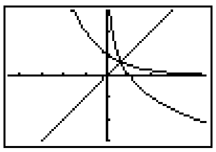
[−6, 6] by [−4, 4]

26.



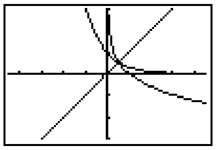
[−6, 6] by [−4, 4]

27.



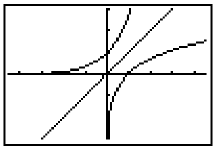
[−4.5, 4.5] by [−3, 3]

28.



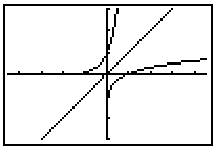
[−4.5, 4.5] by [−3, 3]

29.



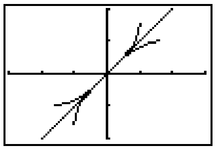
[−4.5, 4.5] by [−3, 3]

30.



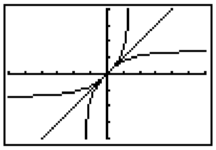
[−4.5, 4.5] by [−3, 3]

31.



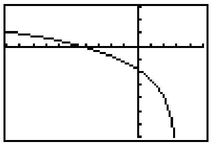
[−3, 3] by [−2, 2]

32.



[−6, 6] by [−4, 4]

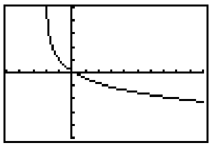
33.



[−10, 5] by [−7, 3]

Domain: $(-\infty, 3)$; Range: all reals

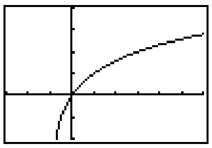
34.



[−5, 10] by [−5, 5]

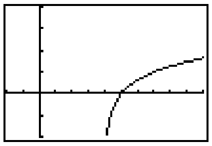
Domain: $(-2, \infty)$; Range: all reals

35.



[−3, 6] by [−2, 4]

Domain: $(-1, \infty)$; Range: all reals

36. 

[−2, 10] by [−2, 4]

Domain: $(4, \infty)$; Range: all reals

37. $t = \dfrac{\ln 2}{\ln 1.045} \approx 15.75$

38. $t = \dfrac{\ln 3}{0.05} \approx 21.97$

39. $x = \ln\left(\dfrac{3 \pm \sqrt{5}}{2}\right) \approx -0.96$ or 0.96

40. $x = \log_2\left(\dfrac{5 \pm \sqrt{21}}{2}\right) \approx -2.26$ or 2.26

41. $y = e^{2t+4}$ **42.** $y = 2xe^x + 1$

43. $f^{-1}(x) = \log_2\left(\dfrac{x}{100 - x}\right)$

44. $f^{-1}(x) = \log_{1.1}\left(\dfrac{x}{50 - x}\right)$

45. (a) $f(f(x)) = \sqrt{1 - (f(x))^2}$
$= \sqrt{1 - (1 - x^2)}$
$= \sqrt{x^2}$
$= x$, since $x \geq 0$

(b) $f(f(x)) = f\left(\dfrac{1}{x}\right) = \dfrac{1}{1/x} = x$ for all $x \neq 0$

46. (a) Amount $= 8\left(\dfrac{1}{2}\right)^{t/12}$

(b) After 36 hours

47. About 14.936 years. (If the interest is only paid annually, it will take 15 years.)

48. After about 44.081 years

49. (a) $y = -2539.852 + 636.896 \ln x$

(b) 209.94 million metric tons

(c) When $x \approx 101.08$ or 2001

50. (a) $y = -590.969 + 152.817 \ln x$

(b) 87.94

(c) When $x \approx 104.84$, in about 2005

51. (a) Suppose that $f(x_1) = f(x_2)$. Then $mx_1 + b = mx_2 + b$, which gives $x_1 = x_2$ since $m \neq 0$.

(b) $f^{-1}(x) = \dfrac{x - b}{m}$; the slopes are reciprocals.

(c) They are also parallel lines with non-zero slope.

(d) They are also perpendicular lines with non-zero slopes.

52. (a) y_2 is a vertical shift (upward) of y_1

(b) Each graph of y_3 is a horizontal line.

(c) The graphs of y_4 and $y = a$ are the same.

(d) $y_1 = \ln x - \ln a$

53. If the graph of $f(x)$ passes the horizontal line test, so will the graph of $g(x) = -f(x)$ since it's the same graph reflected about the x-axis.

54. Suppose that $g(x_1) = g(x_2)$. Then $\dfrac{1}{f(x_1)} = \dfrac{1}{f(x_2)}$,
$f(x_1) = f(x_2)$, and $x_1 = x_2$ since f is one-to-one.

55. (a) Domain: All reals
Range: If $a > 0$, then (d, ∞)
If $a < 0$, then $(-\infty, d)$

(b) Domain: (c, ∞)
Range: All reals

56. (a) Suppose that $f(x_1) = f(x_2)$. Then cross multiplying, expanding and subtracting like terms leaves
$bcx_2 + adx_1 = adx_2 + bcx_1$.
This gives $(ad - bc)x_1 = (ad - bc)x_2$, which means that $x_1 = x_2$ since $(ad - bc) \neq 0$.

(b) $f^{-1}(x) = \dfrac{-dx + b}{cx - a}$

(c) Horizontal asymptote: $y = \dfrac{a}{c}$ $(c \neq 0)$

Vertical asymptote: $x = -\dfrac{d}{c}$ $(c \neq 0)$

(d) Horizontal asymptote: $y = -\dfrac{d}{c}$ $(c \neq 0)$

Vertical asymptote: $x = \dfrac{a}{c}$ $(c \neq 0)$

The horizontal asymptote of f becomes the vertical asymptote of f^{-1} and vice versa due to the reflection of the graph about the line $y = x$.

1.6 Trigonometric Functions (pp. 41–51)

Quick Review 1.6

1. $60°$ **2.** $-\left(\dfrac{450}{\pi}\right)° \approx -143.24°$

3. $-\dfrac{2\pi}{9}$ **4.** $\dfrac{\pi}{4}$

5. $x \approx 0.6435$, $x \approx 2.4981$

6. $x \approx 1.9823$, $x \approx 4.3009$

7. $x \approx 0.7854 \left(\text{or } \dfrac{\pi}{4}\right)$, $x \approx 3.9270 \left(\text{or } \dfrac{5\pi}{4}\right)$

8. $f(-x) = 2(-x)^2 - 3 = 2x^2 - 3 = f(x)$
The graph is symmetric about the y-axis because if a point (a, b) is on the graph, then so is the point $(-a, b)$.

9. $f(-x) = (-x)^3 - 3(-x) = -x^3 + 3x$
$= -(x^3 - 3x) = -f(x)$
The graph is symmetric about the origin because if a point (a, b) is on the graph, then so is the point $(-a, -b)$.

10. $x \geq 0$

Section 1.6 Exercises

1. $\frac{5\pi}{4}$

2. $\frac{72}{7\pi} \approx 3.274$

3. $\frac{1}{2}$ radian or $\approx 28.65°$

4. $\frac{\pi}{4}$ radian or $45°$

5. Possible answers are:
(a) $[0, 4\pi]$ by $[-3, 3]$ **(b)** $[0, 4\pi]$ by $[-3, 3]$
(c) $[0, 2\pi]$ by $[-3, 3]$

6. Possible answers are:
(a) $[0, 4\pi]$ by $[-2, 2]$ **(b)** $[0, 4\pi]$ by $[-2, 2]$
(c) $[0, 2\pi]$ by $[-3, 3]$

7. $\frac{\pi}{6}$ radian or $30°$

8. $-\frac{\pi}{4}$ radian or $-45°$

9. ≈ -1.3734 radians or $-78.6901°$

10. ≈ 0.7954 radian or $45.5730°$

11. (a) π **(b)** 1.5
(c) $[-2\pi, 2\pi]$ by $[-2, 2]$

12. (a) $\frac{2\pi}{3}$ **(b)** 2
(c) $\left[-\frac{2\pi}{3}, \frac{2\pi}{3}\right]$ by $[-4, 4]$

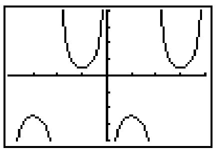
13. (a) π **(b)** 3
(c) $[-2\pi, 2\pi]$ by $[-4, 4]$

14. (a) 4π **(b)** 5
(c) $[-4\pi, 4\pi]$ by $[-10, 10]$

15. (a) 6 **(b)** 4
(c) $[-3, 3]$ by $[-5, 5]$

16. (a) 2 **(b)** 1
(c) $[-4, 4]$ by $[-2, 2]$

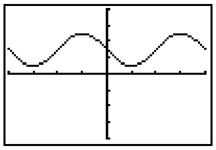
17. (a) $\frac{2\pi}{3}$
(b) $x \neq \frac{k\pi}{3}$, for integers k
(c) $(-\infty, -5] \cup [1, \infty)$
(d)



$\left[-\frac{2\pi}{3}, \frac{2\pi}{3}\right]$ by $[-8, 8]$

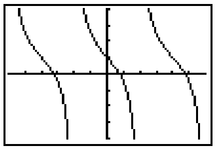
18. (a) $\frac{\pi}{2}$ **(b)** All reals
(c) $[1, 5]$

18. continued
(d)



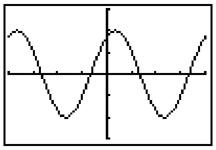
$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ by $[-8, 8]$

19. (a) $\frac{\pi}{3}$
(b) $x \neq \frac{k\pi}{6}$, for odd integers k
(c) All reals
(d)



$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ by $[-8, 8]$

20. (a) π **(b)** All reals
(c) $[-2, 2]$
(d)



$[-\pi, \pi]$ by $[-3, 3]$

21. $\cos\theta = \frac{15}{17}$ $\sin\theta = \frac{8}{17}$ $\tan\theta = \frac{8}{15}$
$\sec\theta = \frac{17}{15}$ $\csc\theta = \frac{17}{8}$ $\cot\theta = \frac{15}{8}$

22. $\cos\theta = \frac{12}{13}$ $\sin\theta = -\frac{5}{13}$ $\tan\theta = -\frac{5}{12}$
$\sec\theta = \frac{13}{12}$ $\csc\theta = -\frac{13}{5}$ $\cot\theta = -\frac{12}{5}$

23. $\cos\theta = -\frac{3}{5}$ $\sin\theta = \frac{4}{5}$ $\tan\theta = -\frac{4}{3}$
$\sec\theta = -\frac{5}{3}$ $\csc\theta = \frac{5}{4}$ $\cot\theta = -\frac{3}{4}$

24. $\cos\theta = -\frac{1}{\sqrt{2}}$ $\sin\theta = \frac{1}{\sqrt{2}}$ $\tan\theta = -1$
$\sec\theta = -\sqrt{2}$ $\csc\theta = \sqrt{2}$ $\cot\theta = -1$

25. $x \approx 1.190$ and $x \approx 4.332$

26. $x \approx 8.629$ and $x \approx 10.220$

27. $x = \frac{\pi}{6}$ and $x = \frac{5\pi}{6}$

28. $x \approx -1.911$ and $x \approx 1.911$

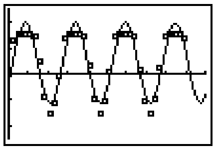
29. $x = \frac{7\pi}{6} + 2k\pi$ and $x = \frac{11\pi}{6} + 2k\pi$, k any integer

30. $x = \frac{3\pi}{4} + k\pi$, k any integer

31. $\frac{\sqrt{72}}{11} \approx 0.771$

32. $\frac{9}{\sqrt{88}} \approx 0.959$

33. **(a)** $y = 1.543 \sin(2468.635x - 0.494) + 0.438$



[0, 0.01] by [−2.5, 2.5]

(b) Frequency = 392.9, so it must be a "G."

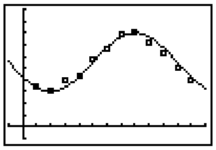
34. **(a)** $b = \frac{\pi}{6}$

(b) It's half of the difference, so $a = 25$

(c) $k = 55$

(d) $h = 5$; $y = 25 \sin\left[\frac{\pi}{6}(t - 5)\right] + 55$

(e)



[−1, 13] by [−10, 100]

35. **(a)** 37 **(b)** 365

(c) 101 **(d)** 25

36. **(a)** Highest: 62°F; lowest: −12°F

(b) Average = 25°F. This is because the average of the highest and lowest values of the (unshifted) sine function is 0.

37. **(a)** $\cot(-x) = \frac{\cos(-x)}{\sin(-x)} = \frac{\cos(x)}{-\sin(x)} = -\cot(x)$

(b) Assume that f is even and g is odd.

Then $\frac{f(-x)}{g(-x)} = \frac{f(x)}{-g(x)} = -\frac{f(x)}{g(x)}$ so $\frac{f}{g}$ is odd.

The situation is similar for $\frac{g}{f}$.

38. **(a)** $\csc(-x) = \frac{1}{\sin(-x)} = \frac{1}{-\sin(x)} = -\csc(x)$

(b) Assume that f is odd.

Then $\frac{1}{f(-x)} = \frac{1}{-f(x)} = -\frac{1}{f(x)}$ so $\frac{1}{f}$ is odd.

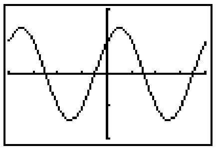
39. Assume that f is even and g is odd. Then $f(-x)\,g(-x) = f(x)[-g(x)] = -f(x)g(x)$ so fg is odd.

40. If (a, b) is the point on the unit circle corresponding to the angle θ, then $(-a, -b)$ is the point on the unit circle corresponding to the angle $(\theta + \pi)$ since it is exactly halfway around the circle. This means that both $\tan\theta$ and $\tan(\theta + \pi)$ have the same value, $\frac{b}{a}$.

41. **(a)** $y = 3.0014 \sin(0.9996x + 2.0012) + 2.9999$

(b) $y = 3 \sin(x + 2) + 3$

42. **(a)**



[−2π, 2π] by [−2, 2]

The graph is a sine/cosine type graph, but it is shifted and has an amplitude greater than 1.

(b) Amplitude ≈ 1.414 or $\sqrt{2}$, period = 2π, horizontal shift ≈ −0.785 $\left(\text{that is, } -\frac{\pi}{4}\right)$ or 5.498 $\left(\text{that is, } \frac{7\pi}{4}\right)$, vertical shift = 0

(c) $\sin\left(x + \frac{\pi}{4}\right) = \sin(x) \cdot \frac{1}{\sqrt{2}} + \cos(x) \cdot \frac{1}{\sqrt{2}}$

$= \frac{1}{\sqrt{2}}(\sin x + \cos x)$

So, $\sin(x) + \cos(x) = \sqrt{2} \sin\left(x + \frac{\pi}{4}\right)$.

43. **(a)** $\sqrt{2} \sin\left(ax + \frac{\pi}{4}\right)$ **(b)** See part **(a)**.

(c) It works.

(d) $\sin\left(ax + \frac{\pi}{4}\right)$

$= \sin(ax) \cdot \frac{1}{\sqrt{2}} + \cos(ax) \cdot \frac{1}{\sqrt{2}}$

$= \frac{1}{\sqrt{2}}(\sin ax + \cos ax)$

So, $\sin(ax) + \cos(ax) = \sqrt{2} \sin\left(ax + \frac{\pi}{4}\right)$.

44. **(a)** One possible answer:

$y = \sqrt{a^2 + b^2} \sin\left(x + \tan^{-1}\frac{b}{a}\right)$

(b) See part **(a)**. **(c)** It works.

(d) $\sin\left(x + \tan^{-1}\frac{b}{a}\right)$

$= \sin(x) \cos\left(\tan^{-1}\frac{b}{a}\right) + \cos(x) \sin\left(\tan^{-1}\frac{b}{a}\right)$

$= \sin(x) \frac{a}{\sqrt{a^2 + b^2}} + \cos(x) \frac{b}{\sqrt{a^2 + b^2}}$

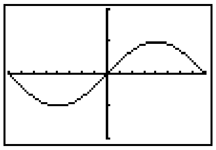
$= \frac{1}{\sqrt{a^2 + b^2}} \cdot a \sin(x) + b \cos(x)$

and multiplying through by the square root gives the result.

45. Since $\sin(x)$ has period 2π, $(\sin(x + 2\pi))^3 = (\sin(x))^3$. This function has period 2π. A graph shows that no smaller number works for the period.

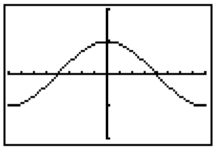
46. Since $\tan(x)$ has period π, $|\tan(x + \pi)| = |\tan(x)|$. This function has period π. A graph shows that no smaller number works for the period.

47. One possible graph:



$\left[-\frac{\pi}{60}, \frac{\pi}{60}\right]$ by $[-2, 2]$

48. One possible graph:



$\left[-\frac{1}{60}, \frac{1}{60}\right]$ by $[-2, 2]$

Chapter 1 Review Exercises (pp. 52–53)

1. $y = 3x - 9$

2. $y = -\frac{1}{2}x + \frac{3}{2}$

3. $x = 0$

4. $y = -2x$

5. $y = 2$

6. $y = -\frac{2}{5}x + \frac{21}{5}$

7. $y = -3x + 3$

8. $y = 2x - 5$

9. $y = -\frac{4}{3}x - \frac{20}{3}$

10. $y = -\frac{5}{3}x - \frac{19}{3}$

11. $y = \frac{2}{3}x + \frac{8}{3}$

12. $y = \frac{5}{3}x - 5$

13. $y = -\frac{1}{2}x + 3$

14. $y = -\frac{2}{7}x - \frac{6}{7}$

15. Origin

16. y-axis

17. Neither

18. y-axis

19. Even

20. Odd

21. Even

22. Odd

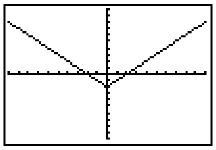
23. Odd

24. Neither

25. Neither

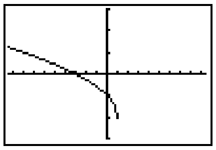
26. Even

27. **(a)** Domain: all reals **(b)** Range: $[-2, \infty)$
(c)



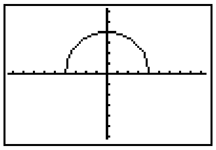
$[-10, 10]$ by $[-10, 10]$

28. **(a)** Domain: $(-\infty, 1]$ **(b)** Range: $[-2, \infty)$
(c)



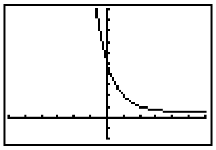
$[-9.4, 9.4]$ by $[-3, 3]$

29. **(a)** Domain: $[-4, 4]$ **(b)** Range: $[0, 4]$
(c)



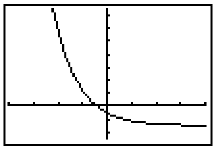
$[-9.4, 9.4]$ by $[-6.2, 6.2]$

30. **(a)** Domain: all reals **(b)** Range: $(1, \infty)$
(c)



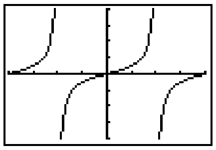
$[-6, 6]$ by $[-4, 20]$

31. **(a)** Domain: all reals **(b)** Range: $(-3, \infty)$
(c)



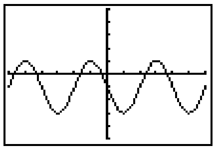
$[-4, 4]$ by $[-5, 15]$

32. **(a)** Domain: $x \neq \frac{k\pi}{4}$, for odd integers k
(b) Range: all reals
(c)



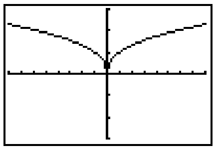
$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ by $[-8, 8]$

33. **(a)** Domain: all reals **(b)** Range: $[-3, 1]$
(c)



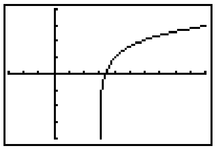
$[-\pi, \pi]$ by $[-5, 5]$

34. **(a)** Domain: all reals **(b)** Range: $[0, \infty)$
(c)



$[-8, 8]$ by $[-3, 3]$

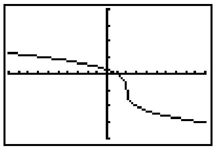
35. **(a)** Domain: $(3, \infty)$ **(b)** Range: all reals
(c)



$[-3, 10]$ by $[-4, 4]$

36. (a) Domain: all reals **(b)** Range: all reals

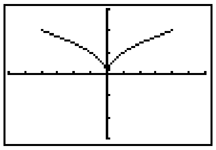
(c)



$[-10, 10]$ by $[-4, 4]$

37. (a) Domain: $[-4, 4]$ **(b)** Range: $[0, 2]$

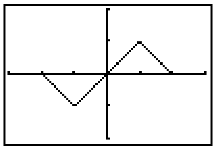
(c)



$[-6, 6]$ by $[-3, 3]$

38. (a) Domain: $[-2, 2]$ **(b)** Range: $[-1, 1]$

(c)



$[-3, 3]$ by $[-2, 2]$

39. $f(x) = \begin{cases} 1 - x, & 0 \le x < 1 \\ 2 - x, & 1 \le x \le 2 \end{cases}$

40. $f(x) = \begin{cases} \dfrac{5x}{2}, & 0 \le x < 2 \\ -\dfrac{5}{2}x + 10, & 2 \le x \le 4 \end{cases}$

41. (a) 1 **(b)** $\dfrac{1}{\sqrt{2.5}} \left(= \sqrt{\dfrac{2}{5}}\right)$

(c) $x, x \neq 0$

(d) $\dfrac{1}{\sqrt{1/\sqrt{x+2} + 2}}$

42. (a) 2 **(b)** 1

(c) x

(d) $\sqrt[3]{\sqrt[3]{x + 1} + 1}$

43. (a) $(f \circ g)(x) = -x, x \ge -2$

$(g \circ f)(x) = \sqrt{4 - x^2}$

(b) Domain $(f \circ g)$: $[-2, \infty)$

Domain $(g \circ f)$: $[-2, 2]$

(c) Range $(f \circ g)$: $(-\infty, 2]$

Range $(g \circ f)$: $[0, 2]$

44. (a) $(f \circ g)(x) = \sqrt[4]{1 - x}$

$(g \circ f)(x) = \sqrt{1 - \sqrt{x}}$

(b) Domain $(f \circ g)$: $(-\infty, 1]$

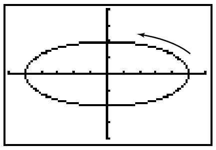
Domain $(g \circ f)$: $[0, 1]$

44. continued

(c) Range $(f \circ g)$: $[0, \infty)$

Range $(g \circ f)$: $[0, 1]$

45. (a)



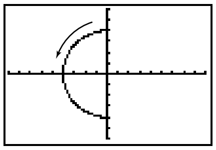
$[-6, 6]$ by $[-4, 4]$

Initial point: (5, 0)

Terminal point: (5, 0)

(b) $\left(\dfrac{x}{5}\right)^2 + \left(\dfrac{y}{2}\right)^2 = 1$; all

46. (a)



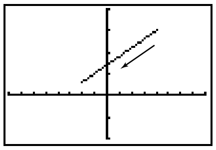
$[-9, 9]$ by $[-6, 6]$

Initial point: (0, 4)

Terminal point: (0, −4)

(b) $x^2 + y^2 = 16$; left half

47. (a)



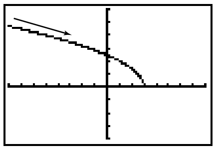
$[-8, 8]$ by $[-10, 20]$

Initial point: (4, 15)

Terminal point: (−2, 3)

(b) $y = 2x + 7$; from (4, 15) to (−2, 3)

48. (a)



$[-8, 8]$ by $[-4, 6]$

Initial point: None

Terminal point: (3, 0)

(b) $y = \sqrt{6 - 2x}$; all

49. Possible answer:

$x = -2 + 6t, y = 5 - 2t, 0 \le t \le 1$

50. Possible answer:

$x = -3 + 7t, y = -2 + t, -\infty < t < \infty$

51. Possible answer:

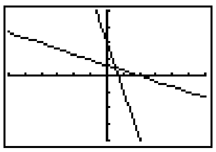
$x = 2 - 3t, y = 5 - 5t, 0 \le t$

52. Possible answer:

$x = t, y = t(t - 4), t \le 2$

53. (a) $f^{-1}(x) = \dfrac{2 - x}{3}$

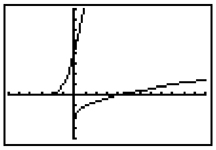
(b)



$[-6, 6]$ by $[-4, 4]$

54. **(a)** $f^{-1}(x) = \sqrt{x} - 2$

(b)



$[-6, 12]$ by $[-4, 8]$

55. ≈ 0.6435 radians or $36.8699°$

56. ≈ -1.1607 radians or $-66.5014°$

57. $\cos\theta = \dfrac{3}{7}$ $\quad \sin\theta = \dfrac{\sqrt{40}}{7}$ $\quad \tan\theta = \dfrac{\sqrt{40}}{3}$

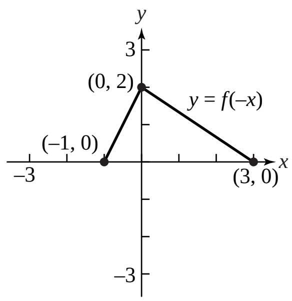
$\sec\theta = \dfrac{7}{3}$ $\quad \csc\theta = \dfrac{7}{\sqrt{40}}$ $\quad \cot\theta = \dfrac{3}{\sqrt{40}}$

58. **(a)** $x \approx 3.3430$ and $x \approx 6.0818$

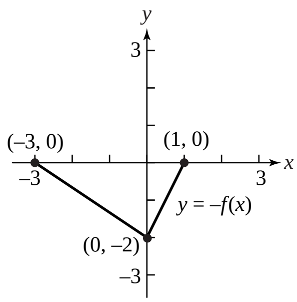
(b) $x \approx 3.3430 + 2k\pi$ and $x \approx 6.0818 + 2k\pi$, k any integer

59. $x = -5 \ln 4$

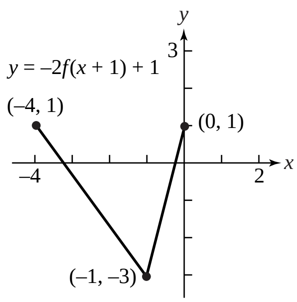
60. **(a)**



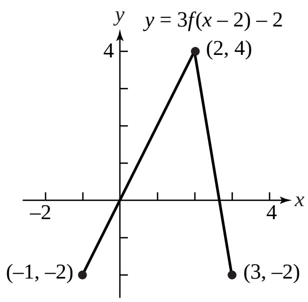
(b)



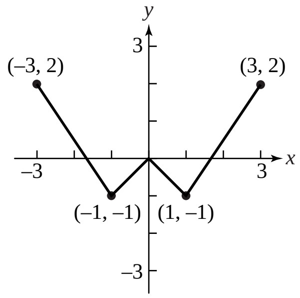
(c)



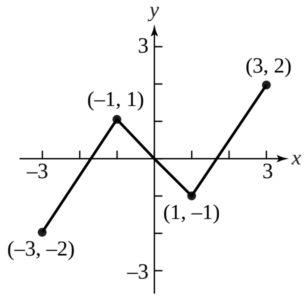
(d)



61. **(a)**



(b)



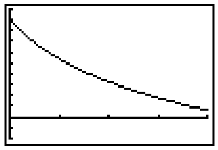
62. **(a)** $V = 100{,}000 - 10{,}000x$, $0 \le x \le 10$

(b) After 4.5 years

63. **(a)** 90 units

(b) $90 - 52 \ln 3 \approx 32.8722$ units

(c)



$[0, 4]$ by $[-20, 100]$

64. After $\dfrac{\ln(10/3)}{\ln 1.08} \approx 15.6439$ years

(If the bank only pays interest at the end of the year, it will take 16 years.)

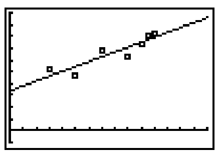
65. **(a)** $N = 4 \cdot 2^t$

(b) 4 days: 64; one week: 512

(c) After $\dfrac{\ln 500}{\ln 2} \approx 8.9658$ days, or after nearly 9 days.

(d) Because it suggests the number of guppies will continue to double indefinitely and become arbitrarily large, which is impossible due to the finite size of the tank and the oxygen supply in the water.

66. **(a)** $y = 20.627x + 338.622$



$[0, 30]$ by $[-100, 1000]$

(b) Approximately 957

(c) Slope is 20.627. It represents the approximate arrival increase in number of doctorates earned by Hispanic Americans per year.

67. **(a)** $y = 14.60175 \cdot 1.00232^x$
(b) Sometime during the year 2132 (when $x \approx 232$)
(c) 0.232%

Chapter 2
Limits and Continuity

2.1 Rates of Change and Limits (pp. 55–65)

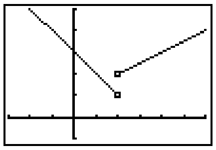
Quick Review 2.1

1. 0 **2.** $\frac{11}{12}$
3. 0 **4.** $\frac{1}{3}$
5. $-4 < x < 4$ **6.** $-c^2 < x < c^2$
7. $-1 < x < 5$ **8.** $c - d^2 < x < c + d^2$
9. $x - 6$ **10.** $\frac{x}{x+1}$

Section 2.1 Exercises

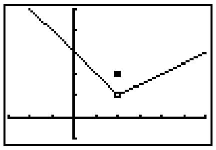
1. **(a)** 3 **(b)** -2
(c) No limit **(d)** 1
2. **(a)** 5 **(b)** 2
(c) No limit **(d)** 2
3. **(a)** -4 **(b)** -4
(c) -4 **(d)** -4
4. **(a)** 3 **(b)** 3
(c) 3 **(d)** 3
5. **(a)** 4 **(b)** -3
(c) No limit **(d)** 4
6. **(a)** 1 **(b)** 1
(c) 1 **(d)** 3
7. $-\frac{3}{2}$ **8.** 1
9. –15 **10.** 5
11. 0 **12.** 0
13. 4 **14.** $\sqrt{5}$
15. 1 **16.** 0
17. Expression not defined at $x = -2$. There is no limit.
18. Expression not defined at $x = 0$. There is no limit.
19. Expression not defined at $x = 0$. There is no limit.
20. Expression not defined at $x = 0$. Limit = 8.
21. $\frac{1}{2}$ **22.** $\frac{1}{4}$
23. $-\frac{1}{2}$ **24.** $-\frac{1}{4}$
25. 12 **26.** 2
27. -1 **28.** 2
29. 0 **30.** 4
31. **(a)** True **(b)** True
(c) False **(d)** True

31. continued
(e) True **(f)** True
(g) False **(h)** False
(i) False **(j)** False
32. **(a)** True **(b)** False
(c) False **(d)** True
(e) True **(f)** True
(g) True **(h)** True
(i) True
33. (c) **34.** (b)
35. (d) **36.** (a)
37. 0 **38.** -1
39. 0 **40.** 1
41. 1 **42.** -1
43. **(a)** 6 **(b)** 0
(c) 9 **(d)** -3
44. **(a)** 4 **(b)** -21
(c) -12 **(d)** $-\frac{7}{3}$
45. **(a)**



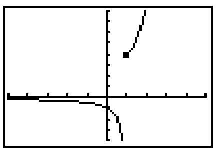
$[-3, 6]$ by $[-1, 5]$

(b) Right-hand: 2
Left-hand: 1
(c) No, because the two one-sided limits are different.
46. **(a)**



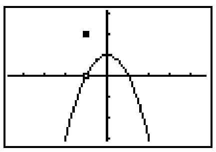
$[-3, 6]$ by $[-1, 5]$

(b) Right-hand: 1
Left-hand: 1
(c) Yes. The limit is 1.
47. **(a)**



$[-5, 5]$ by $[-4, 8]$

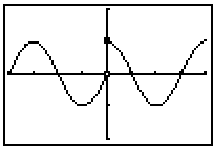
(b) Right-hand: 4
Left-hand: no limit
(c) No, because the left-hand limit doesn't exist.
48. **(a)**



$[-4.7, 4.7]$ by $[-3.1, 3.1]$

(b) Right-hand: 0
Left-hand: 0
(c) Yes. The limit is 0.

49. (a)

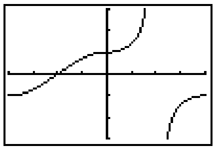


$[-2\pi, 2\pi]$ by $[-2, 2]$

(b) $(-2\pi, 0) \cup (0, 2\pi)$

(c) $c = 2\pi$ **(d)** $c = -2\pi$

50. (a)

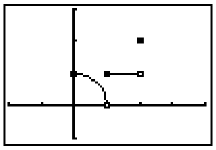


$[-\pi, \pi]$ by $[-3, 3]$

(b) $\left(-\pi, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right)$

(c) $c = \pi$ **(d)** $c = -\pi$

51. (a)

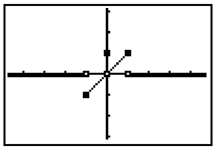


$[-2, 4]$ by $[-1, 3]$

(b) $(0, 1) \cup (1, 2)$ **(c)** $c = 2$

(d) $c = 0$

52. (a)



$[-4.7, 4.7]$ by $[-3.1, 3.1]$

(b) $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

(c) None **(d)** None

53. 0 **54.** 0

55. 0 **56.** 0

57. (a) 14.7 m/sec **(b)** 29.4 m/sec

58. (a) $g = \frac{5}{4}$ **(b)** 5 m/sec

(c) 10 m/sec

59. (a)

x	-0.1	-0.01	-0.001	-0.0001
$f(x)$	-0.054402	-0.005064	-0.000827	-0.000031

(b)

x	0.1	0.01	0.001	0.0001
$f(x)$	-0.054402	-0.005064	-0.000827	-0.000031

The limit appears to be 0.

60. (a)

x	-0.1	-0.01	-0.001	-0.0001
$f(x)$	0.5440	0.5064	-0.8269	0.3056

(b)

x	0.1	0.01	0.001	0.0001
$f(x)$	-0.5440	-0.5064	0.8269	-0.3056

There is no clear indication of a limit.

61. (a)

x	-0.1	-0.01	-0.001	-0.0001
$f(x)$	2.0567	2.2763	2.2999	2.3023

(b)

x	0.1	0.01	0.001	0.0001
$f(x)$	2.5893	2.3293	2.3052	2.3029

The limit appears to be approximately 2.3.

62. (a)

x	-0.1	-0.01	-0.001	-0.0001
$f(x)$	0.074398	-0.009943	0.000585	0.000021

(b)

x	0.1	0.01	0.001	0.0001
$f(x)$	-0.074398	0.009943	-0.000585	-0.000021

The limit appears to be 0.

63. (a) Because the right-hand limit at zero depends only on the values of the function for positive x-values near zero.

(b) Use: area of triangle $= \left(\frac{1}{2}\right)$(base)(height)

area of circular sector $= \frac{(\text{angle})(\text{radius})^2}{2}$

(c) This is how the areas of the three regions compare.

(d) Multiply by 2 and divide by $\sin\theta$.

(e) Take reciprocals, remembering that all of the values involved are positive.

(f) The limits for $\cos\theta$ and 1 are both equal to 1. Since $\frac{\sin\theta}{\theta}$ is between them, it must also have a limit of 1.

(g) $\frac{\sin(-\theta)}{-\theta} = \frac{-\sin(\theta)}{-\theta} = \frac{\sin(\theta)}{\theta}$

(h) If the function is symmetric about the y-axis, and the right-hand limit at zero is 1, then the left-hand limit at zero must also be 1.

(i) The two one-sided limits both exist and are equal to 1.

64. (a) The limit can be found by substitution.

(b) One possible answer: $a = 1.75$, $b = 2.28$

(c) One possible answer: $a = 1.99$, $b = 2.01$

65. (a) $f\left(\frac{\pi}{6}\right) = \frac{1}{2}$

(b) One possible answer: $a = 0.305$, $b = 0.775$

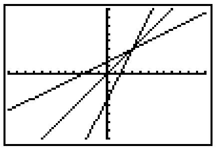
(c) One possible answer: $a = 0.513$, $b = 0.535$

66. $\frac{1}{2}$

2.2 Limits Involving Infinity (pp. 65–73)

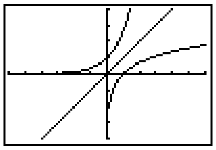
Quick Review 2.2

1. $f^{-1}(x) = \frac{x+3}{2}$



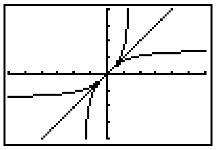
$[-12, 12]$ by $[-8, 8]$

2. $f^{-1}(x) = \ln(x)$



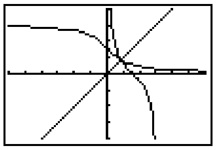
$[-6, 6]$ by $[-4, 4]$

3. $f^{-1}(x) = \tan(x), -\frac{\pi}{2} < x < \frac{\pi}{2}$



$[-6, 6]$ by $[-4, 4]$

4. $f^{-1}(x) = \cot(x), 0 < x < \pi$



$[-6, 6]$ by $[-4, 4]$

5. $q(x) = \frac{2}{3}$
   $r(x) = -3x^2 - \left(\frac{5}{3}\right)x + \frac{7}{3}$
6. $q(x) = 2x^2 + 2x + 1$
   $r(x) = -x^2 - x - 2$
7. **(a)** $f(-x) = \cos x$
   (b) $f\left(\frac{1}{x}\right) = \cos\left(\frac{1}{x}\right)$
8. **(a)** $f(-x) = e^x$
   (b) $f\left(\frac{1}{x}\right) = e^{-1/x}$
9. **(a)** $f(-x) = -\frac{\ln(-x)}{x}$
   (b) $f\left(\frac{1}{x}\right) = -x \ln x$
10. **(a)** $f(-x) = \left(x + \frac{1}{x}\right) \sin x$
    (b) $f\left(\frac{1}{x}\right) = \left(\frac{1}{x} + x\right) \sin\left(\frac{1}{x}\right)$

Section 2.2 Exercises

1. **(a)** 1 **(b)** 1 **(c)** $y = 1$
2. **(a)** 0 **(b)** 0 **(c)** $y = 0$
3. **(a)** 0 **(b)** $-\infty$ **(c)** $y = 0$
4. **(a)** ∞ **(b)** ∞ **(c)** None
5. **(a)** 3 **(b)** -3 **(c)** $y = 3, y = -3$
6. **(a)** 2 **(b)** -2 **(c)** $y = 2, y = -2$
7. **(a)** 1 **(b)** -1 **(c)** $y = 1, y = -1$
8. **(a)** 1 **(b)** 1 **(c)** $y = 1$
9. ∞
10. $-\infty$
11. $-\infty$
12. $-\infty$
13. 0
14. ∞
15. ∞
16. $-\infty$
17. **(a)** $x = -2, x = 2$
    (b) Left-hand limit at -2 is ∞.
    Right-hand limit at -2 is $-\infty$.
    Left-hand limit at 2 is $-\infty$.
    Right-hand limit at 2 is ∞.
18. **(a)** $x = -2$
    (b) Left-hand limit at -2 is $-\infty$.
    Right-hand limit at -2 is ∞.
19. **(a)** $x = -1$
    (b) Left-hand limit at -1 is $-\infty$.
    Right-hand limit at -1 is ∞.
20. **(a)** $x = -\frac{1}{2}, x = 3$
    (b) Left-hand limit at $-\frac{1}{2}$ is ∞.
    Right hand limit at $-\frac{1}{2}$ is $-\infty$.
    Left-hand limit at 3 is ∞.
    Right-hand limit at 3 is $-\infty$.
21. **(a)** $x = k\pi$, k any integer
    (b) At each vertical asymptote:
    Left-hand limit is $-\infty$.
    Right-hand limit is ∞.
22. **(a)** $x = \frac{\pi}{2} + n\pi$, n any integer
    (b) If n is even:
    Left-hand limit is ∞.
    Right-hand limit is $-\infty$.
    If n is odd:
    Left-hand limit is $-\infty$.
    Right-hand limit is ∞.
23. Both are 1
24. Both are 5
25. Both are 1
26. Both are 2
27. Both are 0
28. Both are 0
29. (a)
30. (c)
31. (d)
32. (b)

33. **(a)** $3x^2$ **(b)** None

34. **(a)** $-4x^3$ **(b)** None

35. **(a)** $\frac{1}{2x}$ **(b)** $y = 0$

36. **(a)** 3 **(b)** $y = 3$

37. **(a)** $4x^2$ **(b)** None

38. **(a)** $-x^2$ **(b)** None

39. **(a)** e^x **(b)** $-2x$

40. **(a)** x^2 **(b)** e^{-x}

41. **(a)** x **(b)** x

42. **(a)** x^2 **(b)** x^2

43. At ∞: ∞
At $-\infty$: 0

44. At ∞: 0
At $-\infty$: ∞

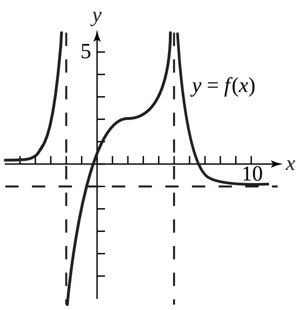
45. At ∞: 0
At $-\infty$: 0

46. At ∞: 1
At $-\infty$: 1

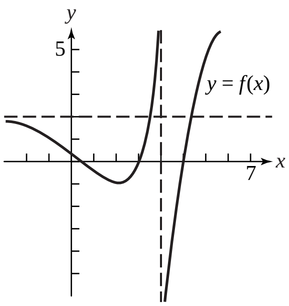
47. **(a)** 0 **(b)** -1
(c) $-\infty$ **(d)** -1

48. **(a)** 1 **(b)** 0
(c) 2 **(d)** ∞

49. One possible answer:



50. One possible answer:



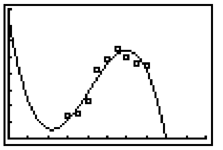
51. $\dfrac{f_1(x)/f_2(x)}{g_1(x)/g_2(x)} = \dfrac{f_1(x)/g_1(x)}{f_2(x)/g_2(x)}$

As x goes to infinity, $\dfrac{f_1}{g_1}$ and $\dfrac{f_2}{g_2}$ both approach 1.

Therefore, using the above equation, $\dfrac{f_1/f_2}{g_1/g_2}$ must also approach 1.

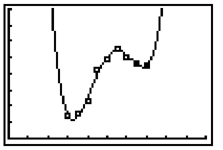
52. Yes. The limit of $(f + g)$ will be the same as the limit of g. This is because adding numbers that are very close to a given real number L will not have a significant effect on the value of $(f + g)$ since the values of g are becoming arbitrarily large.

53. **(a)** Using 1980 as $x = 0$:
$y = -2.2316x^3 + 54.7134x^2 - 351.0933x + 733.2224$



[0, 20] by [0, 800]

(b) Again using 1980 as $x = 0$:
$y = 1.458561x^4 - 60.5740x^3 + 905.8877x^2 - 5706.0943x + 12967.6288$



[0, 20] by [0, 800]

(c) Cubic: approximately -2256 dollars
Quartic: approximately 9979 dollars

(d) Cubic: End behavior model is $-2.2316x^3$. This model predicts that the grants will become negative by 1996.
Quartic: End behavior model is $1.458561x^4$. This model predicts that the size of the grants will grow very rapidly after 1995.
Neither of these seem reasonable. There is no reason to expect the grants to disappear (become negative) based on the data. Similarly, the data give no indication that a period of rapid growth is about to occur.

54. **(a)** $f \to -\infty$ as $x \to 0^-$, $f \to \infty$ as $x \to 0^+$, $g \to 0$, $fg \to 1$

(b) $f \to \infty$ as $x \to 0^-$, $f \to -\infty$ as $x \to 0^+$, $g \to 0$, $fg \to -8$

(c) $f \to -\infty$ as $x \to 2^-$, $f \to \infty$ as $x \to 2^+$, $g \to 0$, $fg \to 0$

(d) $f \to \infty$, $g \to 0$, $fg \to \infty$

(e) Nothing — you need more information to decide.

55. **(a)** This follows from $x - 1 < \text{int } x \leq x$ which is true for all x. Dividing by x gives the result.
(b) 1 **(c)** 1

56. For $x > 0$, $0 < e^{-x} < 1$, so $0 < \dfrac{e^{-x}}{x} < \dfrac{1}{x}$.
Since both 0 and $\dfrac{1}{x}$ approach zero as $x \to \infty$, the Sandwich Theorem states that $\dfrac{e^{-x}}{x}$ must also approach zero.

57. This is because as x approaches infinity, $\sin x$ continues to oscillate between 1 and -1 and doesn't approach any given real number.

58. Limit $= 2$, because $\dfrac{\ln x^2}{\ln x} = \dfrac{2 \ln x}{\ln x} = 2$.

59. Limit = ln(10),
since $\frac{\ln x}{\log x} = \frac{\ln x}{\ln x/\ln 10} = \ln 10$.

60. Limit = 1.
Since $\ln(x+1) = \ln x\left(1 + \frac{1}{x}\right)$
$= \ln x + \ln\left(1 + \frac{1}{x}\right)$,
$\frac{\ln(x+1)}{\ln x} = \frac{\ln x + \ln(1 + 1/x)}{\ln x}$
$= 1 + \frac{\ln(1 + 1/x)}{\ln x}$.
But as $x \to \infty$, $1 + \frac{1}{x}$ approaches 1, so $\ln\left(1 + \frac{1}{x}\right)$ approaches $\ln(1) = 0$. Also, as $x \to \infty$, $\ln x$ approaches infinity. This means the second term above approaches 0 and the limit is 1.

2.3 Continuity (pp. 73–81)

Quick Review 2.3

1. 2
2. (a) −2 **(b)** −1
(c) No limit **(d)** −1
3. (a) 1 **(b)** 2
(c) No limit **(d)** 2
4. $(f \circ g)(x) = \frac{x+2}{6x+1}, x \neq 0$
$(g \circ f)(x) = \frac{3x+4}{2x-1}, x \neq -5$
5. $g(x) = \sin x, x \geq 0$
$(f \circ g)(x) = \sin^2 x, x \geq 0$
6. $f(x) = \frac{1}{x^2} + 1, x > 0$
$(f \circ g)(x) = \frac{x}{x-1}, x > 1$
7. $x = \frac{1}{2}, -5$ **8.** $x \approx 0.453$
9. $x = 1$ **10.** Any c in $[1, 2)$

Section 2.3 Exercises

1. $x = -2$, infinite discontinuity
2. $x = 1$ and $x = 3$, both infinite discontinuities
3. None **4.** None
5. All points not in the domain, i.e., all $x < -\frac{3}{2}$
6. None
7. $x = 0$, jump discontinuity
8. $x = k\pi$ for all integers k, infinite discontinuity
9. $x = 0$, infinite discontinuity
10. All points not in the domain, i.e., all $x < -1$
11. (a) Yes **(b)** Yes
(c) Yes **(d)** Yes
12. (a) Yes **(b)** Yes
(c) No **(d)** No
13. (a) No **(b)** No
14. Everywhere in $[-1, 3)$ except for $x = 0, 1, 2$
15. 0 **16.** 2
17. No, because the right-hand and left-hand limits are not the same at zero.
18. Yes. Assign the value 0 to $f(3)$.
19. (a) $x = 2$
(b) Not removable, the one-sided limits are different.
20. (a) $x = 2$
(b) Removable, assign the value 1 to $f(2)$.
21. (a) $x = 1$
(b) Not removable, it's an infinite discontinuity.
22. (a) $x = -1$
(b) Removable, assign the value 0 to $f(-1)$.
23. (a) All points not in the domain along with $x = 0, 1$
(b) $x = 0$ is a removable discontinuity, assign $f(0) = 0$.
$x = 1$ is not removable, the two-sided limits are different.
24. (a) All points not in the domain along with $x = 1, 2$
(b) $x = 1$ is not removable, the one-sided limits are different.
$x = 2$ is a removable discontinuity, assign $f(2) = 1$.
25. $y = x - 3$
26. $y = \frac{x^2 + x + 1}{x + 1}$
27. $y = \begin{cases} \frac{\sin x}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$
28. $y = \begin{cases} \frac{\sin 4x}{x}, & x \neq 0 \\ 4, & x = 0 \end{cases}$
29. $y = \sqrt{x} + 2$
30. $y = \frac{x^2 - 2x - 15}{x + 2}$

Note: There are different ways to verify the continuity of the functions in 31–34. In each case, one possible answer is given.

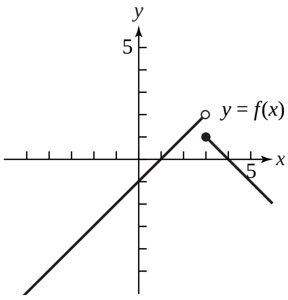
31. Assume $y = x$, constant functions, and the square root function are continuous.
Use the sum, composite, and quotient theorems.
Domain: $(-2, \infty)$

32. Assume $y = x$, constant functions, and the cube root function are continuous.
Use the difference, composite, product, and sum theorems.
Domain: $(-\infty, \infty)$

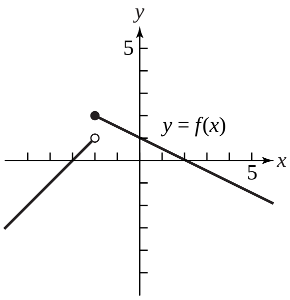
33. Assume $y = x$ and the absolute value function are continuous.
Use the product, constant multiple, difference, and composite theorems.
Domain: $(-\infty, \infty)$

34. Assume $y = x$ and $y = 1$ are continuous.
Use the product, difference, and quotient theorems.
One also needs to verify that the limit of this function as x approaches 1 is 2.
Domain: $(-\infty, \infty)$

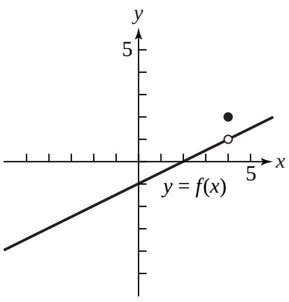
35. One possible answer:



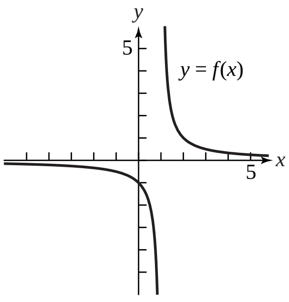
36. One possible answer:



37. One possible answer:



38. One possible answer:



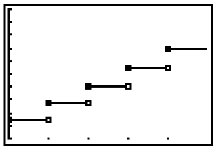
39. $x \approx -0.724$ and $x \approx 1.221$

40. $x \approx -1.521$

41. $a = \dfrac{4}{3}$

42. Consider $f(x) = x - e^{-x}$. f is continuous, $f(0) = -1$,and $f(1) = 1 - \dfrac{1}{e} > 0.5$. By the Intermediate Value Theorem, for some c in $(0, 1)$, $f(c) = 0$ and $e^{-c} = c$.

43. (b)

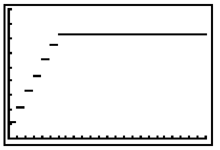


[0, 4.8] by [35000, 45000]

Continuous at all points in the domain [0, 5) except at $t = 1, 2, 3, 4$.

44. (a) $f(x) = \begin{cases} -1.10 \text{ int } (-x), & 0 \le x \le 6 \\ 7.25, & 6 < x \le 24 \end{cases}$

(b)

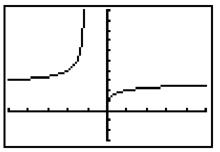


[0, 24] by [0, 9]

This is continuous at all values of x in the domain [0, 24] except for $x = 0, 1, 2, 3, 4, 5, 6$.

45. (a) Domain of f: $(-\infty, -1) \cup (0, \infty)$

(b)



[−5, 5] by [−3, 10]

(c) Because f is undefined there due to division by 0.

(d) $x = 0$: removable, right-hand limit is 0
$x = -1$: not removable, infinite discontinuity

(e) 2.718 or e

46. This is because $\lim_{h\to 0} f(a + h) = \lim_{x\to a} f(x)$.

47. Suppose not. Then f would be negative somewhere in the interval and positive somewhere else in the interval. So, by the Intermediate Value Theorem, it would have to be zero somewhere in the interval, which contradicts the hypothesis.

48. Since the absolute value function is continuous, this follows from the theorem about continuity of composite functions.

49. For any real number a, the limit of this function as x approaches a cannot exist. This is because as x approaches a, the values of the function will continually oscillate between 0 and 1.

2.4 Rates of Change and Tangent Lines (pp. 82–90)

Quick Review 2.4

1. $\Delta x = 8$, $\Delta y = 3$
2. $\Delta x = a - 1$, $\Delta y = b - 3$
3. Slope $= -\frac{4}{7}$
4. Slope $= \frac{2}{3}$
5. $y = \frac{3}{2}x + 6$
6. $y = -\frac{7}{3}x + \frac{25}{3}$
7. $y = -\frac{3}{4}x + \frac{19}{4}$
8. $y = \frac{4}{3}x + \frac{8}{3}$
9. $y = -\frac{2}{3}x + \frac{7}{3}$
10. $b = \frac{19}{3}$

Section 2.4 Exercises:

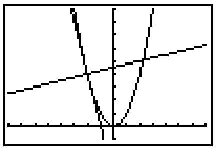
1. **(a)** 19 **(b)** 1
2. **(a)** 1
(b) $\frac{7 - \sqrt{41}}{2} \approx 0.298$
3. **(a)** $\frac{1 - e^{-2}}{2} \approx 0.432$
(b) $\frac{e^3 - e}{2} \approx 8.684$
4. **(a)** $\frac{\ln 4}{3} \approx 0.462$
(b) $\frac{\ln(103/100)}{3} = \frac{\ln 1.03}{3} \approx 0.0099$
5. **(a)** $-\frac{4}{\pi} \approx -1.273$
(b) $-\frac{3\sqrt{3}}{\pi} \approx -1.654$
6. **(a)** $-\frac{2}{\pi} \approx -0.637$
(b) 0
7. Using $Q_1 = (10, 225)$, $Q_2 = (14, 375)$, $Q_3 = (16.5, 475)$, $Q_4 = (18, 550)$, and $P = (20, 650)$
(a)

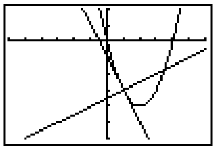
Secant	Slope
PQ_1	43
PQ_2	46
PQ_3	50
PQ_4	50

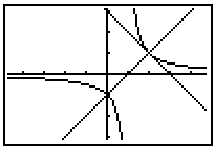
Units are meters/second
(b) Approximately 50 m/sec
8. Using $Q_1 = (5, 20)$, $Q_2 = (7, 38)$, $Q_3 = (8.5, 56)$, $Q_4 = (9.5, 72)$, and $P = (10, 80)$
(a)

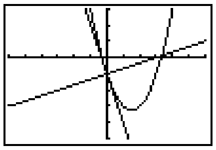
Secant	Slope
PQ_1	12
PQ_2	14
PQ_3	16
PQ_4	16

Units are meters/second
(b) Approximately 16 m/sec

9. **(a)** -4 **(b)** $y = -4x - 4$
(c) $y = \frac{1}{4}x + \frac{9}{2}$
(d)

$[-8, 7]$ by $[-1, 9]$

10. **(a)** -2 **(b)** $y = -2x - 1$
(c) $y = \frac{1}{2}x - \frac{7}{2}$
(d)

$[-6, 6]$ by $[-6, 2]$

11. **(a)** -1 **(b)** $y = -x + 3$
(c) $y = x - 1$
(d)

$[-4.7, 4.7]$ by $[-3.1, 3.1]$

12. **(a)** -3 **(b)** $y = -3x - 1$
(c) $y = \frac{1}{3}x - 1$
(d)

$[-6, 6]$ by $[-5, 3]$

13. **(a)** 1 **(b)** -1
14. -1
15. No. Slope from the left is -2; slope from the right is 2. The two-sided limit of the difference quotient doesn't exist.
16. Yes. The slope is -1.
17. Yes. The slope is $-\frac{1}{4}$.
18. No. The function is discontinuous at $x = \frac{3\pi}{4}$. The left-hand limit of the difference quotient doesn't exist.
19. **(a)** $2a$
(b) The slope of the tangent steadily increases as a increases.
20. **(a)** $-\frac{2}{a^2}$
(b) The slope of the tangent is always negative. The tangents are very steep near $x = 0$ and nearly horizontal as a moves away from the origin.

21. (a) $-\dfrac{1}{(a-1)^2}$

(b) The slope of the tangent is always negative. The tangents are very steep near $x = 1$ and nearly horizontal as a moves away from the origin.

22. (a) $-2a$

(b) The slope of the tangent steadily decreases as a increases.

23. 19.6 m/sec

24. 60 ft/sec

25. 6π in^2/in.

26. 16π in^3/in.

27. 3.72 m/sec

28. 45.76 m/sec

29. $(-2, -5)$

30. $(-2, 7)$

31. (a) At $x = 0$: $y = -x - 1$
At $x = 2$: $y = -x + 3$

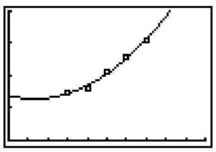
(b) At $x = 0$: $y = x - 1$
At $x = 2$: $y = x - 1$

32. At $x = -1$: $y = 2x + 10$
At $x = 3$: $y = -6x + 18$

33. (a) 0.3 billion dollars per year

(b) 0.5 billion dollars per year

(c) $y = 0.0571x^2 - 0.1514x + 1.3943$

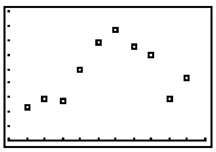


[0, 10] by [0, 4]

(d) 1993 to 1995: 0.31 billion dollars per year
1995 to 1997: 0.53 billion dollars per year

(e) 0.65 billion dollars per year

34. (a)



[7, 18] by [0, 900]

(b)

Q From Year	Slope
1988	23.9
1989	18.9
1990	24.3
1991	−8.8
1992	−48.8
1993	−80.8
1994	−70.3
1995	−80.0
1996	144.0

(c) As Q gets closer to 1997, the slopes do not seem to be approaching a limit value. The years 1995–97 seem to be very unusual and unpredictable.

35. (a) $\dfrac{e^{1+h} - e}{h}$

(b) Limit ≈ 2.718

(c) They're about the same.

35. continued

(d) Yes, it has a tangent whose slope is about e.

36. (a) $\dfrac{2^{1+h} - 2}{h}$

(b) Limit ≈ 1.386

(c) They're about the same.

(d) Yes, it has a tangent whose slope is about ln 4.

37. No

38. Yes

39. Yes

40. No

41. This function has a tangent with slope zero at the origin. It is sandwiched between two functions, $y = x^2$ and $y = -x^2$, both of which have slope zero at the origin.

Looking at the difference quotient,
$-h \le \dfrac{f(0+h) - f(0)}{h} \le h$, so
the Sandwich Theorem tells us that the limit is 0.

42. This function does not have a tangent line at the origin. As the function oscillates between $y = x$ and $y = -x$ infinitely often near the origin, there are an infinite number of difference quotients (secant line slopes) with a value of 1 and with a value of -1. Thus the limit of the difference quotient doesn't exist.

The difference quotient is
$\dfrac{f(0+h) - f(0)}{h} = \sin\left(\dfrac{1}{h}\right)$ which oscillates between 1 and -1 infinitely often near zero.

43. Slope ≈ 0.540

Chapter 2 Review Exercises (pp. 91–93)

1. -15

2. $\dfrac{5}{21}$

3. No limit

4. No limit

5. $-\dfrac{1}{4}$

6. $\dfrac{2}{5}$

7. $+\infty, -\infty$

8. $\dfrac{1}{2}$

9. 2

10. 0

11. 6

12. 5

13. 0

14. 1

15. Limit exists

16. Limit exists

17. Limit exists

18. Doesn't exist

19. Limit exists

20. Limit exists

21. Yes

22. No

23. No

24. Yes

25. (a) 1

(b) 1.5

(c) No

(d) g is discontinuous at $x = 3$ (and points not in domain).

25. continued

(e) Yes, can remove discontinuity at $x = 3$ by assigning the value 1 to $g(3)$.

26. (a) 1.5 **(b)** 0

(c) 0 **(d)** No

(e) k is discontinuous at $x = 1$ (and points not in domain).

(f) Discontinuity at $x = 1$ is not removable because the two one-sided limits are different.

27. (a) Vertical Asymp.: $x = -2$

(b) Left-hand limit $= -\infty$
Right-hand limit $= \infty$

28. (a) Vertical Asymp.: $x = 0$ and $x = -2$

(b) At $x = 0$:
Left-hand limit $= -\infty$
Right-hand limit $= -\infty$
At $x = -2$:
Left-hand limit $= \infty$
Right-hand limit $= -\infty$

29. (a) At $x = -1$:
Left-hand limit $= 1$
Right-hand limit $= 1$
At $x = 0$:
Left-hand limit $= 0$
Right-hand limit $= 0$
At $x = 1$:
Left-hand limit $= -1$
Right-hand limit $= 1$

(b) At $x = -1$:
Yes, the limit is 1.
At $x = 0$:
Yes, the limit is 0.
At $x = 1$:
No, the limit doesn't exist because the two one-sided limits are different.

(c) At $x = -1$:
Continuous because $f(-1) =$ the limit.
At $x = 0$:
Discontinuous because $f(0) \neq$ the limit.
At $x = 1$:
Discontinuous because limit doesn't exist.

30. (a) Left-hand limit $= 3$
Right-hand limit $= -3$

(b) No, because the two one-sided limits are different.

(c) Every place except for $x = 1$

(d) At $x = 1$

31. $x = -2$ and $x = 2$

32. There are no points of discontinuity.

33. (a) $\frac{2}{x}$ **(b)** $y = 0$ (x-axis)

34. (a) 2 **(b)** $y = 2$

35. (a) x^2 **(b)** None

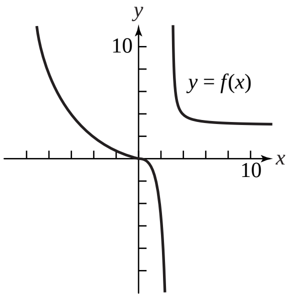
36. (a) x **(b)** None

37. (a) e^x **(b)** x

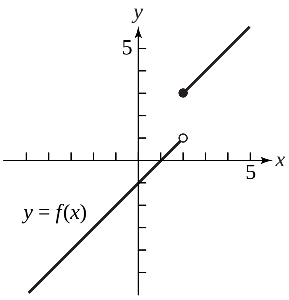
38. (a) $\ln|x|$ **(b)** $\ln|x|$

39. $k = 8$ **40.** $k = \frac{1}{2}$

41. One possible answer:



42. One possible answer:



43. $\frac{2}{\pi}$ **44.** $\frac{2}{3}\pi aH$

45. $12a$ **46.** $2a - 1$

47. (a) -1 **(b)** $y = -x - 1$

(c) $y = x - 3$

48. $\left(\frac{3}{2}, -\frac{9}{4}\right)$

49. (a) 25. Perhaps this is the number of bears placed in the reserve when it was established.

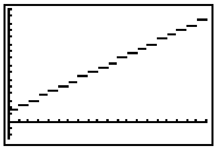
(b) 200

(c) Perhaps this is the maximum number of bears which the reserve can support due to limitations of food, space, or other resources. Or, perhaps the number is capped at 200 and excess bears are moved to other locations.

50. (a)

$$f(x) = \begin{cases} 3.20 - 1.35 \cdot \text{int}(-x + 1), & 0 < x \leq 20 \\ 0, & x = 0 \end{cases}$$

(b)



[0, 20] by [−5, 32]

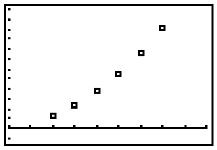
f is discontinuous at integer values of x: 0, 1, 2, ..., 19

51. (a) Cubic: $y = -1.644x^3 + 42.981x^2 - 254.369x + 300.232$
Quartic: $y = 2.009x^4 - 102.081x^3 + 1884.997x^2 - 14918.180x + 43004.464$

(b) Cubic: $-1.644x^3$, predicts spending will go to 0.
Quartic: $2.009x^4$, predicts spending will go to ∞.

52. $\lim_{x\to c} f(x) = \frac{3}{2}, \lim_{x\to c} g(x) = \frac{1}{2}$

53. (a)



[3, 12] by [−2, 24]

(b)

Year of Q	Slope of PQ
1995	3.48
1996	3.825
1997	4.1
1998	4.45
1999	4.9

(c) Approximately 5 billion dollars per year

(d) $y = 0.3214x^2 - 1.3471x + 1.3857$
Predicted rate of change in 2000 is 5.081 billion dollars per year.

Chapter 3
Derivatives

3.1 Derivative of a Function (pp. 95–104)

Quick Review 3.1

1. 4 **2.** $\frac{5}{2}$

3. −1 **4.** 8

5. 0

6. $(-\infty, 0]$ and $[2, \infty)$

7. $\lim_{x\to 1^+} f(x) = 0; \lim_{x\to 1^-} f(x) = 3$

8. 0

9. No, the two one-sided limits are different.

10. No. f is discontinuous at $x = 1$ because the limit doesn't exist there.

Section 3.1 Exercises

1. (a) $y = 5x - 7$

(b) $y = -\frac{1}{5}x + \frac{17}{5}$

2. $-\frac{1}{9}$ **3.** $-\frac{1}{9}$

4. $f'(x) = 3$ **5.** $\frac{dy}{dx} = 7$

6. $2x$ **7.** (b)

8. (a) **9.** (d)

10. (c)

11. $\frac{dy}{dx} = 4x - 13$, tangent line is $y = -x - 13$

12. (a) $y = 3x - 2$

(b) $y = -\frac{1}{3}x + \frac{4}{3}$

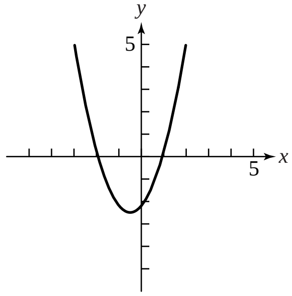
13. (ii) **14.** (iv)

15. (a) Sometime around April 1. The rate then is approximately $\frac{1}{6}$ hour per day.

(b) Yes. Jan. 1 and July 1

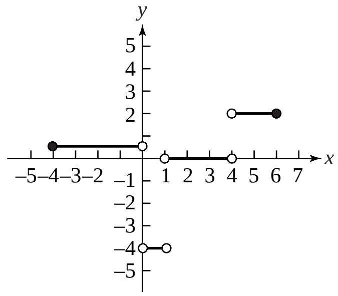
(c) Positive: Jan 1. through July 1
Negative: July 1 through Dec. 31

16.



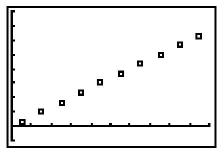
17. (a) 0 and 0 **(b)** 1700 and 1300

18. (a)



(b) $x = 0, 1, 4$

19. Graph of derivative:



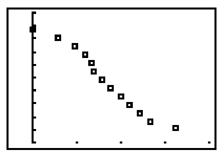
[0, 10] by [−10, 80]

(a) The speed of the skier

(b) Feet per second

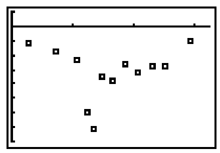
(c) Approximately $D = 6.65t$

20. (a)



[−0.5, 4] by [700, 1700]

(b)



[0, 3.24] by [−800, 100]

(c) Feet per mile

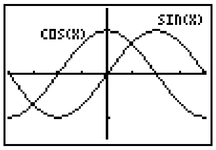
(d) Feet per mile

(e) Look for the steepest part of the curve. This is where the elevation is dropping most rapidly, and therefore the most likely location for significant "rapids".

20. continued

(f) Look for the lowest point on the graph. This is where the elevation is dropping most rapidly, and therefore the most likely location for significant "rapids".

21.



$[-\pi, \pi]$ by $[-1.5, 1.5]$

Cosine could be the derivative of sine. The values of cosine are positive where sine is increasing, zero where sine has horizontal tangents, and negative where sine is decreasing.

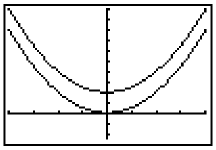
22. We show that the right-hand derivative at 1 does not exist.

$$\lim_{h\to 0^+} \frac{f(1+h)-f(1)}{h} = \lim_{h\to 0^+} \frac{3(1+h)-(1)^3}{h}$$
$$= \lim_{h\to 0^+} \frac{2+3h}{h} = \lim_{h\to 0^+} \left(\frac{2}{h}+3\right) = \infty$$

23. $\lim_{h\to 0^+} \frac{f(0+h)-f(0)}{h} = \lim_{h\to 0^+} \frac{\sqrt{h}}{h} = \lim_{h\to 0^+} \frac{1}{\sqrt{h}} = \infty$

Thus, the right-hand derivative at 0 does not exist.

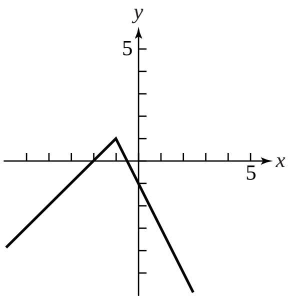
24. Two parabolas are parallel if they have the same derivative at every value of x. This means their tangent lines are parallel at each value of x. Two such parabolas are given by $y = x^2$ and $y = x^2 + 4$. They are graphed below.



$[-4, 4]$ by $[-5, 20]$

The parabolas are "everywhere equidistant", as long as the distance between them is always measured along a vertical line.

25.



26. **(a)** $2x$ **(b)** 2

(c) 2 **(d)** 2

(e) Yes, the two one-sided limits exist and are the same.

(f) 2

(g) Does not exist

(h) It does not exist because the right-hand derivative does not exist.

27. The y-intercept is $b - a$.

28. $k = -2$

29. **(a)** 0.992 **(b)** 0.008

(c) If P is the answer to (b), then the probability of a shared birthday when there are four people is $1 - (1 - P)\frac{362}{365} \approx 0.016$.

(d) No. Clearly, February 29th is a much less likely birth date. Furthermore, census data do not support the assumption that the other 365 birth dates are equally likely. However, this simplifying assumption may still give us some insight into this problem even if the calculated probabilities aren't completely accurate.

3.2 Differentiability (pp. 105–112)

Quick Review 3.2

1. Yes **2.** No
3. Yes **4.** Yes
5. No **6.** All reals
7. $[0, \infty)$ **8.** $[3, \infty)$
9. 3.2 **10.** 5

Section 3.2 Exercises

1. Left-hand derivative $= 0$
Right-hand derivative $= 1$

2. Left-hand derivative $= 0$
Right-hand derivative $= 2$

3. Left-hand derivative $= \frac{1}{2}$
Right-hand derivative $= 2$

4. Left-hand derivative $= 1$
Right-hand derivative $= -1$

5. **(a)** All points in $[-3, 2]$
(b) None **(c)** None

6. **(a)** All points in $[-2, 3]$
(b) None **(c)** None

7. **(a)** All points in $[-3, 3]$ except $x = 0$
(b) None **(c)** $x = 0$

8. **(a)** All points in $[-2, 3]$ except $x = -1, 0, 2$
(b) $x = -1$ **(c)** $x = 0, x = 2$

9. **(a)** All points in $[-1, 2]$ except $x = 0$
(b) $x = 0$ **(c)** None

10. **(a)** All points in $[-3, 3]$ except $x = -2, 2$
(b) $x = -2, x = 2$ **(c)** None

11. Discontinuity **12.** Cusp
13. Corner **14.** Vertical tangent
15. Corner **16.** Cusp
17. All reals except $x = -1, 5$
18. All reals except $x = 2$ **19.** All reals except $x = 0$
20. All reals **21.** All reals except $x = 3$

22. All reals

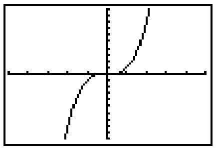
23. (a) $x = 0$ is not in their domains, or, they are both discontinuous at $x = 0$.

(b) For $\frac{1}{x}$: 1,000,000

For $\frac{1}{x^2}$: 0

(c) It returns an incorrect response because even though these functions are not defined at $x = 0$, they are defined at $x = \pm 0.001$.

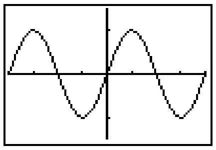
24.



$[-5, 5]$ by $[-10, 10]$

$\frac{dy}{dx} = x^3$

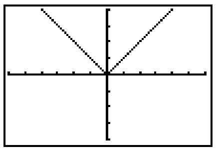
25.



$[-2\pi, 2\pi]$ by $[-1.5, 1.5]$

$\frac{dy}{dx} = \sin x$

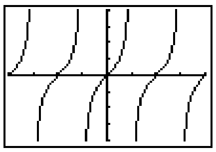
26.



$[-6, 6]$ by $[-4, 4]$

$\frac{dy}{dx} = \text{abs}\,(x)$ or $|x|$

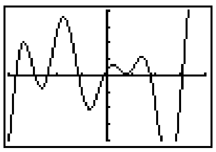
27.



$[-2\pi, 2\pi]$ by $[-4, 4]$

$\frac{dy}{dx} = \tan x$

28.



$[-2\pi, 2\pi]$ by $[-20, 20]$

Does not look like any basic function.

29. (a) $a + b = 2$ **(b)** $a = -3$ and $b = 5$

30. The function $f(x)$ does not have the intermediate value property. Choose some a in $(-1, 0)$ and b in $(0, 1)$. Then $f(a) = 0$ and $f(b) = 1$, but f does not take on any value between 0 and 1.

31. (a) Note that $-x \le x \sin \frac{1}{x} \le x$, for all x, so $\lim_{x\to 0}\left(x \sin \frac{1}{x}\right) = 0$ by the Sandwich Theorem. Therefore, f is continuous at $x = 0$.

(b) $\frac{f(0+h) - f(0)}{h} = \frac{h \sin \frac{1}{h} - 0}{h} = \sin \frac{1}{h}$

(c) The limit does not exist because $\sin \frac{1}{h}$ oscillates between -1 and 1 an infinite number of times arbitrarily close to $h = 0$.

(d) No

(e) $\frac{g(0+h) - g(0)}{h} = \frac{h^2 \sin \frac{1}{h} - 0}{h}$

$= h \sin \frac{1}{h}$

As noted in part (a), the limit of this as x approaches zero is 0, so $g'(0) = 0$.

3.3 Rules for Differentiation (pp. 112–121)

Quick Review 3.3

1. $x + x^2 - 2x^{-1} - 2$ **2.** $x + x^{-1}$

3. $3x^2 - 2x^{-1} + 5x^{-2}$ **4.** $\frac{3}{2}x^2 - x + 2x^{-2}$

5. $x^{-3} + x^{-1} + 2x^{-2} + 2$ **6.** $x^2 + x$

7. Root: $x \approx 1.173$, $500x^6 \approx 1305$
Root: $x \approx 2.394$, $500x^6 \approx 94{,}212$

8. (a) 7 **(b)** 7
(c) 7 **(d)** 0

9. (a) 0 **(b)** 0
(c) 0

10. (a) $f'(x) = \frac{1}{\pi}$ **(b)** $f'(x) = -\pi x^{-2}$

Section 3.3 Exercises

1. $\frac{dy}{dx} = -2x$, $\frac{d^2y}{dx^2} = -2$

2. $\frac{dy}{dx} = x^2 - 1$, $\frac{d^2y}{dx^2} = 2x$

3. $\frac{dy}{dx} = 2$, $\frac{d^2y}{dx^2} = 0$

4. $\frac{dy}{dx} = 2x + 1$, $\frac{d^2y}{dx^2} = 2$

5. $\frac{dy}{dx} = x^2 + x + 1$, $\frac{d^2y}{dx^2} = 2x + 1$

6. $\frac{dy}{dx} = -1 + 2x - 3x^2$, $\frac{d^2y}{dx^2} = 2 - 6x$

7. $\frac{dy}{dx} = 4x^3 - 21x^2 + 4x$, $\frac{d^2y}{dx^2} = 12x^2 - 42x + 4$

8. $\frac{dy}{dx} = 15x^2 - 15x^4$, $\frac{d^2y}{dx^2} = 30x - 60x^3$

9. $\frac{dy}{dx} = -8x^{-3} - 8, \frac{d^2y}{dx^2} = 24x^{-4}$

10. $\frac{dy}{dx} = -x^{-5} + x^{-4} - x^{-3} + x^{-2},$

$\frac{d^2y}{dx^2} = 5x^{-6} - 4x^{-5} + 3x^{-4} - 2x^{-3}$

11. (a) $3x^2 + 2x + 1$

(b) $3x^2 + 2x + 1$

12. (a) $\frac{x(2x) - (x^2 + 3)}{x^2} = \frac{x^2 - 3}{x^2}$

(b) $1 - \frac{3}{x^2}$

13. $-\frac{19}{(3x - 2)^2}$

14. $-\frac{5}{x^2} + \frac{2}{x^3}$

15. $\frac{3}{x^4}$

16. $\frac{x^2 - 2x - 1}{(1 + x^2)^2}$

17. $\frac{x^4 + 2x}{(1 - x^3)^2}$

18. $\frac{1}{\sqrt{x}(\sqrt{x} + 1)^2}$

19. $\frac{12 - 6x^2}{(x^2 - 3x + 2)^2}$

20. (a) Let $f(x) = x$.

$\lim_{h\to 0} \frac{f(x + h) - f(x)}{h} = \lim_{h\to 0} \frac{(x + h) - x}{h}$

$= \lim_{h\to 0} \frac{h}{h} = \lim_{h\to 0} (1) = 1$

(b) Note that $u = u(x)$ is a function of x.

$\lim_{h\to 0} \frac{-u(x + h) - [-u(x)]}{h}$

$= \lim_{h\to 0} \left(-\frac{u(x + h) - u(x)}{h}\right)$

$= -\lim_{h\to 0} \frac{u(x + h) - u(x)}{h} = -\frac{du}{dx}$

21. $\frac{d}{dx}(c \cdot f(x)) = c \cdot \frac{d}{dx} f(x) + f(x) \cdot \frac{d}{dx} c$

$= c \cdot \frac{d}{dx} f(x) + 0 = c \cdot \frac{d}{dx} f(x)$

22. $-\frac{f'(x)}{[f(x)]^2}$

23. (a) 13 **(b)** -7

(c) $\frac{7}{25}$ **(d)** 20

24. (a) 2 **(b)** -10

(c) $\frac{10}{9}$ **(d)** -12

25. (iii) **26.** (iii)

27. $y = -\frac{1}{9}x + \frac{29}{9}$

28. Slope is 4 at $x = \pm 1$:
tangent at $x = -1$: $y = 4x + 2$
tangent at $x = 1$: $y = 4x - 2$
Smallest slope is 1 and occurs at $x = 0$.

29. $(-1, 27)$ and $(2, 0)$

30. x-intercept $= -\frac{4}{3}$, y-intercept $= 16$

31. At $(0, 0)$: $y = 4x$
At $(1, 2)$: $y = 2$

32. $y = -\frac{1}{2}x + 2$

33. $-\frac{nRT}{(V - nb)^2} + \frac{2an^2}{V^3}$

34. $\frac{ds}{dt} = 9.8t, \frac{d^2s}{dt^2} = 9.8$

35. $\frac{dR}{dM} = CM - M^2$

36. If the radius of a circle is changed by a very small amount Δr, the change in the area can be thought of as a very thin strip with length given by the circumference, $2\pi r$, and width Δr. Therefore, the change in the area can be thought of as $(2\pi r)(\Delta r)$, which means that the change in the area divided by the change in the radius is just $2\pi r$.

37. If the radius of a sphere is changed by a very small amount Δr, the change in the volume can be thought of as $(4\pi r^2)(\Delta r)$, which means that the change in the volume divided by the change in the radius is just $4\pi r^2$.

38. 390 bushels of annual production per year.

39. It is going down approximately 20 cents per year. (rate ≈ -0.201 dollars/year)

40. (a) It is insignificant in the limiting case and can be treated as zero (and removed from the expression).

(b) It was "rejected" because it is incomparably smaller than the other terms: $v\,du$ and $u\,dv$.

(c) The product rule given in the text.

(d) Because dx is "infinitely small," and this could be thought of as dividing by zero.

(e) $d\left(\frac{u}{v}\right) = \frac{u + du}{v + dv} - \frac{u}{v}$

$= \frac{(u + du)v - u(v + dv)}{(v + dv)v}$

$= \frac{uv + vdu - uv - udv}{v^2 + vdv}$

$= \frac{vdu - udv}{v^2}.$

3.4 Velocity and Other Rates of Change (pp. 122-133)

Quick Review 3.4

1. Downward **2.** y-intercept $= -256$

3. x-intercepts $= 2, 8$ **4.** $(-\infty, 144]$

5. $(5, 144)$ **6.** $x = 3, 7$

7. $x = \frac{15}{8}$ **8.** $(-\infty, 5)$

9. 64 **10.** -32

Section 3.4 Exercises

1. $3s^2$

2. (a) 10 m **(b)** 2 m/sec

(c) 5 m/sec **(d)** 2 m/sec^2

(e) At $t = \frac{3}{2}$ sec **(f)** At $s = -\frac{1}{4}$ m

3. **(a)** $\text{vel}(t) = 24 - 1.6t$, $\text{accel}(t) = -1.6$
(b) 15 seconds **(c)** 180 meters
(d) About 4.393 seconds
(e) 30 seconds

4. Mars: $t \approx 4.462$ sec
Jupiter: $t \approx 0.726$ sec

5. About 29.388 meters

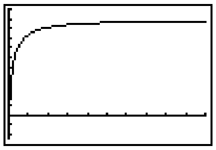
6. Moon: 320 seconds
Earth: 52 seconds

7. For the moon:
$x_1(t) = 3(t < 160) + 3.1(t \geq 160)$
$y_1(t) = 832t - 2.6t^2$
t-values: 0 to 320
window: [0, 6] by [−10,000, 70,000]
For the earth:
$x_1(t) = 3(t < 26) + 3.1(t \geq 26)$
$y_1(t) = 832t - 16t^2$
t-values: 0 to 52
window: [0, 6] by [−1000, 11,000]

8. At $t = 0$: 10,000 bacteria/hour
At $t = 5$: 0 bacteria/hour
At $t = 10$: −10,000 bacteria/hour

9. At the end of 10 minutes: 8000 gallons/minute
Average over first 10 minutes:
10,000 gallons/minute

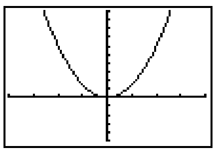
10. **(a)** \$110 per machine
(b) \$80 per machine
(c) \$79.90 for the 101^{st} machine

11. **(a)**

[0, 50] by [−500, 2200]

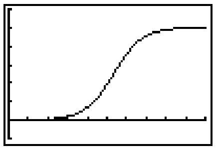
The values of x which make sense are the whole numbers, $x \geq 0$.
(b) $\dfrac{2000}{(x+1)^2}$
(c) Approximately \$55.56
(d) The limit is 0. This means that as x gets large, one reaches a point where very little extra revenue can be expected from selling more desks.

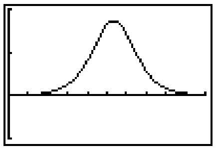
12. At $t = 1$: -6 m/sec^2
At $t = 3$: 6 m/sec^2

13. At $t = 1$: 0 m/sec
At $t = 2$: 1 m/sec

14. **(a)** $g'(x) = h'(x) = t'(x) = 3x^2$
(b)

[−4, 4] by [−10, 20]

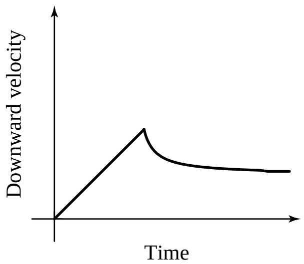
(c) $f(x)$ must be of the form $f(x) = x^3 + c$, where c is a constant.
(d) Yes. $f(x) = x^3$ **(e)** Yes. $f(x) = x^3 + 3$

15. **(a)**

[0, 200] by [−2, 12]

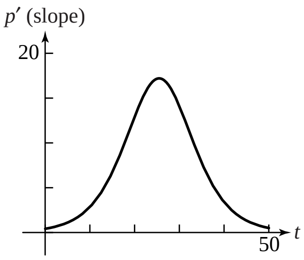
(b) $x \geq 0$ (whole numbers)
(c)

[0, 200] by [−0.1, 0.2]

P seems to be relatively sensitive to changes in x between approximately $x = 60$ and $x = 160$.
(d) The maximum occurs when $x \approx 106.44$. Since x must be an integer, $P(106) \approx 4.924$ thousand dollars or \$4924.
(e) \$13 per package sold, \$165 per package sold, \$118 per package sold, \$31 per package sold, \$6 per package sold, $P'(300) \approx 0$ (on the order of 10^{-6}, or \$0.001 per package sold)
(f) The limit is 10. Maximum possible profit is \$10,000 monthly.
(g) Yes. In order to sell more and more packages, the company might need to lower the price to a point where they won't make any additional profit.

16. **(a)** 190 ft/sec **(b)** 2 seconds
(c) After 8 seconds, and its velocity was 0 ft/sec then
(d) After about 11 seconds, and it was falling 90 ft/sec then
(e) About 3 seconds
(f) Just before the engine stopped;
from $t = 2$ to $t = 11$ while the rocket was in free fall

17. Possible answer:


18. **(a)**



horizontal axis: Days
vertical axis: Flies per day

(b) Fastest: Around the 25th day
Slowest: Day 50 or day 0

19. At $t \approx 2.83$

20. **(a)** It begins at the point $(-5, 2)$ moving in the positive direction. After a little more than one second, it has moved a bit past $(6, 2)$ and it turns back in the negative direction for approximately 2 seconds. At the end of that time, it is near $(-2, 2)$ and it turns back again in the positive direction. After that, it continues moving in the positive direction indefinitely, speeding up as it goes.

(b) Speeds up: $[1.153, 2.167]$ and $[3.180, \infty)$
slows down: $[0, 1.153]$ and $[2.167, 3.180]$

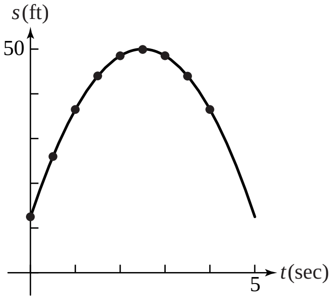
(c) At $t \approx 1.153$ and $t \approx 3.180$

(d) At $t \approx 1.153$ and $t \approx 3.180$ "instantaneously"

(e) The velocity starts out positive but decreasing, it becomes negative, then starts to increase, and becomes positive again and continues to increase.
The speed is decreasing, reaches 0 at $t \approx 1.15$, then increases until $t \approx 2.17$, decreases until $t \approx 3.18$ when it is 0 again, and then increases after that.

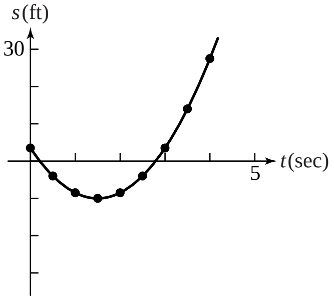
(f) At about 0.745 sec, 1.626 sec, 4.129 sec

21. **(a)**



(b) $s'(1) = 18$, $s'(2.5) = 0$, $s'(3.5) = -12$

22. **(a)**

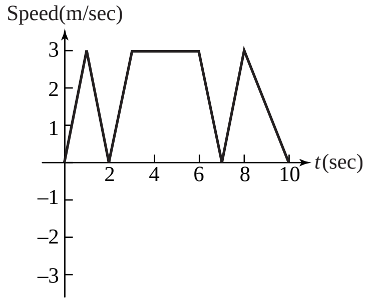


(b) $s'(1) = -6$, $s'(2.5) = 12$, $s'(3.5) = 24$

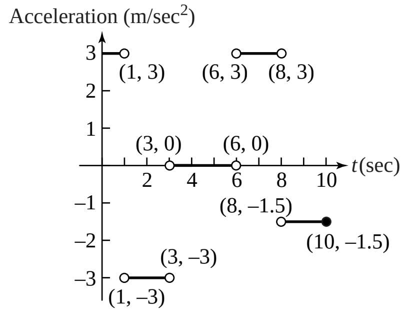
23. **(a)** At $t = 2$ and $t = 7$

(b) Between $t = 3$ and $t = 6$

(c)

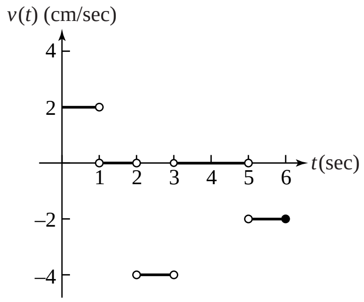


(d)

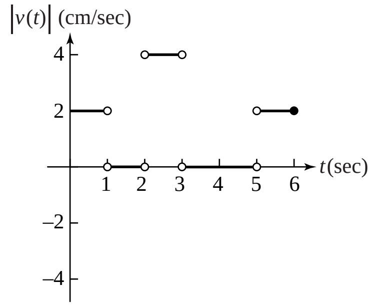


24. **(a)** Left: $2 < t < 3$, $5 < t \leq 6$
Right: $0 \leq t < 1$
Standing still: $1 < t < 2$, $3 < t < 5$

(b) Velocity graph:



Speed graph:



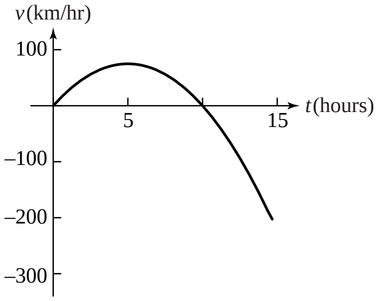
25. **(a)** Move forward: $0 \leq t < 1$ and $5 < t < 7$
move backward: $1 < t < 5$
speed up: $1 < t < 2$ and $5 < t < 6$
slow down: $0 \leq t < 1$, $3 < t < 5$, and $6 < t < 7$

(b) Positive: $3 < t < 6$
negative: $0 \leq t < 2$ and $6 < t < 7$
zero: $2 < t < 3$ and $7 < t \leq 9$

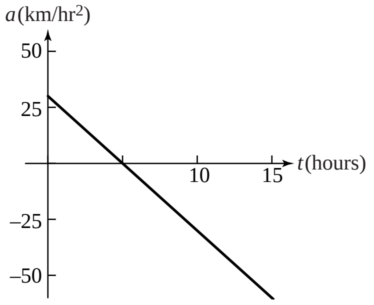
(c) At $t = 0$ and $2 < t < 3$

(d) $7 < t \leq 9$

26. (a) Velocity graph:



Acceleration:



(b) $\frac{ds}{dt} = 30t - 3t^2$ and $\frac{d^2s}{dt^2} = 30 - 6t$.
The graphs are very similar.

27. (a) $\frac{4}{7}$ of a second. Average velocity = 280 cm/sec

(b) Velocity = 560 cm/sec;
acceleration = 980 cm/sec^2

(c) About 28 flashes per second

28. Graph C is position, graph A is velocity, and graph B is acceleration.
A is the derivative of C because it is positive, negative, and zero where C is increasing, decreasing, and has horizontal tangents, respectively. The relationship between B and A is similar.

29. Graph C is position, graph B is velocity, and graph A is acceleration.
B is the derivative of C because it is negative and zero where C is decreasing and has horizontal tangents, respectively.
A is the derivative of B because it is positive, negative, and zero where B is increasing, decreasing, and has horizontal tangents, respectively.

30. (a) $\frac{dy}{dt} = \frac{t}{12} - 1$

(b) Fastest: at $t = 0$

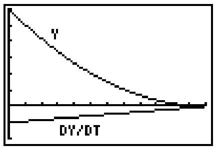
slowest: at $t = 12$

At $t = 0$: $\frac{dy}{dt} = -1$

at $t = 12$: $\frac{dy}{dt} = 0$

30. continued

(c)



[0, 12] by [−2, 6]

y is decreasing and $\frac{dy}{dt}$ is negative over the entire interval. y decreases more rapidly early in the interval, and the magnitude of $\frac{dy}{dt}$ is larger then. $\frac{dy}{dt}$ is 0 at $t = 12$, where y seems to have a horizontal tangent.

31. (a) 16π cubic feet of volume per foot of radius

(b) By about 11.092 cubic feet

32. It will take 25 seconds, and the aircraft will have traveled approximately 694.444 meters.

33. Exit velocity ≈ 348.712 ft/sec ≈ 237.758 mi/h

34. By estimating the slope of the velocity graph at that point.

35. Since profit = revenue − cost, using Rule 4 (the "difference rule"), and taking derivatives, we see that marginal profit
= marginal revenue − marginal cost.

36. (a) 135 seconds

(b) $\frac{5}{73} \approx 0.068$ furlongs/sec

(c) $\frac{1}{13} \approx 0.077$ furlongs/sec

(d) During the last furlong (between the 9th and 10th furlong markers)

(e) During the first furlong (between markers 0 and 1)

37. (a) Assume that f is even. Then,

$$f'(-x) = \lim_{h\to 0} \frac{f(-x+h) - f(-x)}{h} = \lim_{h\to 0} \frac{f(x-h) - f(x)}{h},$$

and substituting $k = -h$,

$$= \lim_{k\to 0} \frac{f(x+k) - f(x)}{-k} = -\lim_{k\to 0} \frac{f(x+k) - f(x)}{k} = -f'(x)$$

So, f' is an odd function.

37. continued

(b) Assume that f is odd. Then,

$$f'(-x) = \lim_{h\to 0} \frac{f(-x+h) - f(-x)}{h}$$
$$= \lim_{h\to 0} \frac{-f(x-h) + f(x)}{h},$$

and substituting $k = -h$,

$$= \lim_{k\to 0} \frac{-f(x+k) + f(x)}{-k}$$
$$= \lim_{k\to 0} \frac{f(x+k) - f(x)}{k} = f'(x)$$

So, f' is an even function.

38. $\frac{d}{dx} fgh = \frac{df}{dx}gh + f\frac{dg}{dx}h + fg\frac{dh}{dx}$

3.5 Derivatives of Trigonometric Functions (pp. 134–141)

Quick Review 3.5

1. $\frac{3\pi}{4} \approx 2.356$

2. $\left(\frac{306}{\pi}\right)^\circ \approx 97.403^\circ$

3. $\frac{\sqrt{3}}{2}$

4. Domain: all reals; range: $[-1, 1]$

5. Domain: $x \neq k\frac{\pi}{2}$, where k is an odd integer; range: all reals

6. 0

7. $\pm\frac{1}{\sqrt{2}}$

8. Multiply by $\frac{1 + \cos h}{1 + \cos h}$ and use $1 - \cos^2 h = \sin^2 h$.

9. $y = 12x - 35$

10. 12

Section 3.5 Exercises

1. $1 + \sin x$

2. $2 \cos x - \sec^2 x$

3. $-\frac{1}{x^2} + 5 \cos x$

4. $x \sec x \tan x + \sec x$

5. $-x^2 \cos x - 2x \sin x$

6. $3 + x \sec^2 x + \tan x$

7. $4 \sec x \tan x$

8. $\frac{1 + \cos x + x \sin x}{(1 + \cos x)^2}$

9. $-\frac{\csc^2 x}{(1 + \cot x)^2} = -\frac{1}{(\sin x + \cos x)^2}$

10. $-\frac{1}{1 + \sin x}$

11. $y = -x + \pi + 3$

12. Approximately $y = -1.081x + 2.122$

13. Approximately $y = -8.063x + 25.460$

14.
$$\lim_{h\to 0} \frac{\cos(x+h) - \cos(x)}{h}$$
$$= \lim_{h\to 0} \frac{(\cos x \cos h - \sin x \sin h) - \cos x}{h}$$
$$= \lim_{h\to 0} \frac{(\cos x)(\cos h - 1) - \sin x \sin h}{h}$$
$$= (\cos x) \lim_{h\to 0} \left(\frac{\cos h - 1}{h}\right) - (\sin x)\left(\lim_{h\to 0} \frac{\sin h}{h}\right)$$
$$= \cos x\,(0) - \sin x\,(1) = -\sin x$$

15. (a)
$$\frac{d}{dx} \tan x = \frac{d}{dx} \frac{\sin x}{\cos x}$$
$$= \frac{(\cos x)(\cos x) - (\sin x)(-\sin x)}{(\cos x)^2}$$
$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$
$$= \frac{1}{\cos^2 x} = \sec^2 x$$

(b)
$$\frac{d}{dx} \sec x = \frac{d}{dx} \frac{1}{\cos x}$$
$$= \frac{(\cos x)(0) - (1)(-\sin x)}{(\cos x)^2}$$
$$= \frac{\sin x}{(\cos x)^2} = \sec x \tan x$$

16. (a)
$$\frac{d}{dx} \cot x = \frac{d}{dx} \frac{\cos x}{\sin x}$$
$$= \frac{(\sin x)(-\sin x) - (\cos x)(\cos x)}{(\sin x)^2}$$
$$= -\frac{\sin^2 x + \cos^2 x}{\sin^2 x}$$
$$= -\frac{1}{\sin^2 x} = -\csc^2 x$$

(b)
$$\frac{d}{dx} \csc x = \frac{d}{dx} \frac{1}{\sin x}$$
$$= \frac{(\sin x)(0) - (1)(\cos x)}{(\sin x)^2}$$
$$= -\frac{\cos x}{(\sin x)^2} = -\csc x \cot x$$

17. $\frac{d}{dx} \sec x = \sec x \tan x$ which is 0 at $x = 0$, so the slope of the tangent line is 0.

$\frac{d}{dx} \cos x = -\sin x$ which is 0 at $x = 0$, so the slope of the tangent line is 0.

18. $\frac{d}{dx} \tan x = \sec^2 x$ which is never 0.

$\frac{d}{dx} \cot x = -\csc^2 x$ which is never 0.

19. Tangent: $y = -x + \frac{\pi}{4} + 1$

normal: $y = x + 1 - \frac{\pi}{4}$

20. $\left(-\frac{\pi}{4}, -1\right), \left(\frac{\pi}{4}, 1\right)$

21. (a) $y = -x + \frac{\pi}{2} + 2$

(b) $y = 4 - \sqrt{3}$

22. **(a)** $y = -4x + \pi + 4$
(b) $y = 2$

23. **(a)** Velocity: $-2\cos t$ m/sec
Speed: $|2\cos t|$ m/sec
Accel.: $2\sin t$ m/sec^2
Jerk: $2\cos t$ m/sec^3

(b) Velocity: $-\sqrt{2}$ m/sec
Speed: $\sqrt{2}$ m/sec
Accel.: $\sqrt{2}$ m/sec^2
Jerk: $\sqrt{2}$ m/sec^3

(c) The body starts at 2, goes to 0 and then oscillates between 0 and 4.
Speed: *Greatest* when $\cos t = \pm 1$ (or $t = k\pi$), at the center of the interval of motion.
Zero when $\cos t = 0 \left(\text{or } t = \frac{k\pi}{2}, k \text{ odd}\right)$, at the endpoints of the interval of motion.
Acceleration: *Greatest* (in magnitude) when $\sin t = \pm 1 \left(\text{or } t = \frac{k\pi}{2}, k \text{ odd}\right)$
Zero when $\sin t = 0$ (or $t = k\pi$)
Jerk: *Greatest* (in magnitude) when $\cos t = \pm 1$ (or $t = k\pi$)
Zero when $\cos t = 0 \left(\text{or } t = \frac{k\pi}{2}, k \text{ odd}\right)$

24. **(a)** Velocity: $\cos t - \sin t$ m/sec
Speed: $|\cos t - \sin t|$ m/sec
Accel.: $-\sin t - \cos t$ m/sec^2
Jerk: $-\cos t + \sin t$ m/sec^3

(b) Velocity: 0 m/sec
Speed: 0 m/sec
Accel.: $-\sqrt{2}$ m/sec^2
Jerk: 0 m/sec^3

(c) The body starts at 1, goes to $\sqrt{2}$ and then oscillates between $\pm\sqrt{2}$.
Speed:
Greatest when $t = \frac{3\pi}{4} + k\pi$
Zero when $t = \frac{\pi}{4} + k\pi$
Acceleration:
Greatest (in magnitude) when $t = \frac{\pi}{4} + k\pi$
Zero when $t = \frac{3\pi}{4} + k\pi$
Jerk:
Greatest (in magnitude) when $t = \frac{3\pi}{4} + k\pi$
Zero when $t = \frac{\pi}{4} + k\pi$

25. **(a)** The limit is $\frac{\pi}{180}$ because this is the conversion factor for changing from degrees to radians.

(b) This limit is still 0.

(c) $\frac{d}{dx}\sin x = \frac{\pi}{180}\cos x$

(d) $\frac{d}{dx}\cos x = -\frac{\pi}{180}\sin x$

(e) $\frac{d^2}{dx^2}\sin x = -\frac{\pi^2}{180^2}\sin x$
$\frac{d^3}{dx^3}\sin x = -\frac{\pi^3}{180^3}\cos x$
$\frac{d^2}{dx^2}\cos x = -\frac{\pi^2}{180^2}\cos x$
$\frac{d^3}{dx^3}\cos x = \frac{\pi^3}{180^3}\sin x$

26. $y'' = \csc^3 x + \csc x \cot^2 x$

27. $y'' = \frac{2 + 2\theta\tan\theta}{\cos^2\theta} = \frac{2\cos\theta + 2\theta\sin\theta}{\cos^3\theta}$

28. Continuous if $b = 1$, because this makes the two one-sided limits equal.
Differentiable: No, because for $b = 1$, the left-hand derivative is 1 and the right-hand derivative is 0.(The left-hand derivative does not exist for other values of b).

29. $\sin x$ **30.** $\cos x$

31. $y = x$

32. **(a)** 0.12
(b) $\sin(0.12) \approx 0.1197122$
The approximation is within 0.0003 of the actual value.

33.
$$\begin{aligned}\frac{d}{dx}\sin 2x &= \frac{d}{dx}2\sin x\cos x\\ &= 2\,[(\sin x)(-\sin x) + (\cos x)(\cos x)]\\ &= 2\,[\cos^2 x - \sin^2 x] = 2\cos 2x\end{aligned}$$

34.
$$\begin{aligned}\frac{d}{dx}\cos 2x &= \frac{d}{dx}[(\cos x)(\cos x) - (\sin x)(\sin x)]\\ &= [2(\cos x)(-\sin x) - 2(\sin x)(\cos x)]\\ &= -4\,(\sin x)(\cos x) = -2(2\sin x\cos x)\\ &= -2\sin 2x\end{aligned}$$

35.
$$\begin{aligned}\lim_{h\to 0}\frac{\cos h - 1}{h} &= \lim_{h\to 0}\frac{(\cos h - 1)(\cos h + 1)}{h(\cos h + 1)}\\ &= \lim_{h\to 0}\frac{\cos^2 h - 1}{h(\cos h + 1)}\\ &= \lim_{h\to 0}\frac{-\sin^2 h}{h(\cos h + 1)}\\ &= -\left(\lim_{h\to 0}\frac{\sin h}{h}\right)\left(\lim_{h\to 0}\frac{\sin h}{\cos h + 1}\right)\\ &= -(1)\left(\frac{0}{2}\right) = 0\end{aligned}$$

36. $A = -\frac{1}{2}, B = 0$

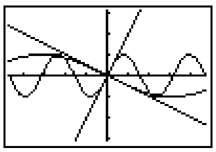
3.6 Chain Rule (pp. 141–149)

Quick Review 3.6

1. $\sin(x^2+1)$ **2.** $\sin(49x^2+1)$
3. $49x^2+1$ **4.** $7x^2+7$
5. $\sin\dfrac{x^2+1}{7x}$ **6.** $g(f(x))$
7. $g(h(f(x)))$ **8.** $h(g(f(x)))$
9. $f(h(h(x)))$ **10.** $f(g(h(x)))$

Section 3.6 Exercises

1. $3\cos(3x+1)$ **2.** $-5\cos(7-5x)$
3. $-\sqrt{3}\sin(\sqrt{3}x)$
4. $(2-3x^2)\sec^2(2x-x^3)$
5. $\dfrac{10}{x^2}\csc^2\left(\dfrac{2}{x}\right)$ **6.** $\dfrac{2\sin x}{(1+\cos x)^2}$
7. $-\sin(\sin x)\cos x$
8. $\sec(\tan x)\tan(\tan x)\sec^2 x$
9. $-2(x+\sqrt{x})^{-3}\left(1+\dfrac{1}{2\sqrt{x}}\right)$
10. $\dfrac{\csc x}{\csc x+\cot x}$
11. $-5\sin^{-6}x\cos x+3\cos^2 x\sin x$
12. $8x^3(2x-5)^3+3x^2(2x-5)^4$
$= x^2(2x-5)^3(14x-15)$
13. $4\sin^3 x\sec^2 4x+3\sin^2 x\cos x\tan 4x$
14. $2\sec x\sqrt{\sec x+\tan x}$
15. $-3(2x+1)^{-3/2}$
16. $(1+x^2)^{-3/2}$
17. $6\sin(3x-2)\cos(3x-2)=3\sin(6x-4)$
18. $-4(1+\cos 2x)\sin 2x$
19. $-42(1+\cos^2 7x)^2\cos 7x\sin 7x$
20. $\dfrac{5}{2}(\tan 5x)^{-1/2}\sec^2 5x$
21. $3\sin\left(\dfrac{\pi}{2}-3t\right)$
22. $4t\sin(\pi-4t)+\cos(\pi-4t)$
23. $\dfrac{4}{\pi}\cos 3t-\dfrac{4}{\pi}\sin 5t$
24. $\dfrac{3\pi}{2}\cos\dfrac{3\pi t}{2}-\dfrac{7\pi}{4}\sin\dfrac{7\pi t}{4}$
25. $-\sec^2(2-\theta)$
26. $2\sec^3 2\theta+2\sec 2\theta\tan^2 2\theta$
27. $\dfrac{\theta\cos\theta+\sin\theta}{2\sqrt{\theta\sin\theta}}$ **28.** $\sqrt{\sec\theta}\,(\theta\tan\theta+2)$
29. $2\sec^2 x\tan x$ **30.** $2\csc^2 x\cot x$
31. $18\csc^2(3x-1)\cot(3x-1)$
32. $2\sec^2\dfrac{x}{3}\tan\dfrac{x}{3}$ **33.** $\dfrac{5}{2}$
34. 1 **35.** $-\dfrac{\pi}{4}$
36. 5π **37.** 0
38. -8
39. **(a)** $-6\sin(6x+2)$ **(b)** $-6\sin(6x+2)$
40. **(a)** $2x\cos(x^2+1)$ **(b)** $2x\cos(x^2+1)$
41. $y=-x+2\sqrt{2}$ **42.** $y=\sqrt{3}x+2$
43. $y=-\dfrac{1}{2}x-\dfrac{1}{2}$ **44.** $y=2x-\sqrt{3}$
45. $y=x+\dfrac{1}{4}$ **46.** $y=x-4$
47. $y=\sqrt{3}x+2-\dfrac{\pi}{\sqrt{3}}$ **48.** $y=2$
49. **(a)** $\dfrac{\cos t}{2t+1}$
(b) $\dfrac{d}{dt}\left(\dfrac{dy}{dx}\right)=-\dfrac{(2t+1)(\sin t)+2\cos t}{(2t+1)^2}$
(c) $\dfrac{d}{dx}\left(\dfrac{dy}{dx}\right)=-\dfrac{(2t+1)(\sin t)+2\cos t}{(2t+1)^3}$
(d) part (c)
50. Since the radius goes through (0, 0) and $(2\cos t, 2\sin t)$, it has slope given by $\tan t$. But $\dfrac{dy}{dx}=\dfrac{dy/dt}{dx/dt}=-\dfrac{\cos t}{\sin t}=-\cot t$, which is the negative reciprocal of $\tan t$. This means that the radius and the tangent are perpendicular.
51. 5 **52.** 3
53. $\dfrac{1}{2}$ **54.** $y=mx$
55. Tangent: $y=\pi x-\pi+2$;
Normal: $y=-\dfrac{1}{\pi}x+\dfrac{1}{\pi}+2$
56. **(a)** $\dfrac{2}{3}$ **(b)** $2\pi+5$
(c) $15-8\pi$ **(d)** $\dfrac{37}{6}$
(e) -1 **(f)** $\dfrac{1}{12\sqrt{2}}$
(g) $\dfrac{5}{32}$ **(h)** $-\dfrac{5}{3\sqrt{17}}$
57. **(a)** 1 **(b)** 6
(c) 1 **(d)** $-\dfrac{1}{9}$
(e) $-\dfrac{40}{3}$ **(f)** -6
(g) $-\dfrac{4}{9}$
58. The slope of $y=\sin(2x)$ at the origin is 2. The slope of $y=-\sin\dfrac{x}{2}$ at the origin is $-\dfrac{1}{2}$. So the lines tangent to the two curves at the origin are perpendicular.



$[-4.7, 4.7]$ by $[-3.1, 3.1]$

59. Because the symbols $\frac{dy}{dx}$, $\frac{dy}{du}$, and $\frac{du}{dx}$ are not fractions. The individual symbols dy, du, and dx do not have numerical values.

60. The amplitude of the velocity is doubled.
The amplitude of the acceleration is quadrupled.
The amplitude of the jerk is multiplied by 8.

61. **(a)** On the 101st day (April 11th)
(b) About 0.637 degrees per day

62. Velocity $= \frac{2}{5}$ m/sec
acceleration $= -\frac{4}{125}$ m/sec^2

63. $\text{Acceleration} = \frac{dv}{dt} = \frac{dv}{ds}\frac{ds}{dt} = \frac{dv}{ds}v$
$= \frac{k}{2\sqrt{s}}(k\sqrt{s}) = \frac{k^2}{2}$

64. Given: $v = \frac{k}{\sqrt{s}}$
acceleration: $= \frac{dv}{dt} = \frac{dv}{ds}\frac{ds}{dt} = \frac{dv}{ds}v$
$= \frac{-k}{2s^{3/2}}\frac{k}{\sqrt{s}} = -\frac{k^2}{2s^2}$

65. $\text{Acceleration} = \frac{dv}{dt} = \frac{d\,f(x)}{dt}$
$= \left[\frac{d\,f(x)}{dx}\right]\left[\frac{dx}{dt}\right]$
$= f'(x)f(x)$

66. $\frac{dT}{du} = \frac{dT}{dL}\frac{dL}{du}$
$= \frac{\pi}{\sqrt{gL}}kL = k\pi\sqrt{\frac{L}{g}} = \frac{kT}{2}$

67. No, this does not contradict the Chain Rule. The Chain Rule states that if two functions are differentiable at the appropriate points, then their composite must also be differentiable. It does not say: If a composite is differentiable, then the functions which make up the composite must all be differentiable.

68. Yes. Either the graph of $y = g(x)$ must have a horizontal tangent at $x = 1$, or the graph of $y = f(u)$ must have a horizontal tangent at $u = g(1)$.This is because $\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$, so the slope of the tangent to the graph of $y = f(g(x)$ at $x = 1$ is given by $f'(g(1))g'(1)$. If this product is zero, then at least one of its factors must be zero.

69. As $h \to 0$, the second curve (the difference quotient) approaches the first $y = 2\cos 2x$.
This is because $2\cos 2x$ is the derivative of $\sin 2x$, and the second curve is the difference quotient used to define the derivative of $\sin 2x$. As $h \to 0$, the difference quotient expression should be approaching the derivative.

70. As $h \to 0$, the second curve (the difference quotient) approaches the first ($y = -2x\sin(x^2)$).
This is because $-2x\sin(x^2)$ is the derivative of $\cos(x^2)$, and the second curve is the difference quotient used to define the derivative of $\cos(x^2)$.
As $h \to 0$, the difference quotient expression should be approaching the derivative.

71. **(a)** Let $f(x) = |x|$.
Then $\frac{d}{dx}|u| = \frac{d}{dx}f(u) = f'(u)\frac{du}{dx}$
$= f'(u)u' = \frac{u}{|u|}u'$.
The derivative of the absolute value function is $+1$ for positive values, -1 for negative values, and undefined at 0. So $f'(u)$ should be $+1$ when $u > 0$ and -1 when $u < 0$. But this is exactly how the expression $\frac{u}{|u|}$ evaluates.

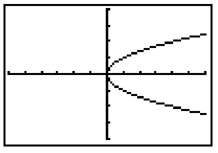
(b) $f'(x) = \frac{(2x)(x^2 - 9)}{|x^2 - 9|}$
$g'(x) = |x|\cos x + \frac{x\sin x}{|x|}$

72. $\frac{dG}{dx} = \frac{d}{dx}\sqrt{uv} = \frac{d}{dx}\sqrt{x^2 + cx} = \frac{2x + c}{2\sqrt{x^2 + cx}}$
$= \frac{x + \frac{c}{2}}{\sqrt{x^2 + cx}}$
$= \frac{A}{G}$, since $A = x + \frac{c}{2}$.

3.7 Implicit Differentiation (pp. 149–157)

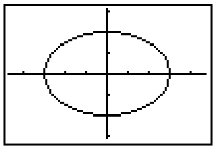
Quick Review 3.7

1. $y_1 = \sqrt{x}, y_2 = -\sqrt{x}$



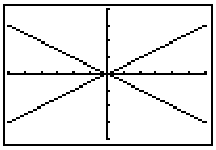
$[-6, 6]$ by $[-4, 4]$

2. $y_1 = \frac{2}{3}\sqrt{9 - x^2}, y_2 = -\frac{2}{3}\sqrt{9 - x^2}$



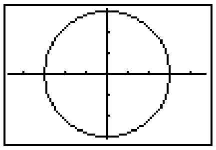
$[-4.7, 4.7]$ by $[-3.1, 3.1]$

3. $y_1 = \frac{x}{2}, y_2 = -\frac{x}{2}$



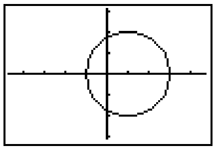
$[-6, 6]$ by $[-4, 4]$

4. $y_1 = \sqrt{9 - x^2}, y_2 = -\sqrt{9 - x^2}$



$[-4.7, 4.7]$ by $[-3.1, 3.1]$

5. $y_1 = \sqrt{2x + 3 - x^2}, y_2 = -\sqrt{2x + 3 - x^2}$



$[-4.7, 4.7]$ by $[-3.1, 3.1]$

6. $y' = \frac{4x - y + 2xy}{x^2}$ **7.** $y' = \frac{y + x\cos x}{\sin x - x}$

8. $y' = \frac{xy^2}{x^2 - y + x}$ **9.** $x^{3/2} - x^{5/6}$

10. $x^{-1/2} + x^{-5/6}$

Section 3.7 Exercises

1. $\frac{9}{4}x^{5/4}$ **2.** $-\frac{3}{5}x^{-8/5}$

3. $\frac{1}{3}x^{-2/3}$ **4.** $\frac{1}{4}x^{-3/4}$

5. $-(2x + 5)^{-3/2}$ **6.** $-4(1 - 6x)^{-1/3}$

7. $x^2(x^2 + 1)^{-1/2} + (x^2 + 1)^{1/2}$

8. $(x^2 + 1)^{-3/2}$ **9.** $-\frac{2xy + y^2}{2xy + x^2}$

10. $\frac{6y - x^2}{y^2 - 6x}$ **11.** $\frac{1}{y(x + 1)^2}$

12. $\frac{y}{x} - (x + y)^2$ or $\frac{1 - 3x^2 - 2xy}{x^2 + 1}$

13. $-\frac{1}{4}(1 - x^{1/2})^{-1/2}x^{-1/2}$ **14.** $x^{-3/2}(2x^{-1/2} + 1)^{-4/3}$

15. $-\frac{9}{2}(\csc x)^{3/2}\cot x$

16. $\frac{5}{4}[\sin(x + 5)]^{1/4}\cos(x + 5)$

17. $\cos^2 y$ **18.** $\sec y$

19. $-\frac{1}{x}\cos^2(xy) - \frac{y}{x}$ **20.** $\frac{1 - y}{x - \cos y}$

21. (b), (c), and (d) **22.** (a) and (c)

23. $\frac{dy}{dx} = -\frac{x}{y}$

$\frac{d^2y}{dx^2} = -\frac{(x^2 + y^2)}{y^3} = -\frac{1}{y^3}$

24. $\frac{dy}{dx} = -\left(\frac{y}{x}\right)^{1/3}$

$\frac{d^2y}{dx^2} = \frac{x^{2/3} + y^{2/3}}{3x^{4/3}y^{1/3}} = \frac{1}{3x^{4/3}y^{1/3}}$

25. $\frac{dy}{dx} = \frac{x + 1}{y}$

$\frac{d^2y}{dx^2} = \frac{y^2 - (x + 1)^2}{y^3} = -\frac{1}{y^3}$

26. $\frac{dy}{dx} = \frac{1}{y + 1}$

$\frac{d^2y}{dx^2} = -\frac{1}{(y + 1)^3}$

27. (a) $y = \frac{7}{4}x - \frac{1}{2}$

(b) $y = -\frac{4}{7}x + \frac{29}{7}$

28. (a) $y = \frac{3}{4}x - \frac{25}{4}$

(b) $y = -\frac{4}{3}x$

29. (a) $y = 3x + 6$

(b) $y = -\frac{1}{3}x + \frac{8}{3}$

30. (a) $y = -x - 1$

(b) $y = x + 3$

31. (a) $y = \frac{6}{7}x + \frac{6}{7}$

(b) $y = -\frac{7}{6}x - \frac{7}{6}$

32. (a) $y = 2$

(b) $x = \sqrt{3}$

33. (a) $y = -\frac{\pi}{2}x + \pi$

(b) $y = \frac{2}{\pi}x - \frac{2}{\pi} + \frac{\pi}{2}$

34. (a) $y = 2x$

(b) $y = -\frac{1}{2}x + \frac{5\pi}{8}$

35. (a) $y = 2\pi x - 2\pi$

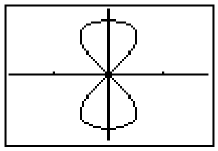
(b) $y = -\frac{x}{2\pi} + \frac{1}{2\pi}$

36. (a) $y = \pi$ **(b)** $x = 0$

37. (a) At $\left(\frac{\sqrt{3}}{4}, \frac{\sqrt{3}}{2}\right)$: Slope $= -1$;

at $\left(\frac{\sqrt{3}}{4}, \frac{1}{2}\right)$: Slope $= \sqrt{3}$

(b)



$[-1.8, 1.8]$ by $[-1.2, 1.2]$

Parameter interval:
$-1 \le t \le 1$

38. (a) Tangent: $y = 2x - 1$

normal: $y = -\frac{1}{2}x + \frac{3}{2}$

(b) One way is to graph the equations

$y = \pm\sqrt{\frac{x^3}{2 - x}}$.

39. (a) $(-1)^3(1)^2 = \cos(\pi)$ is true since both sides equal: -1.

(b) The slope is $\frac{3}{2}$.

40. (a) There are three values: 1, $\frac{-1 \pm \sqrt{5}}{2}$

(b) $f'(2) = 1, f''(2) = -4$

41. The points are $(\pm\sqrt{7}, 0)$.

$\frac{dy}{dx} = -\frac{2x + y}{2y + x}$

At both points, $\frac{dy}{dx} = -2$

42. (a) $\left(\sqrt{\frac{7}{3}}, -2\sqrt{\frac{7}{3}}\right)$ and $\left(-\sqrt{\frac{7}{3}}, 2\sqrt{\frac{7}{3}}\right)$

(b) $\left(-2\sqrt{\frac{7}{3}}, \sqrt{\frac{7}{3}}\right)$ and $\left(2\sqrt{\frac{7}{3}}, -\sqrt{\frac{7}{3}}\right)$

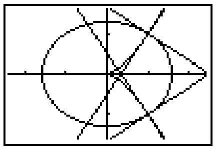
43. First curve: $\frac{dy}{dx} = -\frac{2x}{3y}$

second curve: $\frac{dy}{dx} = \frac{3x^2}{2y}$

At (1, 1), the slopes are $-\frac{2}{3}$ and $\frac{3}{2}$ respectively.

At (1, −1), the slopes are $\frac{2}{3}$ and $-\frac{3}{2}$ respectively.

In both cases, the tangents are perpendicular.



[−2.4, 2.4] by [−1.6, 1.6]

44. Velocity = 36 m/sec;

acceleration = $\frac{27}{4}$ m/sec^2

45. Acceleration = $\frac{dv}{dt} = 4(s - t)^{-1/2}(v - 1)$

$= 32$ ft/sec^2

46. At (3, 2): $\frac{27}{8}$;

at (−3, 2): $-\frac{27}{8}$;

at (−3, −2): $\frac{27}{8}$;

at (3, −2): $-\frac{27}{8}$

47. (a) At (4, 2): $\frac{5}{4}$;

at (2, 4): $\frac{4}{5}$

(b) At $(3\sqrt[3]{2}, 3\sqrt[3]{4}) \approx (3.780, 4.762)$

(c) At $(3\sqrt[3]{4}, 3\sqrt[3]{2}) \approx (4.762, 3.780)$

48. (3, −1)

49. At (−1, −1): $y = -2x - 3$;

at (3, −3): $y = -2x + 3$

50. The normal at the point (b^2, b) is:

$y = -2bx + 2b^3 + b$.

This line intersects the x-axis at $x = b^2 + \frac{1}{2}$, which must be greater than $\frac{1}{2}$ if $b \neq 0$.

The two normals are perpendicular when $a = \frac{3}{4}$.

51. (a) $\frac{dy}{dx} = -\frac{b^2x}{a^2y}$

The tangent line is $y - y_1 = -\frac{b^2x_1}{a^2y_1}(x - x_1)$.

This gives: $a^2y_1y - a^2y_1^{\,2} = -b^2x_1x + b^2x_1^{\,2}$,

$a^2y_1y + b^2x_1x = a^2y_1^{\,2} + b^2x_1^{\,2}$.

But $a^2y_1^{\,2} + b^2x_1^{\,2} = a^2b^2$ since (x_1, y_1) is on the ellipse.

Therefore, $a^2y_1y + b^2x_1x = a^2b^2$, and dividing by a^2b^2 gives $\frac{x_1x}{a^2} + \frac{y_1y}{b^2} = 1$.

(b) $\frac{x_1x}{a^2} - \frac{y_1y}{b^2} = 1$.

52. (a) Solve for y.

(b) Because the limit of $\frac{f(x)}{g(x)}$ as $x \to \infty$ is 1.

(c) Because the limit of $\frac{f(x)}{g(x)}$ as $x \to \infty$ is 1.

3.8 Derivatives of Inverse Trigonometric Functions (pp. 157–163)

Quick Review 3.8

1. Domain: [−1, 1]
Range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
At 1: $\frac{\pi}{2}$

2. Domain: [−1, 1]
Range: $[0, \pi]$
At 1: 0

3. Domain: all reals
Range: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
At 1: $\frac{\pi}{4}$

4. Domain: $(-\infty, -1] \cup [1, \infty)$
Range: $\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$
At 1: 0

5. Domain: all reals
Range: all reals
At 1: 1

6. $f^{-1}(x) = \frac{x + 8}{3}$

7. $f^{-1}(x) = x^3 - 5$

8. $f^{-1}(x) = \frac{8}{x}$

9. $f^{-1}(x) = \frac{2}{3 - x}$

10. $f^{-1}(x) = 3\tan x$, $-\frac{\pi}{2} < x < \frac{\pi}{2}$

Section 3.8 Exercises

1. $-\frac{2x}{\sqrt{1 - x^4}}$

2. $\frac{1}{|x|\sqrt{x^2 - 1}}$

3. $\frac{\sqrt{2}}{\sqrt{1-2t^2}}$

4. $-\frac{1}{\sqrt{2t-t^2}}$

5. $\frac{1}{|2s+1|\sqrt{s^2+s}}$

6. $\frac{1}{|s|\sqrt{25s^2-1}}$

7. $-\frac{2}{(x^2+1)\sqrt{x^2+2}}$

8. $-\frac{2}{|x|\sqrt{x^2-4}}$

9. $-\frac{1}{\sqrt{1-t^2}}$

10. $-\frac{6}{t\sqrt{t^4-9}}$

11. $-\frac{1}{2\sqrt{t}(t+1)}$

12. $-\frac{1}{2t\sqrt{t-1}}$

13. $-\frac{2s^2}{\sqrt{1-s^2}}$

14. $\frac{s|s|-1}{|s|\sqrt{s^2-1}}$

15. $0, x > 1$

16. $0, x \neq 0$

17. $\sin^{-1} x$

18. $-\frac{2}{(\sin^{-1} 2x)^2\sqrt{1-4x^2}}$

19. (a) $y = 2x - \frac{\pi}{2} + 1$

(b) $y = \frac{1}{2}x - \frac{1}{2} + \frac{\pi}{4}$

20. (a) $f(1) = 3, f'(1) = 12$

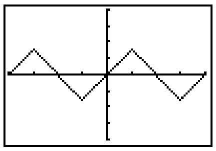
(b) $f^{-1}(3) = 1, (f^{-1})'(3) = \frac{1}{12}$

21. (a) $f'(x) = 3 - \sin x$ and $f'(x) \neq 0$. So f has a differentiable inverse by Theorem 3.

(b) $f(0) = 1, f'(0) = 3$

(c) $f^{-1}(1) = 0, (f^{-1})'(1) = \frac{1}{3}$

22.



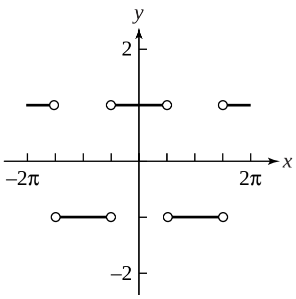
$[-2\pi, 2\pi]$ by $[-4, 4]$

(a) All reals

(b) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

(c) At the points $x = k\frac{\pi}{2}$, where k is an odd integer.

(d)



(e) $f'(x) = \frac{\cos x}{\sqrt{1-\sin^2 x}}$, which is ± 1 depending on whether $\cos x$ is positive or negative.

23. (a) $v(t) = \frac{dx}{dt} = \frac{1}{1+t^2}$ which is always positive.

(b) $a(t) = \frac{dv}{dt} = -\frac{2t}{(1+t^2)^2}$ which is always negative.

(c) $\frac{\pi}{2}$

24.
$$\frac{d}{dx}\cos^{-1}(x) = \frac{d}{dx}\left(\frac{\pi}{2} - \sin^{-1} x\right) = 0 - \frac{d}{dx}\sin^{-1} x = -\frac{1}{\sqrt{1-x^2}}$$

25.
$$\frac{d}{dx}\cot^{-1} x = \frac{d}{dx}\left(\frac{\pi}{2} - \tan^{-1} x\right) = 0 - \frac{d}{dx}\tan^{-1} x = -\frac{1}{1+x^2}$$

26.
$$\frac{d}{dx}\csc^{-1}(x) = \frac{d}{dx}\left(\frac{\pi}{2} - \sec^{-1} x\right) = 0 - \frac{d}{dx}\sec^{-1} x = -\frac{1}{|x|\sqrt{x^2-1}}$$

27. (a) $y = \frac{\pi}{2}$ **(b)** $y = -\frac{\pi}{2}$
(c) None

28. (a) $y = 0$ **(b)** $y = \pi$
(c) None

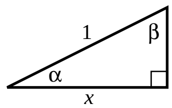
29. (a) $y = \frac{\pi}{2}$ **(b)** $y = \frac{\pi}{2}$
(c) None

30. (a) $y = 0$ **(b)** $y = 0$
(c) None

31. (a) None **(b)** None
(c) None

32. (a) None **(b)** None
(c) None

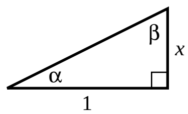
33. (a)



$\alpha = \cos^{-1} x, \beta = \sin^{-1} x$

So $\frac{\pi}{2} = \alpha + \beta = \cos^{-1} x + \sin^{-1} x$.

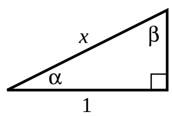
(b)



$\alpha = \tan^{-1} x, \beta = \cot^{-1} x$

So $\frac{\pi}{2} = \alpha + \beta = \tan^{-1} x + \cot^{-1} x$.

(c)



$\alpha = \sec^{-1} x, \beta = \csc^{-1} x$

So $\frac{\pi}{2} = \alpha + \beta = \sec^{-1} x + \csc^{-1} x$.

34. The "straight angle" with the arrows in it is the sum of three angles. Call them A, B, and C, moving clockwise from the upper left to the lower right.
A is equal to $\tan^{-1} 3$ since the opposite side is 3 times as long as the adjacent side.
B is equal to $\tan^{-1} 2$ since the side opposite it is 2 units and the adjacent side is one unit.
C is equal to $\tan^{-1} 1$ since both the opposite and adjacent sides are one unit long.
But the sum of these three angles is the "straight angle," which has measure π radians.

35. If s is the length of a side of the square, and let α, β, γ denote the angles labeled $\tan^{-1} 1$, $\tan^{-1} 2$, and $\tan^{-1} 3$, respectively.

$\tan \alpha = \frac{s}{s} = 1$, so $\alpha = \tan^{-1} 1$ and

$\tan \beta = \frac{s}{\frac{s}{2}} = 2$, so $\beta = \tan^{-1} 2$.

$\gamma = \pi - \alpha - \beta = \pi - \tan^{-1} 1 - \tan^{-1} 2$

$= \tan^{-1} 3$.

3.9 Derivatives of Exponential and Logarithmic Functions (pp. 163–171)

Quick Review 3.9

1. $\frac{\ln 8}{\ln 5}$ **2.** $e^{x \ln 7}$

3. $\tan x$ **4.** $\ln (x - 2)$

5. $3x - 15$ **6.** $\frac{5}{4}$

7. $\ln (4x^4)$ **8.** $x = \frac{\ln 19}{\ln 3} \approx 2.68$

9. $x = \frac{\ln 18 - \ln (\ln 5)}{\ln 5} \approx 1.50$

10. $x = \frac{\ln 3}{\ln 2 - \ln 3} \approx -2.71$

Section 3.9 Exercises

1. $2e^x$ **2.** $2e^{2x}$

3. $-e^{-x}$ **4.** $-5e^{-5x}$

5. $\frac{2}{3}e^{2x/3}$ **6.** $-\frac{1}{4}e^{-x/4}$

7. $e^2 - e^x$ **8.** $x^2e^x + xe^x - e^x$

9. $\frac{e^{\sqrt{x}}}{2\sqrt{x}}$ **10.** $2xe^{(x^2)}$

11. $\pi x^{\pi-1}$ **12.** $(1 + \sqrt{2})x^{\sqrt{2}}$

13. $-\sqrt{2}x^{-\sqrt{2}-1}$ **14.** $(1 - e)x^{-e}$

15. $8^x \ln 8$ **16.** $-9^{-x} \ln 9$

17. $-3^{\csc x}(\ln 3)(\csc x \cot x)$

18. $-3^{\cot x}(\ln 3)(\csc^2 x)$ **19.** $\frac{2\, x^{\ln x} \ln x}{x}$

20. $0, x > 0$ **21.** $\frac{2}{x}$

22. $\frac{2 \ln x}{x}$ **23.** $-\frac{1}{x}, x > 0$

24. $-\frac{1}{x}, x > 0$ **25.** $\frac{1}{x+2}, x > -2$

26. $\frac{1}{x+1}, x > -1$ **27.** $\frac{\sin x}{2 - \cos x}$

28. $\frac{2x}{x^2 + 1}$ **29.** $\frac{1}{x \ln x}$

30. $\ln x$ **31.** $\frac{2}{x \ln 4} = \frac{1}{x \ln 2}$

32. $\frac{1}{2x \ln 5}, x > 0$ **33.** $\frac{3}{(3x + 1) \ln 2}, x > -\frac{1}{3}$

34. $\frac{1}{2(x + 1) \ln 10}, x > -1$ **35.** $-\frac{1}{x \ln 2}, x > 0$

36. $-\frac{1}{x (\ln 2)(\log_2 x)^2}$ **37.** $\frac{1}{x}, x > 0$

38. $\frac{1}{1 + x \ln 3}, x > -\frac{1}{\ln 3}$ **39.** $\frac{1}{\ln 10}$

40. $\ln 10$ **41.** $y = ex$

42. $y = -x$

43. $(\sin x)^x [x \cot x + \ln (\sin x)]$

44. $x^{\tan x}\left[\frac{\tan x}{x} + (\ln x)(\sec^2 x)\right]$

45.
$$\left(\frac{(x - 3)^4(x^2 + 1)}{(2x + 5)^3}\right)^{1/5}\left(\frac{4}{5(x - 3)} + \frac{2x}{5(x^2 + 1)} - \frac{6}{5(2x + 5)}\right)$$

46. $\left(\frac{x\sqrt{x^2 + 1}}{(x + 1)^{2/3}}\right)\left(\frac{1}{x} + \frac{x}{x^2 + 1} - \frac{2}{3(x + 1)}\right)$

47. rate ≈ 0.098 grams/day

48. (a) $\frac{d}{dx} \ln (kx) = \frac{1}{kx} \frac{d}{dx} kx = \frac{k}{kx} = \frac{1}{x}$

(b) $\frac{d}{dx} \ln (kx) = \frac{d}{dx} (\ln k + \ln x)$
$= 0 + \frac{d}{dx} \ln x = \frac{1}{x}$

49. (a) $\ln 2$ **(b)** $f'(0) = \lim_{h\to 0} \frac{2^h - 1}{h}$

(c) $\ln 2$ **(d)** $\ln 7$

50. Recall that a point (a, b) is on the graph of $y = e^x$ if and only if the point (b, a) is on the graph of $y = \ln x$. Since there are points (x, e^x) on the graph of $y = e^x$ with arbitrarily large x-coordinates, there will be points $(x, \ln x)$ on the graph of $y = \ln x$ with arbitrarily large y-coordinates.

51. (a) The graph of y_4 is a horizontal line at $y = a$.

(b) The graph of y_3 is a horizontal line at $y = \ln a$.

(c) $\frac{d}{dx} a^x = a^x$ if and only if $y_3 = \frac{y_2}{y_1} = 1$.
So if $y_3 = \ln a$, then $\frac{d}{dx} a^x$ will equal a^x if and only if $\ln a = 1$, or $a = e$.

51. continued

(d) $y_2 = \frac{d}{dx} a^x = a^x \ln a$. This will equal $y_1 = a^x$ if and only if $\ln a = 1$, or $a = e$.

52. $\frac{d}{dx}\left(-\frac{1}{2}x^2 + k\right) = -x$ and $\frac{d}{dx}(\ln x + c) = \frac{1}{x}$. Therefore, at any given value of x, these two curves will have perpendicular tangent lines.

53. (a) $y = \frac{1}{e}x$

(b) Because the graph of $\ln x$ lies below the graph of the line for all positive $x \neq e$.

(c) Multiplying by e, $e(\ln x) < x$, or $\ln x^e < x$.

(d) Exponentiate both sides of the inequality in (c).

(e) Let $x = \pi$ to see that $\pi^e < e^\pi$.

Chapter 3 Review Exercises (pp. 172–175)

1. $5x^4 - \frac{x}{4} + \frac{1}{4}$

2. $-21x^2 + 21x^6$

3. $-2\cos^2 x + 2\sin^2 x = 2\cos 2x$

4. $-\frac{4}{(2x-1)^2}$

5. $2\sin(1 - 2t)$

6. $\frac{2}{t^2}\csc^2\frac{2}{t}$

7. $\frac{1}{2\sqrt{x}} - \frac{1}{2x^{3/2}}$

8. $\frac{3x+1}{\sqrt{2x+1}}$

9. $3\sec(1 + 3\theta)\tan(1 + 3\theta)$

10. $-4\theta\tan(3 - \theta^2)\sec^2(3 - \theta^2)$

11. $-5x^2\csc 5x\cot 5x + 2x\csc 5x$

12. $\frac{1}{2x}$, $x > 0$

13. $\frac{e^x}{1 + e^x}$

14. $-xe^{-x} + e^{-x}$

15. e

16. $\cot x$, where x is in an interval of the form $(k\pi, (k+1)\pi)$, k even

17. $-\frac{1}{\cos^{-1}x\sqrt{1-x^2}}$

18. $\frac{2}{\theta\ln 2}$

19. $\frac{1}{(t-7)\ln 5}$, $t > 7$

20. $-8^{-t}\ln 8$

21. $\frac{2(\ln x)x^{\ln x}}{x}$

22. $\frac{(2 \cdot 2^x)[x^3\ln 2 + x\ln 2 + 1]}{(x^2+1)^{3/2}}$ or

$\frac{(2x)2^x}{\sqrt{x^2+1}}\left(\frac{1}{x} + \ln 2 + \frac{x}{x^2+1}\right)$

23. $\frac{e^{\tan^{-1}}}{1+x^2}$

24. $-\frac{u}{\sqrt{u^2-u^4}} = -\frac{u}{|u|\sqrt{1-u^2}}$

25. $\frac{t}{|t|\sqrt{t^2-1}} + \sec^{-1}t - \frac{1}{2t}$

26. $-\frac{2+2t^2}{1+4t^2} + 2t\cot^{-1}2t$

27. $\cos^{-1}z$

28. $-\frac{1}{x} + \frac{\csc^{-1}\sqrt{x}}{\sqrt{x-1}}$

29. $-\frac{\sin x}{|\sin x|} = -\text{sign}(\sin x)$, $x \neq \frac{\pi}{2}, \pi, \frac{3\pi}{2}$;

or $\begin{cases} -1, & 0 \le x < \pi, \quad x \neq \frac{\pi}{2} \\ 1, & \pi < x \le 2\pi, \quad x \neq \frac{3\pi}{2} \end{cases}$

30. $2\left(\frac{1+\sin\theta}{1-\cos\theta}\right)\left(\frac{\cos\theta - \sin\theta - 1}{(1-\cos\theta)^2}\right)$

31. For all $x \neq 0$

32. For all real x

33. For all $x < 1$

34. For all $x \neq \frac{7}{2}$

35. $-\frac{y+2}{x+3}$

36. $-\frac{1}{3}(xy)^{-1/5}$

37. $-\frac{y}{x}$ or $-\frac{1}{x^2}$

38. $\frac{1}{2y(x+1)^2}$

39. $-\frac{2x}{y^5}$

40. $-\frac{1+2xy^2}{x^4y^3}$

41. $-2\frac{(3y^2+1)^2\cos x + 12y\sin^2 x}{(3y^2+1)^3}$

42. $\frac{2}{3}x^{-4/3}y^{1/3} + \frac{2}{3}x^{-5/3}y^{2/3} = \frac{8}{3}x^{-5/3}y^{1/3}$

43. $y' = 2x^3 - 3x - 1$,

$y'' = 6x^2 - 3$,

$y''' = 12x$,

$y^{(4)} = 12$, and the rest are all zero.

44. $y' = \frac{x^4}{24}$,

$y'' = \frac{x^3}{6}$,

$y''' = \frac{x^2}{2}$,

$y^{(4)} = x$,

$y^{(5)} = 1$, and the rest are all zero.

45. (a) $y = \frac{2}{\sqrt{3}}x - \sqrt{3}$

(b) $y = -\frac{\sqrt{3}}{2}x + \frac{5\sqrt{3}}{2}$

46. (a) $y = -x + \frac{\pi}{2} + 2$

(b) $y = x - \frac{\pi}{2} + 2$

47. (a) $y = -\frac{1}{4}x + \frac{9}{4}$

(b) $y = 4x - 2$

48. (a) $y = -\frac{5}{4}x + 6$

(b) $y = \frac{4}{5}x - \frac{11}{5}$

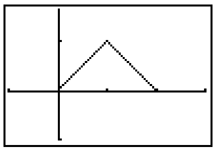
49. $y = x - 2\sqrt{2}$

50. $y = \frac{4}{3}x + 4\sqrt{2}$

51. $y = \frac{10}{3}x - 5\sqrt{3}$

52. $y = (1 + \sqrt{2})x - \sqrt{2} - 1 - \frac{\pi}{4}$
or $y \approx 2.414x - 3.200$

53. (a)



$[-1, 3]$ by $[-1, 5/3]$

(b) Yes, because both of the one-sided limits as $x \to 1$ are equal to $f(1) = 1$.

(c) No, because the left-hand derivative at $x = 1$ is $+1$ and the right-hand derivative at $x = 1$ is -1.

54. (a) The function is continuous for all values of m, because the right-hand limit as $x \to 0$ is equal to $f(0) = 0$ for any value of m.

(b) The left-hand derivative at $x = 0$ is 2, and the right-hand derivative at $x = 0$ is m, so in order for the function to be differentiable at $x = 0$, m must be 2.

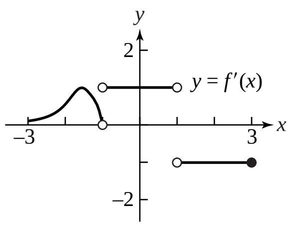
55. (a) For all $x \neq 0$ **(b)** At $x = 0$
(c) Nowhere

56. (a) For all x **(b)** Nowhere
(c) Nowhere

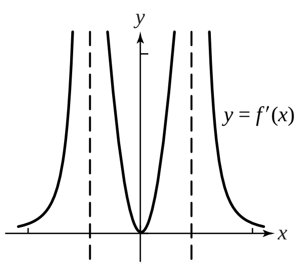
57. (a) $[-1, 0) \cup (0, 4]$ **(b)** At $x = 0$
(c) Nowhere in its domain

58. (a) $[-2, 0) \cup (0, 2]$ **(b)** Nowhere
(c) Nowhere in its domain

59.

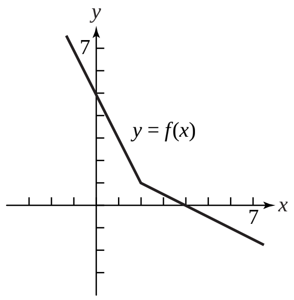


60.

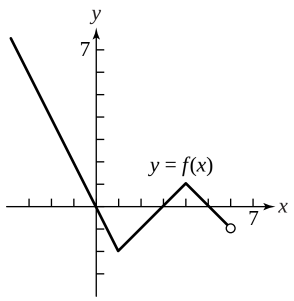


61. (a) iii **(b)** i
(c) ii

62.

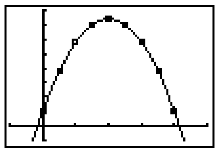


63.



64. Answer is **D**: **i** and **iii** only could be true

65. (a)

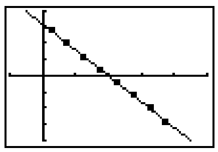


$[-1, 5]$ by $[-10, 80]$

(b)

t interval	avg. vel.
[0, 0.5]	56
[0.5, 1]	40
[1, 1.5]	24
[1.5, 2]	8
[2, 2.5]	−8
[2.5, 3]	−24
[3, 3.5]	−40
[3.5, 4]	−56

(c)



$[-1, 5]$ by $[-80, 80]$

(d) Average velocity is a good approximation to velocity.

66. (a) $-\frac{13}{10}$ **(b)** $-\frac{1}{3}$
(c) $\frac{1}{10}$ **(d)** -1
(e) $-\frac{2}{3}$ **(f)** -12

67. (a) 5 **(b)** 0
(c) 8 **(d)** 2
(e) 6 **(f)** -1

68. $\sqrt{3}$

69. $-\frac{1}{6}$

70. (a) One possible answer:
$x(t) = 10\cos\left(t + \frac{\pi}{4}\right)$
$y(t) = 1$

(b) $5\sqrt{2}$

(c) $s = -10$ and $s = 10$

(d) At $t = \frac{\pi}{4}$:
Velocity $= -10$
Speed $= 10$
Acceleration $= 0$

71. (a) $\frac{ds}{dt} = 64 - 32t$
$\frac{d^2s}{dt^2} = -32$

(b) 2 sec

(c) 64 ft/sec

(d) $\frac{64}{5.2} \approx 12.3$ sec;
$5\left(\frac{64}{5.2}\right) \approx 393.8$ ft

72. (a) $\frac{4}{7}$ sec; 280 cm/sec

(b) 560 cm/sec; 980 cm/sec^2

73. $\pi(20x - x^2)$

74. (a) $r(x) = \left(3 - \frac{x}{40}\right)^2 x = 9x - \frac{3}{20}x^2 + \frac{1}{1600}x^3$

(b) 40 people; $4.00

(c) One possible answer:
Probably not, since the company charges less overall for 60 passengers than it does for 40 passengers.

75. (a) -0.6 km/sec

(b) $\frac{18}{\pi} \approx 5.73$ revolutions/min

76. Yes

77. $y'(r) = -\frac{1}{2r^2 l}\sqrt{\frac{T}{\pi d}}$, so increasing r decreases the frequency.

$y'(l) = -\frac{1}{2rl^2}\sqrt{\frac{T}{\pi d}}$, so increasing l decreases the frequency.

$y'(d) = -\frac{1}{4rl}\sqrt{\frac{T}{\pi d^3}}$, so increasing d decreases the frequency.

$y'(T) = \frac{1}{4rl\sqrt{\pi T d}}$, so increasing T increases the frequency.

78. (a) $P(0) \approx 1.339$, so initially, one student was infected

(b) 200

(c) After 5 days, when the rate is 50 students/day

79. (a) $x \neq k\frac{\pi}{4}$, where k is an odd integer

79. continued

(b) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(c) Where it's not defined, at $x = k\frac{\pi}{4}$, k an odd integer

(d) It has period $\frac{\pi}{2}$ and continues to repeat the pattern seen in this window.

80. $-\frac{1}{3\sqrt{3}}$

Chapter 4
Applications of Derivatives

4.1 Extreme Values of Functions (pp. 177–185)

Quick Review 4.1

1. $\frac{-1}{2\sqrt{4 - x}}$

2. $\frac{3}{4}x^{-1/4}$

3. $\frac{2x}{(9 - x^2)^{3/2}}$

4. $\frac{-2x}{3(x^2 - 1)^{4/3}}$

5. $\frac{2x}{x^2 + 1}$

6. $-\frac{\sin(\ln x)}{x}$

7. $2e^{2x}$

8. 1

9. ∞

10. ∞

11. (a) 1 **(b)** 1
(c) Undefined

12. (a) $x \neq 2$

(b) $f'(x) = \begin{cases} 3x^2 - 2, & x < 2 \\ 1, & x > 2 \end{cases}$

Section 4.1 Exercises

1. Maximum at $x = b$, minimum at $x = c_2$;
Extreme Value Theorem applies, so both the max and min exist.

2. Maximum at $x = c$, minimum at $x = b$;
Extreme Value Theorem applies, so both the max and min exist.

3. Maximum at $x = c$, no minimum;
Extreme Value Theorem doesn't apply, since the function isn't defined on a closed interval.

4. No maximum, no minimum;
Extreme Value Theorem doesn't apply, since the function isn't continuous or defined on a closed interval.

5. Maximum at $x = c$, minimum at $x = a$;
Extreme Value Theorem doesn't apply, since the function isn't continuous.

6. Maximum at $x = a$, minimum at $x = c$;
Extreme Value Theorem doesn't apply, since the function isn't continuous.

7. Local minimum at $(-1, 0)$, local maximum at $(1, 0)$

8. Minima at $(-2, 0)$ and $(2, 0)$, maximum at $(0, 2)$
9. Maximum at $(0, 5)$
10. Local maximum at $(-3, 0)$, local minimum at $(2, 0)$, maximum at $(1, 2)$, minimum at $(0, -1)$
11. Maximum value is $\frac{1}{4} + \ln 4$ at $x = 4$;
minimum value is 1 at $x = 1$;
local maximum at $\left(\frac{1}{2}, 2 - \ln 2\right)$
12. Maximum value is e at $x = -1$;
minimum value is $\frac{1}{e}$ at $x = 1$.
13. Maximum value is $\ln 4$ at $x = 3$;
minimum value is 0 at $x = 0$.
14. Maximum value is 1 at $x = 0$
15. Maximum value is 1 at $x = \frac{\pi}{4}$;
minimum value is -1 at $x = \frac{5\pi}{4}$;
local minimum at $\left(0, \frac{1}{\sqrt{2}}\right)$;
local maximum at $\left(\frac{7\pi}{4}, 0\right)$
16. Local minimum at $(0, 1)$;
local maximum at $(\pi, -1)$
17. Maximum value is $3^{2/5}$ at $x = -3$;
minimum value is 0 at $x = 0$
18. Maximum value is $3^{3/5}$ at $x = 3$
19. Minimum value is 1 at $x = 2$.
20. Local maximum at
$\left(-\sqrt{\frac{2}{3}}, 4 + \frac{4\sqrt{6}}{9}\right) \approx (-0.816, 5.089)$;
Local minimum at
$\left(\sqrt{\frac{2}{3}}, 4 - \frac{4\sqrt{6}}{9}\right) \approx (0.816, 2.911)$
21. Local maximum at $(-2, 17)$;
local minimum at $\left(\frac{4}{3}, -\frac{41}{27}\right)$
22. There are none.
23. Minimum value is 0 at $x = -1$ and $x = 1$.
24. Local maximum at $(0, -1)$
25. Minimum value is 1 at $x = 0$.
26. Local minimum at $(0, 1)$
27. Maximum value is 2 at $x = 1$;
minimum value is 0 at $x = -1$ and $x = 3$.
28. Minimum value is $-\frac{115}{2}$ at $x = -3$;
local maximum at $(0, 10)$;
local minimum at $\left(1, \frac{13}{2}\right)$
29. Maximum value is $\frac{1}{2}$ at $x = 1$;
minimum value is $-\frac{1}{2}$ at $x = -1$.
30. Maximum value is $\frac{1}{2}$ at $x = 0$;
minimum value is $-\frac{1}{2}$ at $x = -2$.
31. Maximum value is 11 at $x = 5$;
minimum value is 5 on the interval $[-3, 2]$;
local maximum at $(-5, 9)$
32. Maximum value is 4 on the interval $[5, 7]$;
minimum value is -4 on the interval $[-2, 1]$.
33. Maximum value is 5 on the interval $[3, \infty)$;
minimum value is -5 on the interval $(-\infty, -2]$.
34. Minimum value is 4 on the interval $[-1, 3]$.
35. **(a)** No
(b) The derivative is defined and nonzero for $x \neq 2$. Also, $f(2) = 0$, and $f(x) > 0$ for all $x \neq 2$.
(c) No, because $(-\infty, \infty)$ is not a closed interval.
(d) The answers are the same as (a) and (b) with 2 replaced by a.
36. **(a)** No **(b)** No
(c) No
(d) Minimum value is 0 at $x = -3$, $x = 0$, and $x = 3$;
local maxima at $(-\sqrt{3}, 6\sqrt{3})$ and $(\sqrt{3}, 6\sqrt{3})$
37.

crit. pt.	derivative	extremum	value
$x = -\frac{4}{5}$	0	local max	$\frac{12}{25}10^{1/3} \approx 1.034$
$x = 0$	undefined	local min	0

38.

crit. pt.	derivative	extremum	value
$x = -1$	0	minimum	-3
$x = 0$	undefined	local max	0
$x = 1$	0	minimum	-3

39.

crit. pt.	derivative	extremum	value
$x = -2$	undefined	local max	0
$x = -\sqrt{2}$	0	minimum	-2
$x = \sqrt{2}$	0	maximum	2
$x = 2$	undefined	local min	0

40.

crit. pt.	derivative	extremum	value
$x = 0$	0	minimum	0
$x = \frac{12}{5}$	0	local max	$\frac{144}{125}15^{1/2} \approx 4.462$
$x = 3$	undefined	minimum	0

41.

crit. pt.	derivative	extremum	value
$x = 1$	undefined	minimum	2

42.

crit. pt.	derivative	extremum	value
$x = 0$	undefined	local min	3
$x = 1$	0	local max	4

43.

crit. pt.	derivative	extremum	value
$x = -1$	0	maximum	5
$x = 1$	undefined	local min	1
$x = 3$	0	maximum	5

44.

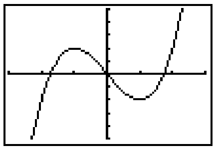
crit. pt.	derivative	extremum	value
$x = -1$	0	local max	4
$x \approx 3.155$	0	local max	≈ -3.079

45. (c) **46.** (b)
47. (d) **48.** (a)

49. (a) Maximum value is 144 at $x = 2$.
(b) The largest volume of the box is 144 cubic units and it occurs when $x = 2$.

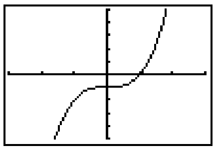
50. (a) Minimum value is 40 at $x = 10$.
(b) The smallest perimeter of the rectangle is 40 units and it occurs when $x = 10$, which makes it a 10 by 10 square.

51. (a) $f'(x) = 3ax^2 + 2bx + c$ is a quadratic, so it can have 0, 1, or 2 zeros, which would be the critical points of f. Examples:



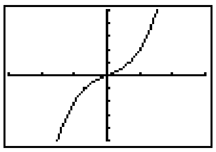
$[-3, 3]$ by $[-5, 5]$

The function $f(x) = x^3 - 3x$ has two critical points at $x = -1$ and $x = 1$.



$[-3, 3]$ by $[-5, 5]$

The function $f(x) = x^3 - 1$ has one critical point at $x = 0$.



$[-3, 3]$ by $[-5, 5]$

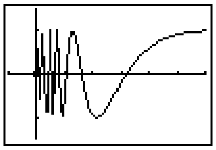
The function $f(x) = x^3 + x$ has no critical points.

(b) Two or none

52. (a) By definition of local maximum, there is an open interval containing c where $f(x) \le f(c)$, so $f(x) - f(c) \le 0$.
(b) Because $x \to c^+$, $(x - c) > 0$, and the sign of the quotient must be negative (or zero). This means the limit is nonpositive.

52. continued
(c) Because as $x \to c^-$, $(x - c) < 0$, and the sign of the quotient must be positive (or zero). This means the limit is nonnegative.
(d) Assuming that $f'(c)$ exists, the one-sided limits in (b) and (c) above must exist and be equal. Since one is nonpositive and one is nonnegative, the only possible common value is 0.
(e) There will be an open interval containing c where $f(x) - f(c) \ge 0$.
The difference quotient for the left-hand derivative will have to be negative (or zero), and the difference quotient for the right-hand derivative will have to be positive (or zero). Taking the limit, the left-hand derivative will be nonpositive, and the right-hand derivative will be nonnegative. Therefore, the only possible value for $f'(c)$ is 0.

53. (a)



$[-0.1, 0.6]$ by $[-1.5, 1.5]$

$f(0) = 0$ is not a local extreme value because in any open interval containing $x = 0$, there are infinitely many points where $f(x) = 1$ and where $f(x) = -1$.

(b) One possible answer, on the interval $[0, 1]$:

$$f(x) = \begin{cases} (1 - x)\cos \dfrac{1}{1 - x}, & 0 \le x < 1 \\ 0, & x = 1 \end{cases}$$

This function has no local extreme value at $x = 1$. Note that it is continuous on $[0, 1]$.

4.2 Mean Value Theorem (pp. 186–194)

Quick Review 4.2

1. $(-\sqrt{3}, \sqrt{3})$
2. $(-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$
3. $[-2, 2]$
4. For all x in its domain, or, $[-2, 2]$
5. On $(-2, 2)$ **6.** $x \ne \pm 1$
7. For all x in its domain, or, for all $x \ne \pm 1$
8. For all x in its domain, or, for all $x \ne \pm 1$
9. $C = 3$ **10.** $C = -4$

Section 4.2 Exercises

1. (a) Local maximum at $\left(\dfrac{5}{2}, \dfrac{25}{4}\right)$
(b) On $\left(-\infty, \dfrac{5}{2}\right]$ **(c)** On $\left[\dfrac{5}{2}, \infty\right)$

2. (a) Local minimum at $\left(\frac{1}{2}, -\frac{49}{4}\right)$

(b) On $\left[\frac{1}{2}, \infty\right)$ **(c)** On $\left(-\infty, \frac{1}{2}\right]$

3. (a) None **(b)** None

(c) On $(-\infty, 0)$ and $(0, \infty)$

4. (a) None **(b)** On $(-\infty, 0)$

(c) On $(0, \infty)$

5. (a) None **(b)** On $(-\infty, \infty)$

(c) None

6. (a) None **(b)** None

(c) On $(-\infty, \infty)$

7. (a) Local maximum at $(-2, 4)$

(b) None **(c)** On $[-2, \infty)$

8. (a) Local maximum at $(0, 9)$; local minima at $(-\sqrt{5}, -16)$ and $(\sqrt{5}, -16)$

(b) On $[-\sqrt{5}, 0]$ and $[\sqrt{5}, \infty)$

(c) On $(-\infty, -\sqrt{5}]$ and $[0, \sqrt{5}]$

9. (a) Local maximum at $\approx (2.67, 3.08)$; local minimum at $(4, 0)$

(b) On $\left(-\infty, \frac{8}{3}\right]$ **(c)** On $\left[\frac{8}{3}, 4\right]$

10. (a) Local minimum at $\approx (-2, -7.56)$

(b) On $[-2, \infty)$ **(c)** On $(-\infty, -2]$

11. (a) Local maximum at $\left(-2, \frac{1}{4}\right)$; local minimum at $\left(2, -\frac{1}{4}\right)$

(b) On $(-\infty, -2]$ and $[2, \infty)$

(c) On $[-2, 2]$

12. (a) None **(b)** None

(c) On $(-\infty, -2)$, $(-2, 2)$, and $(2, \infty)$

13. (a) Local maximum at $\approx (-1.126, -0.036)$; local minimum at $\approx (0.559, -2.639)$

(b) On $(-\infty, -1.126]$ and $[0.559, \infty)$

(c) On $[-1.126, 0.559]$

14. (a) None **(b)** On $(-\infty, \infty)$

(c) None

15. (a) f is continuous on $[0, 1]$ and differentiable on $(0, 1)$.

(b) $c = \frac{1}{2}$

16. (a) f is continuous on $[0, 1]$ and differentiable on $(0, 1)$.

(b) $c = \frac{8}{27}$

17. (a) f is continuous on $[-1, 1]$ and differentiable on $[-1, 1]$.

(b) $c \approx \pm 0.771$

18. (a) f is continuous on $[2, 4]$ and differentiable on $(2, 4)$.

(b) $c \approx 2.820$

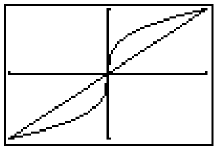
19. (a) $y = \frac{5}{2}$ **(b)** $y = 2$

20. (a) $y = \frac{1}{\sqrt{2}}x - \frac{1}{\sqrt{2}}$, or $y \approx 0.707x - 0.707$

(b) $y = \frac{1}{\sqrt{2}}x - \frac{1}{2\sqrt{2}}$, or $y \approx 0.707x - 0.354$

21. (a) Not differentiable at $x = 0$

(b)

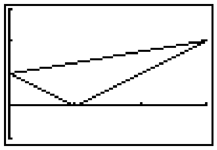


$[-1, 1]$ by $[-1, 1]$

(c) $c = \pm 3^{-3/2} \approx \pm 0.192$

22. (a) Not differentiable at $x = 1$

(b)

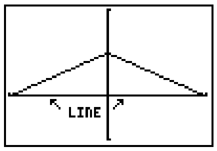


$[0, 3]$ by $[-1, 3]$

(c) There are none.

23. (a) Not differentiable at $x = 0$

(b)

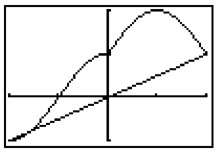


$[-1, 1]$ by $[-1, 2]$

(c) There are none

24. (a) Not differentiable at $x = 0$

(b)



$[-\pi, \pi]$ by $[-1, 2]$

(c) $c \approx -2.818, -0.324, 1.247$

25. $\frac{x^2}{2} + C$ **26.** $2x + C$

27. $x^3 - x^2 + x + C$ **28.** $-\cos x + C$

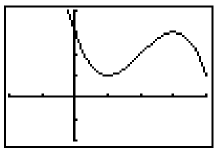
29. $e^x + C$ **30.** $\ln(x - 1) + C$

31. $\frac{1}{x} + \frac{1}{2}, x > 0$ **32.** $x^{1/4} - 3$

33. $\ln(x + 2) + 3$ **34.** $x^2 + x - \sin x + 3$

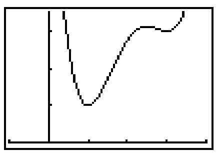
35. Possible answers:

(a)



$[-2, 4]$ by $[-2, 4]$

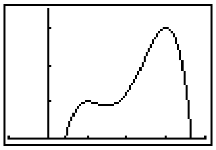
(b)



$[-1, 4]$ by $[0, 3.5]$

35. continued

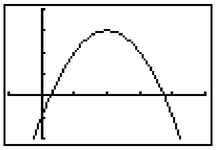
(c)



$[-1, 4]$ by $[0, 3.5]$

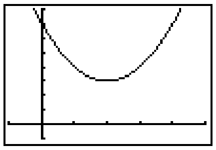
36. Possible answers:

(a)



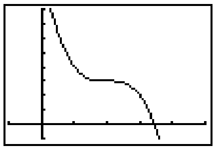
$[-1, 5]$ by $[-2, 4]$

(b)



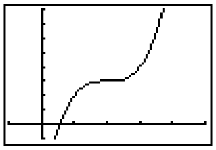
$[-1, 5]$ by $[-1, 8]$

(c)



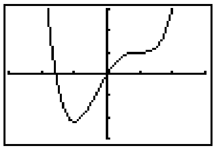
$[-1, 5]$ by $[-1, 8]$

(d)



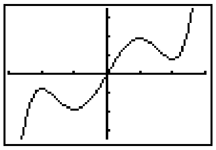
$[-1, 5]$ by $[-1, 8]$

37. One possible answer:



$[-3, 3]$ by $[-15, 15]$

38. One possible answer:



$[-3, 3]$ by $[-70, 70]$

39. Because the trucker's average speed was 79.5 mph, and by the Mean Value Theorem, the trucker must have been going that speed at least once during the trip.

40. Let $f(t)$ denote the temperature indicated after t seconds. We assume that $f'(t)$ is defined and continuous for $0 \le t \le 20$. The average rate of change is 10.6°F/sec. Therefore, by the Mean Value Theorem, $f'(c) = 10.6$°F/sec for some value of c in $[0, 20]$. Since the temperature was constant before $t = 0$, we also know that $f'(0) = 0$°F/min. But f' is continuous,so by the Intermediate Value Theorem, the rate of change $f'(t)$ must have been 10.1°F/sec at some moment during the interval.

41. Because its average speed was approximately 7.667 knots, and by the Mean Value Theorem, it must have been going that speed at least once during the trip.

42. The runner's average speed for the marathon was approximately 11.909 mph. Therefore, by the Mean Value Theorem, the runner must have been going that speed at least once during the marathon. Since the initial speed and final speed are both 0 mph and the runner's speed is continuous, by the Intermediate Value Theorem, the runner's speed must have been 11 mph at least twice.

43. **(a)** 48 m/sec **(b)** 720 meters

(c) After about 27.604 seconds, and it will be going about 48.166 m/sec

44. **(a)** 14 m/sec

(b) $10\sqrt{2}$ m/sec, or, about 14.142 m/sec

45. Because the function is not continuous on $[0, 1]$.

46. Because the Mean Value Theorem applies to the function $y = \sin x$ on any interval, and $y = \cos x$ is the derivative of $\sin x$. So, between any two zeros of $\sin x$, its derivative, $\cos x$, must be zero at least once.

47. $f(x)$ must be zero at least once between a and b by the Intermediate Value Theorem.

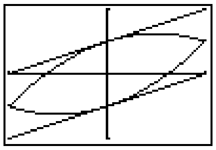
Now suppose that $f(x)$ is zero twice between a and b. Then by the Mean Value Theorem, $f'(x)$ would have to be zero at least once between the two zeros of $f(x)$, but this can't be true since we are given that $f'(x) \ne 0$ on this interval.

Therefore, $f(x)$ is zero once and only once between a and b.

48. Let $f(x) = x^4 + 3x + 1$. Then $f(x)$ is continuous and differentiable everywhere. $f'(x) = 4x^3 + 3$, which is never zero between $x = -2$ and $x = -1$. Since $f(-2) = 11$ and $f(-1) = -1$, exercise 47 applies, and $f(x)$ has exactly one zero between $x = -2$ and $x = -1$.

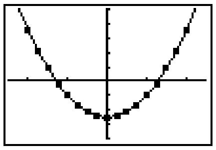
49. Let $f(x) = x + \ln(x + 1)$. Then $f(x)$ is continuous and differentiable everywhere on [0, 3].
$f'(x) = 1 + \frac{1}{x+1}$, which is never zero on [0, 3].
Now $f(0) = 0$, so $x = 0$ is one solution of the equation. If there were a second solution, $f(x)$ would be zero twice in [0, 3], and by the Mean Value Theorem, $f'(x)$ would have to be zero somewhere between the two zeros of $f(x)$. But this can't happen, since $f'(x)$ is never zero on [0, 3]. Therefore, $f(x) = 0$ has exactly one solution in the interval [0, 3].

50. Consider the function $k(x) = f(x) - g(x)$. $k(x)$ is continuous and differentiable on $[a, b]$, and since
$k(a) = f(a) - g(a) = 0$ and
$k(b) = f(b) - g(b) = 0$, by the Mean Value Theorem, there must be a point c in (a, b) where $k'(c) = 0$. But since $k'(c) = f'(c) - g'(c)$, this means that $f'(c) = g'(c)$, and c is a point where the graphs of f and g have parallel or identical tangent lines.



51. (a) Increasing: $[-2, -1.3]$ and $[1.3, 2]$;
decreasing: $[-1.3, 1.3]$;
local max: $x \approx -1.3$
local min: $x \approx 1.3$

(b) Regression equation: $y = 3x^2 - 5$



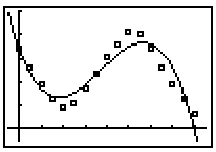
$[-2.5, 2.5]$ by $[-8, 10]$

(c) $f(x) = x^3 - 5x$

52. (a) Toward: $0 < t < 2$ and $5 < t < 8$;
away: $2 < t < 5$

(b) A local extremum in this problem is a time/place where Priya changes the direction of her motion.

(c) Regression equation:
$y = -0.0820x^3 + 0.9163x^2 - 2.5126x + 3.3779$



$[-0.5, 8.5]$ by $[-0.5, 5]$

(d) $f'(t) = -0.2459t^2 + 1.8324t - 2.5126$
toward: $0 < t < 1.81$ and $5.64 < t < 8$;
away: $1.81 < t < 5.64$

53. $\frac{f(b) - f(a)}{b - a} = \frac{\frac{1}{b} - \frac{1}{a}}{b - a} = -\frac{1}{ab}$
$f'(c) = -\frac{1}{c^2}$, so $-\frac{1}{c^2} = -\frac{1}{ab}$ and $c^2 = ab$.
Thus, $c = \sqrt{ab}$.

54. $\frac{f(b) - f(a)}{b - a} = \frac{b^2 - a^2}{b - a} = b + a$
$f'(c) = 2c$, so $2c = b + a$ and $c = \frac{a+b}{2}$.

55. By the Mean Value Theorem,
$\sin b - \sin a = (\cos c)(b - a)$ for some c between a and b. Taking the absolute value of both sides and using $|\cos c| \le 1$ gives the result.

56. Apply the Mean Value Theorem to f on $[a, b]$.
Since $f(b) < f(a)$, $\frac{f(b) - f(a)}{b - a}$ is negative, and hence $f'(x)$ must be negative at some point between a and b.

57. Let $f(x)$ be a monotonic function defined on an interval D. For any two values in D, we may let x_1 be the smaller value and let x_2 be the larger value, so $x_1 < x_2$. Then either $f(x_1) < f(x_2)$ (if f is increasing), or $f(x_1) > f(x_2)$ (if f is decreasing), which means $f(x_1) \ne f(x_2)$. Therefore, f is one-to-one.

4.3 Connecting f' and f'' with the Graph of f (pp. 194–206)

Quick Review 4.3

1. $(-3, 3)$
2. $(-2, 0) \cup (2, \infty)$
3. f: all reals
f': all reals
4. f: all reals
f': $x \ne 0$
5. f: $x \ne 2$
f': $x \ne 2$
6. f: all reals
f': $x \ne 0$
7. $y = 0$
8. $y = 0$
9. $y = 0$ and $y = 200$
10. $y = 0$ and $y = 375$

Section 4.3 Exercises

1. (a) Zero: $x = \pm 1$;
positive: $(-\infty, -1)$ and $(1, \infty)$;
negative: $(-1, 1)$

(b) Zero: $x = 0$;
positive: $(0, \infty)$;
negative: $(-\infty, 0)$

2. (a) Zero: $x \approx 0, \pm 1.25$;
positive: $(-1.25, 0)$ and $(1.25, \infty)$;
negative: $(-\infty, -1.25)$ and $(0, 1.25)$

(b) Zero: $x \approx \pm 0.7$;
positive: $(-\infty, -0.7)$ and $(0.7, \infty)$;
negative: $(-0.7, 0.7)$

3. (a) $(-\infty, -2]$ and $[0, 2]$
(b) $[-2, 0]$ and $[2, \infty)$
(c) Local maxima: $x = -2$ and $x = 2$; local minimum: $x = 0$

4. (a) $[-2, 2]$ **(b)** $(-\infty, -2]$ and $[2, \infty)$
(c) Local maximum: $x = 2$; local minimum: $x = -2$

5. (a) $[0, 1]$, $[3, 4]$, and $[5.5, 6]$
(b) $[1, 3]$ and $[4, 5.5]$
(c) Local maxima: $x = 1$, $x = 4$ (if f is continuous at $x = 4$), and $x = 6$; local minima: $x = 0$, $x = 3$, and $x = 5.5$

6. If f is continuous on the interval $[0, 3]$:
(a) $[0, 3]$ **(b)** Nowhere
(c) Local maximum: $x = 3$; local minimum: $x = 0$

7. (a) $\left[\frac{1}{2}, \infty\right)$ **(b)** $\left(-\infty, \frac{1}{2}\right]$
(c) $(-\infty, \infty)$ **(d)** Nowhere
(e) Local minimum at $\left(\frac{1}{2}, -\frac{5}{4}\right)$
(f) None

8. (a) $[0, 2]$ **(b)** $(-\infty, 0]$ and $[2, \infty)$
(c) $(-\infty, 1)$ **(d)** $(1, \infty)$
(e) Local maximum: $(2, 5)$; local minimum: $(0, -3)$
(f) At $(1, 1)$

9. (a) $[-1, 0]$ and $[1, \infty)$
(b) $(-\infty, -1]$ and $[0, 1]$
(c) $\left(-\infty, -\frac{1}{\sqrt{3}}\right)$ and $\left(\frac{1}{\sqrt{3}}, \infty\right)$
(d) $\left(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$
(e) Local maximum: $(0, 1)$; local minima: $(-1, -1)$ and $(1, -1)$
(f) $\left(\pm\frac{1}{\sqrt{3}}, -\frac{1}{9}\right)$

10. (a) $(-\infty, 0)$ and $[1, \infty)$
(b) $(0, 1]$ **(c)** $(0, \infty)$
(d) $(-\infty, 0)$ **(e)** Local minimum: $(1, e)$
(f) None

11. (a) $[-2, 2]$
(b) $[-\sqrt{8}, -2]$ and $[2, \sqrt{8}]$
(c) $(-\sqrt{8}, 0)$ **(d)** $(0, \sqrt{8})$
(e) Local maxima: $(-\sqrt{8}, 0)$ and $(2, 4)$; local minima: $(-2, -4)$ and $(\sqrt{8}, 0)$
(f) $(0, 0)$

12. (a) $(-\infty, 0)$ and $[0, \infty)$ **(b)** None
(c) $(0, \infty)$ **(d)** $(-\infty, 0)$
(e) Local minimum: $(0, 1)$
(f) None

13. (a) $(-\infty, -2]$ and $\left[-\frac{3}{2}, \infty\right)$
(b) $\left[-2, -\frac{3}{2}\right]$ **(c)** $\left(-\frac{7}{4}, \infty\right)$

13. continued
(d) $\left(-\infty, -\frac{7}{4}\right)$
(e) Local maximum: $(-2, -40)$; local minimum: $\left(-\frac{3}{2}, -\frac{161}{4}\right)$
(f) $\left(-\frac{7}{4}, -\frac{321}{8}\right)$

14. (a) $(-\infty, -0.53]$ and $[0.65, 2.88]$
(b) $[-0.53, 0.65]$ and $[2.88, \infty)$
(c) $(0, 2)$ **(d)** $(-\infty, 0)$ and $(2, \infty)$
(e) Local maxima: $(-0.53, 2.45)$ and $(2.88, 16.23)$; local minimum: $(0.65, -0.68)$
(f) $(0,1)$ and $(2, 9)$

15. (a) $(-\infty, \infty)$ **(b)** None
(c) $(-\infty, 0)$ **(d)** $(0, \infty)$
(e) None **(f)** $(0, 3)$

16. (a) None **(b)** $(-\infty, \infty)$
(c) $(0, \infty)$ **(d)** $(-\infty, 0)$
(e) None **(f)** $(0, 5)$

17. (a) $(-\infty, \infty)$ **(b)** None
(c) $(-\infty, 5 \ln 3) \approx (-\infty, 5.49)$
(d) $(5 \ln 3, \infty) \approx (5.49, \infty)$
(e) None
(f) $\left(5 \ln 3, \frac{5}{2}\right) \approx (5.49, 2.50)$

18. (a) $(-\infty, \infty)$ **(b)** None
(c) $\left(-\infty, 2 \ln \frac{5}{2}\right) \approx (-\infty, 1.83)$
(d) $\left(2 \ln \frac{5}{2}, \infty\right) \approx (1.83, \infty)$
(e) None
(f) $\left(2 \ln \frac{5}{2}, 2\right) \approx (1.83, 2)$

19. (a) $(-\infty, 1)$ **(b)** $[1, \infty)$
(c) None **(d)** $(1, \infty)$
(e) None **(f)** None

20. (a) $[0, 2\pi]$ **(b)** None
(c) $(0, 2\pi)$ **(d)** None
(e) Local maximum: $(2\pi, e^{2\pi})$ local minimum: $(0, 1)$
(f) None

21. (a) $(-\infty, -\sqrt{2}]$ and $[\sqrt{2}, \infty)$
(b) $[-\sqrt{2}, 0)$ and $(0, \sqrt{2}]$
(c) $(0, \infty)$ **(d)** $(-\infty, 0)$
(e) Local maximum: $(-\sqrt{2}, -\sqrt{2e}) \approx (-1.41, -2.33)$; local minimum: $(\sqrt{2}, \sqrt{2e}) \approx (1.41, 2.33)$
(f) None

22. (a) $[-3, -\sqrt{6}]$ and $[0, \sqrt{6}]$ or, $\approx[-3, -2.45]$ and $[0, 2.45]$
(b) $[-\sqrt{6}, 0]$ and $[\sqrt{6}, 3]$ or, $\approx[-2.45, 0]$ and $[2.45, 3]$
(c) Approximately $(-1.56, 1.56)$
(d) Approximately $(-3, -1.56)$ and $(1.56, 3)$

22. continued

(e) Local maxima: $(\pm\sqrt{6}, 6\sqrt{3})$
$\approx (\pm 2.45, 10.39)$;
local minima: $(0, 0)$ and $(\pm 3, 0)$

(f) $\approx(\pm 1.56, 6.25)$

23. (a) $(-\infty, \infty)$ **(b)** None
(c) $(-\infty, 0)$ **(d)** $(0, \infty)$
(e) None **(f)** $(0, 0)$

24. (a) $\left[0, \frac{15}{7}\right]$ **(b)** $\left[\frac{15}{7}, \infty\right)$
(c) None **(d)** $(0, \infty)$

(e) Local maximum:
$\left(\frac{15}{7}, \left(\frac{15}{7}\right)^{3/4} \cdot \frac{20}{7}\right) \approx \left(\frac{15}{7}, 5.06\right)$;
local minimum: $(0, 0)$

(f) None

25. (a) $[1, \infty)$ **(b)** $(-\infty, 1]$
(c) $(-\infty, -2)$ and $(0, \infty)$
(d) $(-2, 0)$
(e) Local minimum: $(1, -3)$
(f) $\approx (-2, 7.56)$ and $(0, 0)$

26. (a) $[0, \infty)$ **(b)** None
(c) $\left(\frac{9}{5}, \infty\right)$ **(d)** $\left(0, \frac{9}{5}\right)$
(e) Local minimum: $(0, 0)$
(f) $\approx \left(\frac{9}{5}, 5.56\right)$

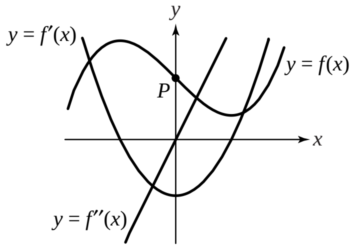
27. (a) Approximately $[0.15, 1.40]$ and $[2.45, \infty)$
(b) Approximately $(-\infty, 0.15]$, $[1.40, 2)$, and $(2, 2.45]$
(c) $(-\infty, 1)$ and $(2, \infty)$ **(d)** $(1, 2)$
(e) Local maximum: $\approx(1.40, 1.29)$;
local minima: $\approx(0.15, 0.48)$ and $(2.45, 9.22)$
(f) $(1, 1)$

28. (a) $[-1, 1]$
(b) $(-\infty, -1]$ and $[1, \infty)$
(c) $(-\sqrt{3}, 0)$ and $(\sqrt{3}, \infty)$
(d) $(-\infty, -\sqrt{3})$ and $(0, \sqrt{3})$
(e) Local maximum: $\left(1, \frac{1}{2}\right)$;
local minimum: $\left(-1, -\frac{1}{2}\right)$
(f) $(0, 0)$, $\left(\sqrt{3}, \frac{\sqrt{3}}{4}\right)$, and $\left(-\sqrt{3}, -\frac{\sqrt{3}}{4}\right)$

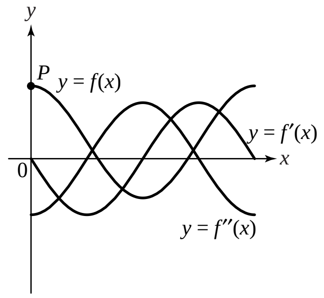
29. (a) None **(b)** At $x = 2$
(c) At $x = 1$ and $x = \frac{5}{3}$

30. (a) At $x = 2$ **(b)** At $x = 4$
(c) At $x = 1$, $x \approx 1.63$, $x \approx 3.37$

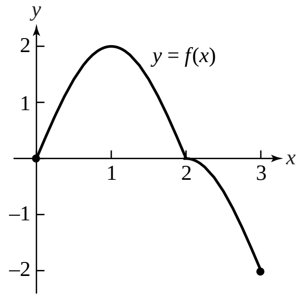
31.



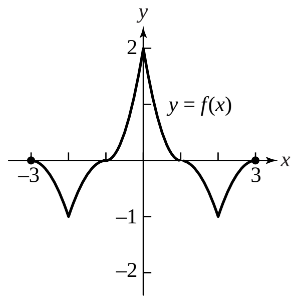
32.



33. (a) Absolute maximum at $(1, 2)$;
absolute minimum at $(3, -2)$
(b) None
(c) One possible answer:

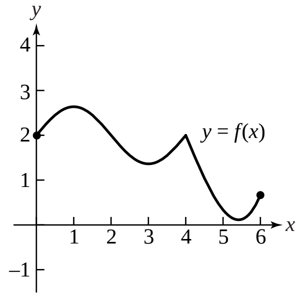


34. (a) Absolute maximum at $(0, 2)$;
absolute minimum at $(2, -1)$ and $(-2, -1)$
(b) At $(1, 0)$ and $(-1, 0)$
(c) One possible answer:

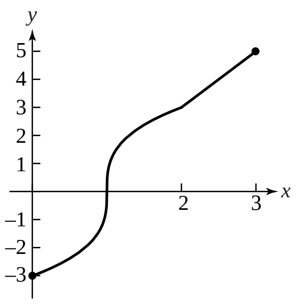


(d) $f(3) = f(-3)$, and $-1 < f(3) \leq 0$.

35.



36.



37. **(a)** $v(t) = 2t - 4$ **(b)** $a(t) = 2$

(c) It begins at position 3 moving in a negative direction. It moves to position -1 when $t = 2$, and then changes direction, moving in a positive direction thereafter.

38. **(a)** $v(t) = -2 - 2t$ **(b)** $a(t) = -2$

(c) It begins at position 6 and moves in the negative direction thereafter.

39. **(a)** $v(t) = 3t^2 - 3$ **(b)** $a(t) = 6t$

(c) It begins at position 3 moving in a negative direction. It moves to position 1 when $t = 1$, and then changes direction, moving in a positive direction thereafter.

40. **(a)** $v(t) = 6t - 6t^2$ **(b)** $a(t) = 6 - 12t$

(c) It begins at position 0. It starts moving in the positive direction until it reaches position 1 when $t = 1$, and then it changes direction. It moves in the negative direction thereafter.

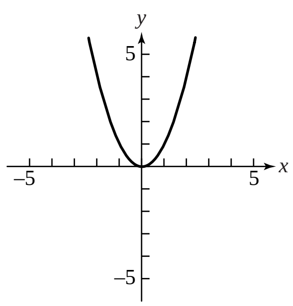
41. **(a)** $t = 2.2, 6, 9.8$ **(b)** $t = 4, 8, 11$

42. **(a)** $t = -0.2, 4, 12$ **(b)** $t = 1.5, 5.2, 8, 11, 13$

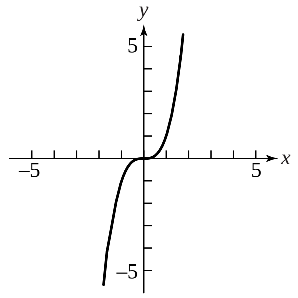
43. No. f must have a horizontal tangent line at that point, but it could be increasing (or decreasing) on both sides of the point, and there would be no local extremum.

44. No. $f''(x)$ could still be positive (or negative) on both sides of $x = c$, in which case the concavity of the function wouldn't change at $x = c$.

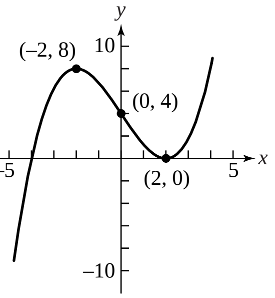
45. One possible answer:



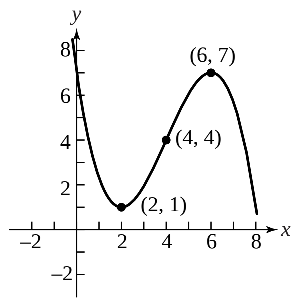
46. One possible answer:



47. One possible answer:

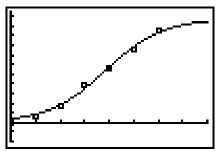


48. One possible answer:



49. **(a)** Regression equation:

$$y = \frac{2161.4541}{1 + 28.1336e^{-0.8627x}}$$



[0, 8] by [−400, 2300]

(b) At approximately $x = 3.868$ (late in 1996), when the sales are about 1081 million dollars/year

(c) 2161.45 million dollars/year

50. **(a)** In exercise 13, $a = 4$ and $b = 21$, so $-\frac{b}{3a} = -\frac{7}{4}$, which is the x-value where the point of inflection occurs. The local extrema are at $x = -2$ and $x = -\frac{3}{2}$, which are symmetric about $x = -\frac{7}{4}$.

50. continued

(b) In exercise 8, $a = -2$ and $b = 6$, so $-\frac{b}{3a} = 1$, which is the x-value where the point of inflection occurs. The local extrema are at $x = 0$ and $x = 2$, which are symmetric about $x = 1$.

(c) $f'(x) = 3ax^2 + 2bx + c$ and $f''(x) = 6ax + 2b$.
The point of inflection will occur where $f''(x) = 0$, which is at $x = -\frac{b}{3a}$.
If there are local extrema, they will occur at the zeros of $f'(x)$. Since $f'(x)$ is quadratic, its graph is a parabola and any zeros will be symmetric about the vertex which will also be where $f''(x) = 0$.

51. (a) $f'(x) = \frac{abce^{bx}}{(e^{bx} + a)^2}$, so the sign of $f'(x)$ is the same as the sign of the product abc.

(b) $f''(x) = -\frac{ab^2ce^{bx}(e^{bx} - a)}{(e^{bx} + a)^3}$. Since $a > 0$, this changes sign when $x = \frac{\ln a}{b}$ due to the $e^{bx} - a$ factor in the numerator, and there is a point of inflection at that location.

52. (a) Since $f''(x)$ is quadratic it must have 0, 1, or 2 zeros. If $f''(x)$ has 0 or 1 zeros, it will not change sign and the concavity of $f(x)$ will not change, so there is no point of inflection.
If $f''(x)$ has 2 zeros, it will change sign twice, and $f(x)$ will have 2 points of inflection.

(b) $f(x)$ has two points of inflection if and only if $3b^2 > 8ac$.

4.4 Modeling and Optimization (pp. 206–220)

Quick Review 4.4

1. None
2. Local maximum: $(-2, 17)$; local minimum: $(1, -10)$
3. $\frac{200\pi}{3}$ cm^3
4. $r \approx 4.01$ cm and $h \approx 19.82$ cm, or, $r \approx 7.13$ cm and $h \approx 6.26$ cm
5. $-\sin \alpha$
6. $\cos \alpha$
7. $\sin \alpha$
8. $-\cos \alpha$
9. $x = 1$ and $y = \sqrt{3}$, or, $x = -1$ and $y = -\sqrt{3}$
10. $x = 0$ and $y = 3$, or, $x = -\frac{24}{13}$ and $y = \frac{15}{13}$

Section 4.4 Exercises

1. (a) As large as possible: 0 and 20; as small as possible: 10 and 10

(b) As large as possible: $\frac{79}{4}$ and $\frac{1}{4}$; as small as possible: 0 and 20

2. Largest area $= \frac{25}{4}$, dimensions are $\frac{5}{\sqrt{2}}$ cm by $\frac{5}{\sqrt{2}}$ cm

3. Smallest perimeter = 16 in., dimensions are 4 in. by 4 in.

4. $A(x) = x(4 - x)$, $0 < x < 4$. $A'(x) = 4 - 2x$, so there is an absolute maximum at $x = 2$. If $x = 2$, then the length of the second side is also 2, so the rectangle with the largest area is a square.

5. (a) $y = 1 - x$ **(b)** $A(x) = 2x(1 - x)$

(c) Largest area $= \frac{1}{2}$, dimensions are 1 by $\frac{1}{2}$

6. Largest area = 32, dimensions are 4 by 8

7. Largest volume is $\frac{2450}{27} \approx 90.74$ in^3; dimensions: $\frac{5}{3}$ in. by $\frac{14}{3}$ in. by $\frac{35}{3}$ in.

8. Since $a^2 + b^2 = 400$, Area $= \frac{1}{2}a(400 - a^2)^{1/2}$.
$\frac{d}{da}\text{Area} = \frac{200 - a^2}{(400 - a^2)^{1/2}}$.
Thus the maximum area occurs when $a^2 = 200$, but then $b^2 = 200$ as well, so $a = b$.

9. Largest area = 80,000 m^2; dimensions: 200 m (perpendicular to river) by 400 m (parallel to river)

10. Dimensions: 12 m (divider is this length) by 18 m; total length required: 72 m

11. (a) 10 ft by 10 ft by 5 ft

(b) Assume that the weight is minimized when the total area of the bottom and the 4 sides is minimized.

12. (a) $x = 15$ ft and $y = 5$ ft

(b) The material for the tank costs 5 dollars/sq ft and the excavation charge is 10 dollars for each square foot of the cross-sectional area of one wall of the hole.

13. 18 in. high by 9 in. wide

14. (a) 96 ft/sec

(b) 256 feet at $t = 3$ seconds

(c) -128 ft/sec

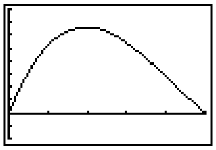
15. $\theta = \frac{\pi}{2}$

16. Radius = height $= 10\pi^{-1/3}$ cm ≈ 6.83 cm
In Example 2, because of the top on the can, the "best" design is less big around and taller.

17. $\frac{8}{\pi}$ to 1

18. (a) $V(x) = 2x^3 - 25x^2 + 75x$

(b) Domain: (0, 5)



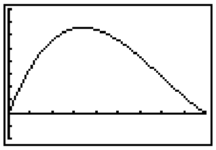
[0, 5] by [−20, 80]

(c) Maximum volume ≈ 66.02 in^3 when $x \approx 1.96$ in.

(d) $V'(x) = 6x^2 - 50x + 75$, so the critical point is at $x = \dfrac{25 - 5\sqrt{7}}{6}$, which confirms the result in (c).

19. (a) $V(x) = 2x(24 - 2x)(18 - 2x)$

(b) Domain: (0, 9)



[0, 9] by [−400, 1600]

(c) Maximum volume ≈ 1309.95 in^3 when $x \approx 3.39$ in.

(d) $V'(x) = 24x^2 - 336x + 864$, so the critical point is at $x = 7 - \sqrt{13}$, which confirms the result in (c).

(e) $x = 2$ in. or $x = 5$ in.

(f) The dimensions of the resulting box are $2x$ in., $(24 - 2x)$ in., and $(18 - 2x)$ in. Each of these measurements must be positive, so that gives the domain of (0, 9).

20. $\dfrac{4}{\sqrt{21}} \approx 0.87$ miles down the shoreline from the point nearest her boat.

21. Dimensions: width ≈ 3.44, height ≈ 2.61; maximum area ≈ 8.98

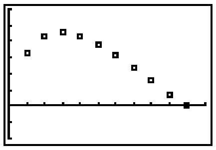
22. Dimensions: Radius $= 10\sqrt{\dfrac{2}{3}} \approx 8.16$ cm, height $= \dfrac{20}{\sqrt{3}} \approx 11.55$ cm; maximum volume $= \dfrac{4000\pi}{3\sqrt{3}} \approx 2418.40\ \text{cm}^3$

23. (a) At $x = 1$

(b)

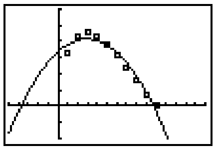
a	b	A
0.1	3.72	0.33
0.2	2.86	0.44
0.3	2.36	0.46
0.4	2.02	0.43
0.5	1.76	0.38
0.6	1.55	0.31
0.7	1.38	0.23
0.8	1.23	0.15
0.9	1.11	0.08
1.0	1.00	0.00

(c)



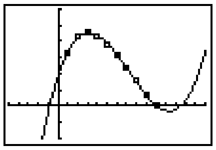
[0, 1.1] by [−0.2, 0.6]

(d) Quadratic:
$A \approx -0.91a^2 + 0.54a + 0.34$



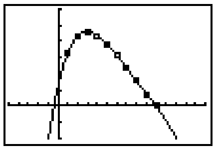
[−0.5, 1.5] by [−0.2, 0.6]

Cubic:
$A \approx 1.74a^3 - 3.78a^2 + 1.86a + 0.19$



[−0.5, 1.5] by [−0.2, 0.6]

Quartic:
$A \approx -1.92a^4 + 5.96a^3 - 6.87a^2 + 2.71a + 0.12$



[−0.5, 1.5] by [−0.2, 0.6]

(e) Quadratic: $A \approx 0.42$;
cubic: $A \approx 0.45$;
quartic: $A \approx 0.46$

24. (a) $f'(x)$ is a quadratic polynomial, and as such it can have 0, 1, or 2 zeros. If it has 0 or 1 zeros, then its sign never changes, so $f(x)$ has no local extrema.
If $f'(x)$ has 2 zeros, then its sign changes twice, and $f(x)$ has 2 local extrema at those points.

24. continued

(b) Possible answers:
No local extrema: $y = x^3$;
2 local extrema: $y = x^3 - 3x$

25. 18 in. by 18 in. by 36 in.

26. (a) $x = 12$ cm and $y = 6$ cm
(b) $x = 12$ cm and $y = 6$ cm

27. Radius $= \sqrt{2}$ m, height $= 1$ m, volume $\frac{2\pi}{3}$ m^3

28. (a) $a = 16$ **(b)** $a = -1$

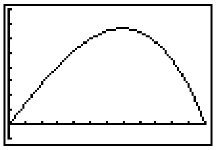
29. $f'(x) = \frac{2x^3 - a}{x^2}$, so the only sign change in $f'(x)$ occurs at $x = \left(\frac{a}{2}\right)^{1/3}$, where the sign changes from negative to positive. This means there is a local minimum at that point, and there are no local maxima.

30. (a) $a = -3$ and $b = -9$
(b) $a = -3$ and $b = -24$

31. $\frac{32\pi}{3}$ cubic units

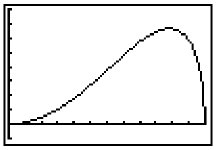
32. (a) $4\sqrt{3}$ in. wide by $4\sqrt{6}$ in. deep

(b)



[0, 12] by [−100, 800]

(c)

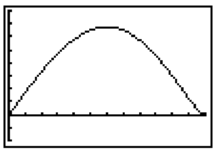


[0, 12] by [−100, 800]

Changing the value of k changes the maximum strength, but not the dimensions of the strongest beam. The graphs for different values of k look the same except that the vertical scale is different.

33. (a) 6 in. wide by $6\sqrt{3}$ in. deep

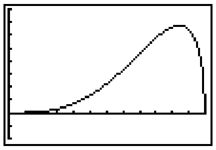
(b)



[0, 12] by [−2000, 8000]

33. continued

(c)



[0, 12] by [−2000, 8000]

$y = x^3(144 - x^2)^{1/2}$

Changing the value of k changes the maximum stiffness, but not the dimensions of the stiffest beam. The graphs for different values of k look the same except that the vertical scale is different.

34. (a) Maximum speed $= 10\pi$ cm/sec;
maximum speed is at $t = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}$ seconds;
position at those times is $s = 0$ cm (rest position);
acceleration at those times is 0 cm/sec^2

(b) The magnitude of the acceleration is greatest when the cart is at positions $s = \pm 10$ cm; The speed of the cart is 0 cm/sec at those times.

35. $2\sqrt{2}$ amps

36. The minimum distance is $\frac{\sqrt{5}}{2}$.

37. The minimum distance is 2.

38. No. It has an absolute minimum at the point $\left(\frac{1}{2}, \frac{3}{4}\right)$.

39. (a) Because $f(x)$ is periodic with period 2π.
(b) No. It has an absolute minimum at the point $(\pi, 0)$.

40. (a) Whenever t is an integer multiple of π sec.
(b) The distance is greatest when $t = \frac{2\pi}{3}$ and $\frac{4\pi}{3}$ sec. The distance at those times is $\frac{3\sqrt{3}}{2}$ m.

41. (a) At $t = \frac{\pi}{3}$ sec and at $t = \frac{4\pi}{3}$ sec
(b) The maximum distance between particles is 1 m.
(c) Near $t = \frac{\pi}{3}$ sec and near $t = \frac{4\pi}{3}$ sec

42. $\theta = \frac{\pi}{6}$

43. (a) Answers will vary. **(b)** $x = \frac{51}{8} = 6.375$ in.
(c) Minimum length ≈ 11.04 in.

44. $M = \frac{C}{2}$

45. $x = \frac{c + 100}{2} = 50 + \frac{c}{2}$

46. Let P be the foot of the perpendicular from A to the mirror, and Q be the foot of the perpendicular from B to the mirror. Suppose the light strikes the mirror at point R on the way from A to B. Let:

a = distance from A to P
b = distance from B to Q
c = distance from P to Q
x = distance from P to R

To minimize the time is to minimize the total distance the light travels going from A to B. The total distance is

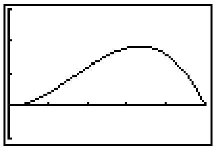
$$D(x) = (x^2 + a^2)^{1/2} + ((c - x)^2 + b^2)^{1/2}.$$

Then $D'(x) = 0$ and $D(x)$ has it minimum when $x = \frac{ac}{a + b}$, or, $\frac{x}{a} = \frac{c}{a + b}$. It follows that $c - x = \frac{bc}{a + b}$, or $\frac{c - x}{b} = \frac{c}{a + b}$. This means that the two triangles APR and BQR are similar, and the two angles must be equal.

47. The rate v is maximum when $x = \frac{a}{2}$.
The rate then is $\frac{ka^2}{4}$.

48. **(a)** $\frac{dv}{dr} = cr\,(2r_0 - 3r)$ which is zero when $r = \frac{2}{3}r_0$.

(b)



[0, 0.5] by [−0.01, 0.03]

49. 67 people

50. **(a)** $q = \sqrt{\frac{2km}{h}}$

(b) $q = \sqrt{\frac{2\text{ km}}{h}}$ (the same amount as in (a))

51. $p(x) = 6x - (x^3 - 6x^2 + 15x)$, $x \geq 0$. This function has its maximum value at the points (0, 0) and (3, 0).

52. $x = 10$ items

53. **(a)** $y'(0) = 0$ **(b)** $y'(-L) = 0$

(c) $y(0) = 0$, so $d = 0$. $y'(0) = 0$, so $c = 0$.
Then $y(-L) = -aL^3 + bL^2 = H$ and $y'(-L) = 3aL^2 - 2bL = 0$.
Solving, $a = 2\frac{H}{L^3}$ and $b = 3\frac{H}{L^2}$, which gives the equation shown.

54. **(a)** $V(x) = \frac{\pi}{3}\left(\frac{2\pi a - x}{2\pi}\right)^2\sqrt{a^2 - \left(\frac{2\pi a - x}{2\pi}\right)^2}$

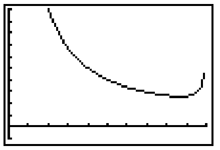
(b) When $a = 4$: $r = \frac{4\sqrt{6}}{3}$, $h = \frac{4\sqrt{3}}{3}$;
when $a = 5$: $r = \frac{5\sqrt{6}}{3}$, $h = \frac{5\sqrt{3}}{3}$;
when $a = 6$: $r = 2\sqrt{6}$, $h = 2\sqrt{3}$;
when $a = 8$: $r = \frac{8\sqrt{6}}{3}$, $h = \frac{8\sqrt{3}}{3}$

(c) $\frac{r}{h} = \sqrt{2}$

55. **(a)** The x- and y-intercepts of the line through R and T are $x - \frac{a}{f'(x)}$ and $a - x\,f'(x)$ respectively.

The area of the triangle is the product of these two values.

(b) Domain: (0, 10)



[0, 10] by [−100, 1000]

The vertical asymptotes at $x = 0$ and $x = 10$ correspond to horizontal or vertical tangent lines, which do not form triangles.

(c) Height = 15, which is 3 times the y-coordinate of the center of the ellipse.

(d) Part (a) remains unchanged.
The domain is $(0, C)$ and the graph is similar.
The minimum area occurs when $x^2 = \frac{3C^2}{4}$.
From this, it follows that the triangle has minimum area when its height is $3B$.

4.5 Linearization and Newton's Method (pp. 220–232)

Quick Review 4.5

1. $2x\cos(x^2 + 1)$

2. $\frac{1 - \cos x - (x + 1)\sin x}{(x + 1)^2}$

3. $x \approx -0.567$

4. $x \approx -0.322$

5. $y = x + 1$

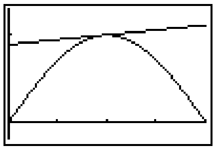
6. $y = 2ex + e + 1$

7. **(a)** $x = -1$ **(b)** $x = -\frac{e + 1}{2e} \approx -0.684$

8.

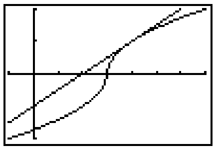
x	$f(x)$	$g(x)$
0.7	−1.457	−1.7
0.8	−1.688	−1.8
0.9	−1.871	−1.9
1.0	−2	−2
1.1	−2.069	−2.1
1.2	−2.072	−2.2
1.3	−2.003	−2.3

9.



$[0, \pi]$ by $[-0.2, 1.3]$

10.



$[-1, 7]$ by $[-2, 2]$

Section 4.5 Exercises

1. (a) $L(x) = 10x - 13$
(b) Differs from the true value in absolute value by less than 10^{-1}

2. (a) $L(x) = -\frac{4}{5}x + \frac{9}{5}$
(b) Differs from the true value in absolute value by less than 10^{-3}

3. (a) $L(x) = 2$
(b) Differs from the true value in absolute value by less than 10^{-2}

4. (a) $L(x) = x$
(b) Differs from the true value in absolute value by less than 10^{-2}

5. (a) $L(x) = x - \pi$
(b) Differs from the true value in absolute value by less than 10^{-3}

6. (a) $L(x) = -x + \frac{\pi}{2}$
(b) Differs from the true value in absolute value by less than 10^{-3}

7. $f(0) = 1$. Also, $f'(x) = k(1 + x)^{k-1}$, so $f'(0) = k$. This means the linearization at $x = 0$ is $L(x) = 1 + kx$.

8. (a) $1 - 6x$ **(b)** $2 + 2x$
(c) $1 - \frac{x}{2}$ **(d)** $\sqrt{2}\left(1 + \frac{x^2}{4}\right)$
(e) $4^{1/3}\left(1 + \frac{x}{4}\right)$ **(f)** $1 - \frac{2}{6 + 3x}$

9. The linearization is $1 + \frac{3x}{2}$. It is the sum of the two individual linearizations.

10. (a) ≈ 1.2, $|1.002^{100} - 1.2| < 10^{-1}$
(b) ≈ 1.003, $|\sqrt[3]{1.009} - 1.003| < 10^{-5}$

11. Center $= -1$, $L(x) = -5$

12. Center $= 8$, $L(x) = \frac{x}{12} + \frac{4}{3}$

13. Center $= 1$, $L(x) = \frac{x}{4} + \frac{1}{4}$, or
Center $= 1.5$, $L(x) = \frac{4x}{25} + \frac{9}{25}$

14. Center $= \frac{\pi}{2}$, $L(x) = -x + \frac{\pi}{2}$

15. $x \approx 0.682328$

16. $x \approx -1.452627, 1.164035$

17. $x \approx 0.386237, 1.961569$

18. $x \approx \pm 1.189207$

19. (a) $dy = (3x^2 - 3)\, dx$
(b) $dy = 0.45$ at the given values

20. (a) $dy = \frac{2 - 2x^2}{(1 + x^2)^2}\, dx$
(b) $dy = -0.024$ at the given values

21. (a) $dy = (2x \ln x + x)\, dx$
(b) $dy = 0.01$ at the given values

22. (a) $dy = \frac{1 - 2x^2}{(1 - x^2)^{1/2}}\, dx$
(b) $dy = -0.2$ at the given values

23. (a) $dy = (\cos x)\, e^{\sin x}\, dx$
(b) $dy = 0.1$ at the given values

24. (a) $dy = \csc\left(1 - \frac{x}{3}\right) \cot\left(1 - \frac{x}{3}\right) dx$
(b) $dy \approx 0.205525$ at the given values

25. (a) $dy = \frac{dx}{(x + 1)^2}$
(b) $dy = 0.01$ at the given values

26. (a) $dy = 2x \sec(x^2 - 1) \tan(x^2 - 1)\, dx$
(b) $dy \approx 1.431663$ at the given values

27. (a) 0.21 **(b)** 0.2
(c) 0.01

28. (a) 0.231 **(b)** 0.2
(c) 0.031

29. (a) $-\frac{2}{11}$ **(b)** $-\frac{1}{5}$
(c) $\frac{1}{55}$

30. (a) 0.04060401 **(b)** 0.04
(c) 0.00060401

31. $4\pi a^2\, dr$ **32.** $8\pi a\, dr$

33. $3a^2\, dx$ **34.** $12a\, dx$

35. $2\pi ah\, dr$ **36.** $2\pi r\, dh$

37. (a) $x + 1$ **(b)** $f(0.1) \approx 1.1$
(c) The actual value is less than 1.1, since the derivative is decreasing over the interval [0, 0.1].

38. (a) $0.08\pi \approx 0.2513$ **(b)** 2%

39. The diameter grew $\frac{2}{\pi} \approx 0.6366$ in.
The cross section area grew about 10 in^2.

40. 3%

41. The side should be measured to within 1%.

42. $180\pi \approx 565.5\ \text{in}^3$

43. The angle should be measured to within 0.76%. This is also ± 0.01 radian or $\approx \pm 0.57$ degree.

44. The height should be measured to within $\frac{1}{3}$%

45. (a) Within 0.5% **(b)** Within 5%

46. The variation of the radius should not exceed $\frac{1}{2000}$ of the ideal radius, that is, 0.05% of the ideal radius.

47. About 37.87 to 1

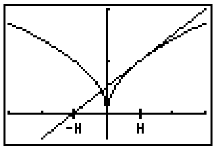
48. (a) $dT = -\pi L^{1/2} g^{-3/2}\, dg$

(b) If g increases, T decreases and the clock speeds up. This can be seen from the fact that dT and dg have opposite signs.

(c) $dg \approx -0.9765$, so $g \approx 979.0235$

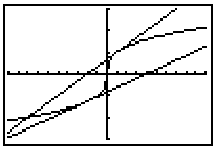
49. If $f'(x_1) \neq 0$, then x_2 and all later approximations are equal to x_1.

50. If $x_1 = h$, then $f'(x_1) = \frac{1}{2h^{1/2}}$ and $x_2 = h - \frac{h^{1/2}}{\frac{1}{2h^{1/2}}}$ $= h - 2h = -h$. If $x_1 = -h$, then $f'(x_1) = -\frac{1}{2h^{1/2}}$ and $x_2 = -h + 2h = h$.



[−3, 3] by [−0.5, 2]

51. $x_2 = -2$, $x_3 = 4$, $x_4 = -8$, and $x_5 = 16$; $|x_n| = 2^{n-1}$.



[−10, 10] by [−3, 3]

52. (a) $b_0 = f(a)$, $b_1 = f'(a)$, and $b_2 = \frac{f''(a)}{2}$.

(b) $1 + x + x^2$

(c) As one zooms in, the two graphs quickly become indistinguishable. They appear to be identical.

(d) The quadratic approximation is $1 - (x - 1) + (x - 1)^2$. As one zooms in, the two graphs quickly become indistinguishable. They appear to be identical.

(e) The quadratic approximation is $1 + \frac{x}{2} - \frac{x^2}{8}$. As one zooms in, the two graphs quickly become indistinguishable. They appear to be identical.

52. continued

(f) For f: $1 + x$;
for g: $1 - (x - 1) = 2 - x$;
for h: $1 + \frac{x}{2}$

53. Just multiply the corresponding derivative formulas by dx.

54.
$$\lim_{x\to 0} \frac{\tan x}{x} = \lim_{x\to 0} \frac{\sin x/\cos x}{x} = \lim_{x\to 0}\left(\frac{1}{\cos x}\,\frac{\sin x}{x}\right) = \left(\lim_{x\to 0}\frac{1}{\cos x}\right)\left(\lim_{x\to 0}\frac{\sin x}{x}\right) = (1)(1) = 1.$$

55. $g(a) = c$, so if $E(a) = 0$, then $g(a) = f(a)$ and $c = f(a)$. Then $E(x) = f(x) - g(x) = f(x) - f(a) - m(x - a)$. Thus, $\frac{E(x)}{x - a} = \frac{f(x) - f(a)}{x - a} - m$. $\lim_{x\to a} \frac{f(x) - f(a)}{x - a} = f'(a)$, so if the limit of $\frac{E(x)}{x - a}$ is zero, then $m = f'(a)$ and $g(x) = L(x)$.

4.6 Related Rates (pp. 232–241)

Quick Review 4.6

1. $\sqrt{74}$

2. $\sqrt{a^2 + b^2}$

3. $\frac{1 - 2y}{2x + 2y - 1}$

4. $-\frac{y + \sin y}{x + x\cos y}$

5. $2x\cos^2 y$

6. $2x + 2y - 1$

7. One possible answer: $x = -2 + 6t$, $y = 1 - 4t$, $0 \le t \le 1$.

8. One possible answer: $x = 5t$, $y = -4 + 4t$, $0 \le t \le 1$.

9. One possible answer: $\frac{\pi}{2} \le t \le \frac{3\pi}{2}$

10. One possible answer: $\frac{3\pi}{2} \le t \le 2\pi$

Section 4.6 Exercises

1. $\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

2. $\frac{dS}{dt} = 8\pi r \frac{dr}{dt}$

3. (a) $\frac{dV}{dt} = \pi r^2 \frac{dh}{dt}$ **(b)** $\frac{dV}{dt} = 2\pi rh \frac{dr}{dt}$

(c) $\frac{dV}{dt} = \pi r^2 \frac{dh}{dt} + 2\pi rh \frac{dr}{dt}$

4. (a) $\frac{dP}{dt} = 2RI\frac{dI}{dt} + I^2\frac{dR}{dt}$

(b) $0 = 2RI\frac{dI}{dt} + I^2\frac{dR}{dt}$, or,

$\frac{dR}{dt} = \left(-\frac{2R}{I}\right)\left(\frac{dI}{dt}\right) = \left(-\frac{2P}{I^3}\right)\left(\frac{dI}{dt}\right)$

5. $\frac{ds}{dt} = \frac{x\frac{dx}{dt} + y\frac{dy}{dt} + z\frac{dz}{dt}}{\sqrt{x^2 + y^2 + z^2}}$

6. $\frac{dA}{dt} = \frac{1}{2}\left(b\sin\theta\frac{da}{dt} + a\sin\theta\frac{db}{dt} + ab\cos\theta\frac{d\theta}{dt}\right)$

7. (a) 1 volt/sec **(b)** $-\frac{1}{3}$ amp/sec

(c) $\frac{dV}{dt} = I\frac{dR}{dt} + R\frac{dI}{dt}$

(d) $\frac{dR}{dt} = \frac{3}{2}$ ohms/sec. R is increasing since $\frac{dR}{dt}$ is positive.

8. π cm^2/sec

9. (a) $\frac{dA}{dt} = 14$ cm^2/sec **(b)** $\frac{dP}{dt} = 0$ cm/sec

(c) $\frac{dD}{dt} = -\frac{14}{13}$ cm/sec

(d) The area is increasing, because its derivative is positive.
The perimeter is not changing, because its derivative is zero.
The diagonal length is decreasing, because its derivative is negative.

10. (a) 2 m^3/sec **(b)** 0 m^2/sec

(c) 0 m/sec

11. $\frac{dx}{dt} = \frac{3000}{\sqrt{51}}$ mph $\approx$ 420.08 mph

12. $\frac{1}{16}$ ft/min

13. (a) 12 ft/sec **(b)** $-\frac{119}{2}$ ft^2/sec

(c) -1 radian/sec

14. 20 ft/sec

15. $\frac{19\pi}{2500} \approx 0.0239$ in^3/min

16. (a) $\frac{1125}{32\pi} \approx 11.19$ cm/min

(b) $\frac{375}{8\pi} \approx 14.92$ cm/min

17. (a) $\frac{32}{9\pi} \approx 1.13$ cm/min

(b) $-\frac{80}{3\pi} \approx -8.49$ cm/min

18. (a) $-\frac{1}{24\pi} \approx -0.01326$ m/min

or $-\frac{25}{6\pi} \approx -1.326$ cm/min

(b) $r = \sqrt{169 - (13 - y)^2} = \sqrt{26y - y^2}$

18. continued

(c) $-\frac{5}{288\pi} \approx -0.00553$ m/min

or $-\frac{125}{72\pi} \approx -0.553$ cm/min

19. $V = \frac{4}{3}\pi r^3$, so $\frac{dV}{dt} = 4\pi r^2\frac{dr}{dt}$. But $S = 4\pi r^2$, so we are given that $\frac{dV}{dt} = kS = 4k\pi r^2$. Substituting, $4k\pi r^2 = 4\pi r^2\frac{dr}{dt}$ which gives $\frac{dr}{dt} = k$.

20. (a) 1 ft/min **(b)** 40π ft^2/min

21. (a) $\frac{5}{2}$ ft/sec **(b)** $-\frac{3}{20}$ radian/sec

22. 11 ft/sec

23. (a) $\frac{dc}{dt} = 0.3$, $\frac{dr}{dt} = 0.9$, $\frac{dp}{dt} = 0.6$

(b) $\frac{dc}{dt} = -1.5625$, $\frac{dr}{dt} = 3.5$, $\frac{dp}{dt} = 5.0625$

24. (a) $\frac{10}{9\pi} \approx 0.354$ in./min

(b) $\frac{8}{5\pi} \approx 0.509$ in./min

25. $\frac{dy}{dt} = \frac{466}{1681} \approx 0.277$ L/min^2

26. 1 radian/sec **27.** $\frac{2}{5}$ radian/sec

28. -5 m/sec **29.** -3 ft/sec

30. -1500 ft/sec

31. In front: 2 radians/sec;

Half second later: 1 radian/sec

32. 1.6 cm^2/min **33.** 80 mph

34. 7.1 in./min **35.** -6 deg/sec

36. 29.5 knots

37. (a) $\frac{24}{5}$ cm/sec **(b)** 0 cm/sec

(c) $-\frac{1200}{160{,}801} \approx -0.00746$ cm/sec

38. (a) -46 cm/sec **(b)** 2 cm/sec

(c) -88 cm/sec

39. (a) The point being plotted would correspond to a point on the edge of the wheel as the wheel turns.

(b) One possible answer:
$\theta = 16\pi t$, where t is in seconds.

39. continued

(c) Assuming counterclockwise motion, the rates are as follows.

$\theta = \frac{\pi}{4}$: $\frac{dx}{dt} \approx -71.086$ ft/sec

$\frac{dy}{dt} \approx 71.086$ ft/sec

$\theta = \frac{\pi}{2}$: $\frac{dx}{dt} \approx -100.531$ ft/sec

$\frac{dy}{dt} = 0$ ft/sec

$\theta = \pi$: $\frac{dx}{dt} = 0$ ft/sec

$\frac{dy}{dt} \approx -100.531$ ft/sec

40. (a) One possible answer: $x = 30\cos\theta$, $y = 40 + 30\sin\theta$

(b) Assuming counterclockwise motion, the rates are as follows.

When $t = 5$: $\frac{dx}{dt} = 0$ ft/sec

$\frac{dy}{dt} \approx -18.850$ ft/sec

When $t = 8$: $\frac{dx}{dt} \approx 17.927$ ft/sec

$\frac{dy}{dt} \approx 5.825$ ft/sec

41. (a) 9% per year
(b) Increasing at 1% per year

Chapter 4 Review Exercises (pp. 242–245)

1. Maximum: $\frac{4\sqrt{6}}{9}$ at $x = \frac{4}{3}$; minimum: -4 at $x = -2$

2. No global extrema

3. (a) $[-1, 0)$ and $[1, \infty)$
(b) $(-\infty, -1]$ and $(0, 1]$
(c) $(-\infty, 0)$ and $(0, \infty)$
(d) None
(e) Local minima at $(1, e)$ and $(-1, e)$
(f) None

4. (a) $[-\sqrt{2}, \sqrt{2}]$
(b) $[-2, -\sqrt{2}]$ and $[\sqrt{2}, 2]$
(c) $(-2, 0)$ **(d)** $(0, 2)$
(e) Local max: $(-2, 0)$ and $(\sqrt{2}, 2)$; local min: $(2, 0)$ and $(-\sqrt{2}, -2)$
(f) $(0, 0)$

5. (a) Approximately $(-\infty, 0.385]$
(b) Approximately $[0.385, \infty)$
(c) None **(d)** $(-\infty, \infty)$
(e) Local maximum at $\approx (0.385, 1.215)$
(f) None

6. (a) $[1, \infty)$ **(b)** $(-\infty, 1]$
(c) $(-\infty, \infty)$ **(d)** None
(e) Local minimum at $(1, 0)$
(f) None

7. (a) $[0, 1)$ **(b)** $(-1, 0]$
(c) $(-1, 1)$ **(d)** None
(e) Local minimum at $(0, 1)$
(f) None

8. (a) $(-\infty, -2^{-1/3}] \approx (-\infty, -0.794]$
(b) $[-2^{-1/3}, 1) \approx [-0.794, 1)$ and $(1, \infty)$
(c) $(-\infty, -2^{1/3}) \approx (-\infty, -1.260)$ and $(1, \infty)$
(d) $(-1.260, 1)$
(e) Local maximum at $\left(-2^{-1/3}, \frac{2}{3} \cdot 2^{-1/3}\right) \approx (-0.794, 0.529)$
(f) $\left(-2^{1/3}, \frac{1}{3} \cdot 2^{1/3}\right) \approx (-1.260, 0.420)$

9. (a) None **(b)** $[-1, 1]$
(c) $(-1, 0)$ **(d)** $(0, 1)$
(e) Local maximum at $(-1, \pi)$; local minimum at $(1, 0)$
(f) $\left(0, \frac{\pi}{2}\right)$

10. (a) $[-\sqrt{3}, \sqrt{3}]$
(b) $(-\infty, -\sqrt{3}]$ and $[\sqrt{3}, \infty)$
(c) Approximately $(-2.584, -0.706)$ and $(3.290, \infty)$
(d) Approximately $(-\infty, -2.584)$ and $(-0.706, 3.290)$
(e) Local maximum at $\left(\sqrt{3}, \frac{\sqrt{3}-1}{4}\right) \approx (1.732, 0.183)$; local minimum at $\left(-\sqrt{3}, \frac{-\sqrt{3}-1}{4}\right) \approx (-1.732, -0.683)$
(f) $\approx(-2.584, -0.573)$, $(-0.706, -0.338)$, and $(3.290, 0.161)$

11. (a) $(0, 2]$ **(b)** $[-2, 0)$
(c) None **(d)** $(-2, 0)$ and $(0, 2)$
(e) Local maxima at $(-2, \ln 2)$ and $(2, \ln 2)$
(f) None

12. (a) Approximately $[0, 0.176]$, $\left[0.994, \frac{\pi}{2}\right]$, $[2.148, 2.965]$, $\left[3.834, \frac{3\pi}{2}\right]$, and $[5.591, 2\pi]$
(b) Approximately $[0.176, 0.994]$, $\left[\frac{\pi}{2}, 2.148\right]$, $[2.965, 3.834]$, and $\left[\frac{3\pi}{2}, 5.591\right]$
(c) Approximately $(0.542, 1.266)$, $(1.876, 2.600)$, $(3.425, 4.281)$, and $(5.144, 6.000)$
(d) Approximately $(0, 0.542)$, $(1.266, 1.876)$, $(2.600, 3.425)$, $(4.281, 5.144)$, and $(6.000, 2\pi)$

12. continued

(e) Local maxima at $\approx(0.176, 1.266)$, $\left(\frac{\pi}{2}, 0\right)$ and $(2.965, 1.266)$, $\left(\frac{3\pi}{2}, 2\right)$, and $(2\pi, 1)$; local minima at $\approx(0, 1)$, $(0.994, -0.513)$, $(2.148, -0.513)$, $(3.834, -1.806)$, and $(5.591, -1.806)$

Note that the local extrema at $x \approx 3.834$, $x = \frac{3\pi}{2}$, and $x \approx 5.591$ are also absolute extrema.

(f) $\approx(0.542, 0.437)$, $(1.266, -0.267)$, $(1.876, -0.267)$, $(2.600, 0.437)$, $(3.425, -0.329)$, $(4.281, 0.120)$, $(5.144, 0.120)$, and $(6.000, -0.329)$

13. (a) $\left(0, \frac{2}{\sqrt{3}}\right]$

(b) $(-\infty, 0]$ and $\left[\frac{2}{\sqrt{3}}, \infty\right)$

(c) $(-\infty, 0)$ **(d)** $(0, \infty)$

(e) Local maximum at $\left(\frac{2}{\sqrt{3}}, \frac{16}{3\sqrt{3}}\right) \approx (1.155, 3.079)$

(f) None

14. (a) Approximately $[-0.578, 1.692]$

(b) Approximately $(-\infty, -0.578]$ and $[1.692, \infty)$

(c) Approximately $(-\infty, 1.079)$

(d) Approximately $(1.079, \infty)$

(e) Local maximum at $\approx (1.692, 20.517)$; local minimum at $\approx (-0.578, 0.972)$

(f) $\approx(1.079, 13.601)$

15. (a) $\left[0, \frac{8}{9}\right]$ **(b)** $(-\infty, 0]$ and $\left[\frac{8}{9}, \infty\right)$

(c) $\left(-\infty, -\frac{2}{9}\right)$ **(d)** $\left(-\frac{2}{9}, 0\right)$ and $(0, \infty)$

(e) Local maximum at $\approx (0.889, 1.011)$; local minimum at $(0, 0)$

(f) $\approx \left(-\frac{2}{9}, 0.667\right)$

16. (a) Approximately $(-\infty, 0.215]$

(b) Approximately $[0.215, 2)$ and $(2, \infty)$

(c) Approximately $(2, 3.710)$

(d) $(-\infty, 2)$ and approximately $(3.710, \infty)$

(e) Local maximum at $\approx (0.215, -2.417)$

(f) $\approx(3.710, -3.420)$

17. (a) None **(b)** At $x = -1$

(c) At $x = 0$ and $x = 2$

18. (a) At $x = -1$ **(b)** At $x = 2$

(c) At $x = \frac{1}{2}$

19. $f(x) = -\frac{1}{4}x^{-4} - e^{-x} + C$

20. $f(x) = \sec x + C$

21. $f(x) = 2 \ln x + \frac{1}{3}x^3 + x + C$

22. $f(x) = \frac{2}{3}x^{3/2} + 2x^{1/2} + C$

23. $f(x) = -\cos x + \sin x + 2$

24. $f(x) = \frac{3}{4}x^{4/3} + \frac{x^3}{3} + \frac{x^2}{2} + x - \frac{31}{12}$

25. $s(t) = 4.9t^2 + 5t + 10$

26. $s(t) = 16t^2 + 20t + 5$

27. $L(x) = 2x + \frac{\pi}{2} - 1$

28. $L(x) = \sqrt{2}x - \frac{\pi\sqrt{2}}{4} + \sqrt{2}$

29. $L(x) = -x + 1$ **30.** $L(x) = 2x + 1$

31. Global minimum value of $\frac{1}{2}$ at $x = 2$

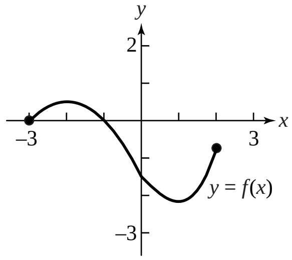
32. (a) T **(b)** P

33. (a) $(0, 2]$ **(b)** $[-3, 0)$

(c) Local maxima at $(-3, 1)$ and $(2, 3)$

34. The 24th day

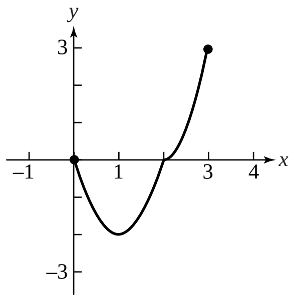
35.



36. (a) Absolute minimum is -2 at $x = 1$; absolute maximum is 3 at $x = 3$

(b) None

(c)



37. (a) $f(x)$ is continuous on $[0.5, 3]$ and differentiable on $(0.5, 3)$.

(b) $c \approx 1.579$ **(c)** $y \approx 1.457x - 1.075$

(d) $y \approx 1.457x - 1.579$

38. (a) $v(t) = -3t^2 - 6t + 4$

(b) $a(t) = -6t - 6$

(c) The particle starts at position 3 moving in the positive direction, but decelerating. At approximately $t = 0.528$, it reaches position 4.128 and changes direction, beginning to move in the negative direction. After that, it continues to accelerate while moving in the negative direction.

39. **(a)** $L(x) = -1$

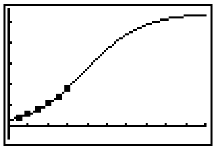
(b) Using the linearization, $f(0.1) \approx -1$

(c) Greater than the approximation in (b), since $f'(x)$ is actually positive over the interval $(0, 0.1)$ and the estimate is based on the derivative being 0.

40. **(a)** $dy = (2x - x^2)e^{-x}\,dx$

(b) $dy \approx 0.00368$

41. **(a)** $y = \dfrac{2701.73}{1 + 17.28e^{-0.36x}}$



[0, 20] by [−300, 2800]

(b) In 1998. There are approximately 1351 million transactions in that year.

(c) Approximately 2702 million transactions per year

42. $x \approx 0.828361$

43. 1200 m/sec

44. 1162.5 m

45. $r = 25$ ft and $s = 50$ ft

46. 54 square units

47. Base is 6 ft by 6 ft, height = 3 ft

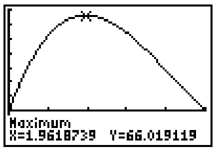
48. Base is 4 ft by 4 ft, height = 2 ft

49. Height = 2, radius = $\sqrt{2}$

50. $r = h = 4$ ft

51. **(a)** $V(x) = x(15 - 2x)(5 - x)$

(b) $0 < x < 5$



[0, 5] by [−10, 70]

(c) Maximum volume ≈ 66.019 in^3 when $x \approx 1.962$ in.

(d) $V'(x) = 6x^2 - 50x + 75$ which is zero at $x = \dfrac{25 - 5\sqrt{7}}{6} \approx 1.962.$

52. 29.925 square units

53. $x = \dfrac{48}{\sqrt{7}} \approx 18.142$ mi and $y = \dfrac{36}{\sqrt{7}} \approx 13.607$ mi

54. $x = 100$ m and $r = \dfrac{100}{\pi}$ m

55. 276 grade A and 553 grade B tires

56. **(a)** 0.765 units

(b) When $t = \dfrac{7\pi}{8} \approx 2.749$ (plus multiples of π if they keep going)

57. Dimensions: base is 6 in. by 12 in., height = 2 in.; maximum volume = 144 in^3

58. -40m^2/sec

59. 5 m/sec

60. Increasing 1 cm/min

61. $\dfrac{dx}{dt} = 4$ units/second

62. **(a)** $h = \dfrac{5r}{2}$ **(b)** $\dfrac{125}{144\pi} \approx 0.276$ ft/min

63. 5 radians/sec

64. Not enough speed. Duck!

65. $dV \approx \dfrac{2\pi ah}{3}\,dr$

66. **(a)** Within 1% **(b)** Within 3%

67. **(a)** Within 4% **(b)** Within 8%

(c) Within 12%

68. Height = 14 feet, estimated error = $\pm\dfrac{2}{45}$ feet

69. $\dfrac{dy}{dx} = 2\sin x\cos x - 3.$

Since $\sin x$ and $\cos x$ are both between 1 and -1, $2\sin x\cos x$ is never greater than 2, and therefore $\dfrac{dy}{dx} \le 2 - 3 = -1$ for all values of x.

Chapter 5
The Definite Integral

5.1 Estimating with Finite Sums (pp. 247–257)

Quick Review 5.1

1. 400 miles

2. 144 miles

3. 100 ft/sec $\approx$ 68.18 mph

4. 9.46×10^{12} km

5. 28 miles

6. 1200 gallons

7. $-3°$

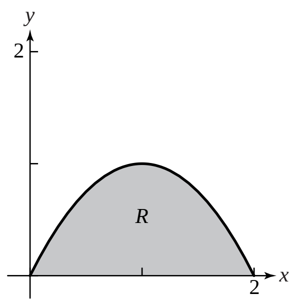
8. 25,920,000 ft^3

9. 17,500 people

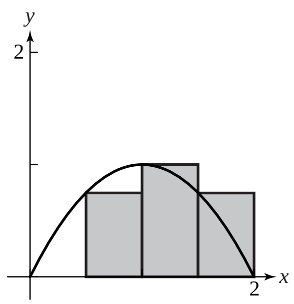
10. 176,400 times

Section 5.1 Exercises

1. **(a)**

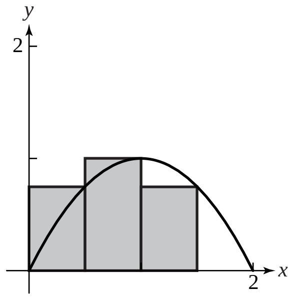


(b)

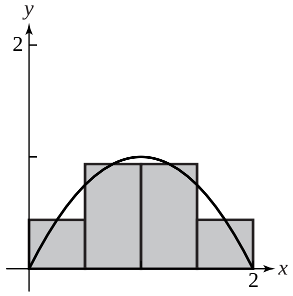


LRAM = 1.25

2.



RRAM = 1.25



LRAM = 1.25

3.

n	LRAM_n	MRAM_n	RRAM_n
10	1.32	1.34	1.32
50	1.3328	1.3336	1.3328
100	1.3332	1.3334	1.3332
500	1.333328	1.333336	1.333328

4. The area is $\frac{4}{3}$. **5.** 13.5
6. 1.0986 **7.** 0.8821
8. 2 **9.** ≈44.8; ≈6.7 L/min
10. (a) 87 in. = 7.25 ft **(b)** 87 in. = 7.25 ft
11. (a) 5220 m **(b)** 4920 m
12. 3665 ft
13. (a) 0.969 mi
(b) 0.006 h = 21.6 sec
116 mph

14.

n	MRAM
10	526.21677
20	524.25327
40	523.76240
80	523.63968
160	523.60900

15.

n	error	% error
10	2.61799	0.5
20	0.65450	0.125
40	0.16362	3.12×10^{-2}
80	0.04091	7.8×10^{-3}
160	0.01023	2×10^{-3}

16. (a) $S_8 \approx 146.08406$
Overestimate, because each rectangle extends beyond the curve.
(b) 9%
17. (a) $S_8 \approx 120.95132$
Underestimate
(b) 10%
18. (a) 372.27873 m^3 **(b)** 11%
19. (a) 15,465 ft^3 **(b)** 16,515 ft^3
20. 31.41593 **21.** 39.26991
22. (a) 74.65 ft/sec **(b)** 45.28 ft/sec
(c) 146.59 ft
23. (a) 240 ft/sec
(b) 1520 ft with RRAM and $n = 5$
24. (a) Upper: 758 gal; lower: 543 gal
(b) Upper: 2363 gal; lower: 1693 gal
(c) Worst case: 31.44 h; best case: 32.37 h
25. (a) Upper: 60.9 tons; lower: 46.8 tons
(b) By the end of October
26. The area of the region is the total number of sales, in millions of units, over the 10-year period.
The area units are
(millions of units/year)years = millions of units.
27. (a) 2 **(b)** $2\sqrt{2} \approx 2.828$
(c) $8 \sin\left(\frac{\pi}{8}\right) \approx 3.061$
(d) Each area is less than the area of the circle, π. As n increases, the polygon area approaches π.
28. False; look at $f(x) = x^2$, $0 \le x \le 1$, $n = 1$.
29. $\text{RRAM}_n f = \text{LRAM}_n f + f(x_n)\Delta x - f(x_0)\Delta x$
Since $f(a) = f(b)$, or $f(x_0) = f(x_n)$, we have $\text{RRAM}_n f = \text{LRAM}_n f$.
30. (a) $A_T = \frac{1}{2}\sin\left(\frac{2\pi}{n}\right)$
(b) $A_P = \frac{n}{2}\sin\left(\frac{2\pi}{n}\right)$
$\lim_{n\to\infty} A_P = \pi$
(c) $A_T = \frac{1}{2}r^2\sin\left(\frac{2\pi}{n}\right)$
$A_p = \frac{n}{2}r^2\sin\left(\frac{2\pi}{n}\right)$
$\lim_{n\to\infty} A_P = \pi r^2$

5.2 Definite Integrals (pp. 258–268)

Quick Review 5.2

1. 55 **2.** 20
3. 5500 **4.** $\sum_{k=1}^{99} k$
5. $\sum_{k=0}^{25} 2k$ **6.** $\sum_{k=1}^{500} 3k^2$

7. $\sum_{x=1}^{50} (2x^2 + 3x)$ **8.** $\sum_{k=0}^{20} x^k$

9. $\sum_{k=0}^{n} (-1)^k = 0$ if n is odd.

10. $\sum_{k=0}^{n} (-1)^k = 1$ if n is even.

Section 5.2 Exercises

1. $\int_0^2 x^2\,dx$ **2.** $\int_{-7}^{5} (x^2 - 3x)\,dx$

3. $\int_1^4 \frac{1}{x}\,dx$ **4.** $\int_2^3 \frac{1}{1-x}\,dx$

5. $\int_0^1 \sqrt{4 - x^2}\,dx$ **6.** $\int_{-\pi}^{\pi} \sin^3 x\,dx$

7. 15 **8.** -80

9. -480 **10.** $\frac{3\pi}{2}$

11. 2.75 **12.** 4

13. 21 **14.** 2

15. $\frac{9\pi}{2}$ **16.** 4π

17. $\frac{5}{2}$ **18.** 1

19. 3 **20.** $2 + \frac{\pi}{2}$

21. $\frac{3\pi^2}{2}$ **22.** 24

23. $\frac{1}{2}b^2$ **24.** $2b^2$

25. $b^2 - a^2$ **26.** $\frac{3}{2}(b^2 - a^2)$

27. $\frac{3}{2}a^2$ **28.** a^2

29. 0 **30.** $\frac{13}{4}$

31. $\frac{1}{4}$ **32.** $\frac{1}{2}$

33. $\frac{3}{4}$ **34.** $-\frac{1}{4}$

35. $\frac{1}{2}$ **36.** 0

37. $-\frac{3}{4}$ **38.** $\frac{3}{4}$

39. ≈ 0.9905 **40.** ≈ 4.3863

41. $\frac{32}{3}$ **42.** ≈ 1.8719

43. (a) 0 **(b)** 1

44. (a) $-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5$
(b) -88

45. (a) -1 **(b)** $-\frac{7}{2}$

46. (a) 3 **(b)** $-\frac{77}{2}$

47. (a) $f \to +\infty$

(b) Using right endpoints we have

$$\int_0^1 \frac{1}{x^2}\,dx = \lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{n}\frac{n^2}{k^2}$$
$$= \lim_{n\to\infty} \sum_{k=1}^{n} \frac{n}{k^2} = \lim_{n\to\infty} n\left[1 + \frac{1}{2^2} + \cdots + \frac{1}{n^2}\right].$$

$n\left(1 + \frac{1}{2^2} + \cdots + \frac{1}{n^2}\right) > n$ and $n \to \infty$

so $n\left(1 + \frac{1}{2^2} + \cdots + \frac{1}{n^2}\right) \to \infty.$

48. (a)
$$\text{RRAM} = \left(\frac{1}{n}\right)\cdot\frac{1}{n} + \left(\frac{2}{n}\right)^2\cdot\frac{1}{n} + \cdots + \left(\frac{n}{n}\right)^2\Bigg]\cdot\frac{1}{n}$$
$$= \sum_{k=1}^{n}\left[\left(\frac{k}{n}\right)^2\cdot\frac{1}{n}\right]$$

(b) $\sum_{k=1}^{n}\left[\left(\frac{k}{n}\right)^2\cdot\frac{1}{n}\right] = \sum_{k=1}^{n}\left(\frac{1}{n^3}\cdot k^2\right) = \frac{1}{n^3}\sum_{k=1}^{n} k^2$

(c)
$$\frac{1}{n^3}\sum_{k=1}^{n} k^2 = \frac{1}{n^3}\cdot\frac{n(n+1)(2n+1)}{6}$$
$$= \frac{n(n+1)(2n+1)}{6n^3}$$

(d)
$$\lim_{n\to\infty}\sum_{k=1}^{\infty}\left[\left(\frac{k}{n}\right)^2\cdot\frac{1}{n}\right] = \lim_{n\to\infty}\frac{n(n+1)(2n+1)}{6n^3}$$
$$= \frac{1}{3}$$

(e) Because $\int_0^1 x^2\,dx$ equals the limit of any Riemann sum as $n \to \infty$ over the interval $[0, 1]$.

5.3 Definite Integrals and Antiderivatives (pp. 268–276)

Quick Review 5.3

1. $\sin x$ **2.** $\cos x$

3. $\tan x$ **4.** $\cot x$

5. $\sec x$ **6.** $\ln(x)$

7. x^n **8.** $-\frac{2^x \ln 2}{(2^x+1)^2}$

9. $xe^x + e^x$ **10.** $\frac{1}{x^2+1}$

Section 5.3 Exercises

1. (a) 0 **(b)** -8
(c) -12 **(d)** 10
(e) -2 **(f)** 16

2. **(a)** 2 **(b)** 9
(c) −2 **(d)** 1
(e) −6 **(f)** 1

3. **(a)** 5 **(b)** $5\sqrt{3}$
(c) −5 **(d)** −5

4. **(a)** $-\sqrt{2}$ **(b)** $\sqrt{2}$
(c) $-\sqrt{2}$ **(d)** 1

5. **(a)** 4 **(b)** −4

6. **(a)** 6 **(b)** 6

7. −14 **8.** 10

9. 1 **10.** −2

11. −1 **12.** $-\frac{7}{4}$

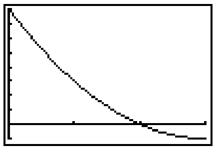
13. $\frac{\pi}{2}$ **14.** $\frac{\pi}{3}$

15. $e^2 - 1 \approx 6.389$ **16.** $6 \ln 2 \approx 4.159$

17. $\frac{16}{3}$ **18.** 16

19. $\frac{19}{3}$ **20.** $\frac{8}{3}$

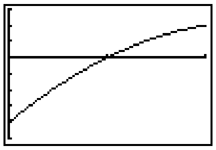
21.



[0, 3] by [−1, 8]

(a) 6 **(b)** $\frac{22}{3}$

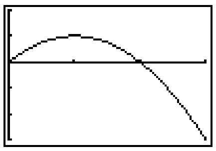
22.



[0, 2] by [−5, 3]

(a) $-\frac{2}{3}$ **(b)** 3

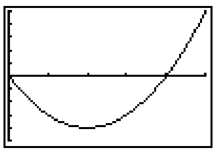
23.



[0, 3] by [−3, 2]

(a) 0 **(b)** $\frac{8}{3}$

24.



[0, 5] by [−5, 5]

(a) $-\frac{25}{3}$ **(b)** 13

25. 0, at $x = 1$ **26.** $-\frac{3}{2}$, at $x = \sqrt{3}$

27. −2, at $x = \frac{1}{\sqrt{3}}$ **28.** 1, at $x = 0$, $x = 2$

29. $\frac{3}{2}$ **30.** $\frac{4-\pi}{4}$

31. 0 **32.** 0

33. $\frac{1}{2} \le \int_0^1 \frac{1}{1+x^4}\,dx \le 1$

34. $\frac{8}{17} \le \int_0^{0.5} \frac{1}{1+x^4}\,dx \le \frac{1}{2}$

$\frac{1}{4} \le \int_{0.5}^1 \frac{1}{1+x^4}\,dx \le \frac{8}{17}$

$\frac{49}{68} \le \int_0^1 \frac{1}{1+x^4}\,dx \le \frac{33}{34}$

35. $0 \le \int_0^1 \sin(x^2)\,dx \le \sin(1) < 1$

36. $\int_0^1 \sqrt{8}\,dx \le \int_0^1 \sqrt{x+8}\,dx \le \int_0^1 \sqrt{9}\,dx$

$2\sqrt{2} \le \int_0^1 \sqrt{x+8}\,dx \le 3$

37. $0 \le (b-a)\min f(x) \le \int_a^b f(x)\,dx$

38. $\int_a^b f(x)\,dx \le (b-a)\max f(x) \le 0$

39. Yes; $av(f) = \frac{1}{b-a}\int_a^b f(x)\,dx$, therefore $\int_a^b f(x)\,dx$

$= av(f)(b-a) = \int_a^b av(f)\,dx.$

40. **(a)** 300 miles **(b)** 8 hours
(c) 37.5 mph

(d) Average speed is $\frac{\text{total distance travelled}}{\text{time}}$ for the whole trip.

$\frac{d_1 + d_2}{t_1 + t_2} \ne \frac{1}{2}\left(\frac{d_1}{t_1} + \frac{d_2}{t_2}\right)$

41. Avg rate $= \frac{\text{total amount released}}{\text{total time}}$

$= \frac{2000 \text{ m}^3}{100 \text{ min} + 50 \text{ min}} = 13\frac{1}{3} \text{ m}^3/\text{min}$

42. $\frac{1}{2}$ **43.** $\frac{7}{6}$

44. **(a)** $A = \frac{1}{2}(b)(h) = \frac{1}{2}bh$

(b) $\frac{h}{2b}x^2 + C$

(c) $\int_0^b y(x)\,dx = \frac{h}{2b}x^2\Big]_0^b = \frac{hb^2}{2b} = \frac{1}{2}bh$

45. $k \approx 2.39838$

46. $\int_a^b F'(x)\,dx = \int_a^b G'(x)\,dx \rightarrow F(b) - F(a) = G(b) - G(a)$

5.4 Fundamental Theorem of Calculus (pp. 277–288)

Quick Review 5.4

1. $2x \cos x^2$ **2.** $2 \sin x \cos x$

3. 0
4. 0
5. $2^x \ln 2$
6. $\frac{1}{2\sqrt{x}}$
7. $\frac{-x \sin x - \cos x}{x^2}$
8. $-\cot t$
9. $\frac{y+1}{2y-x}$
10. $\frac{1}{3x}$

Section 5.4 Exercises

1. $5 - \ln 6 \approx 3.208$
2. $-\frac{26}{3 \ln 3} \approx -7.889$
3. 1
4. $10\sqrt{5} \approx 22.361$
5. $\frac{5}{2}$
6. 1
7. 2
8. π
9. $2\sqrt{3}$
10. $2\sqrt{3}$
11. 0
12. 4
13. $\frac{8}{3}$
14. 0
15. $\frac{5}{2}$
16. 12
17. $\frac{1}{2}$
18. 8

19. (a) No, $f(x) = \frac{x^2-1}{x+1}$ is discontinuous at $x = -1$.
(b) $-\frac{5}{2}$. $f(x)$ is bounded with only one discontinuity. Split it up at $x = -1$, or use area.

20. (a) No, $f(x) = \frac{9-x^2}{3x-9}$ is discontinuous at $x = 3$.
(b) $-\frac{55}{6}$. See 19(b).

21. (a) No, $f(x) = \tan x$ is discontinuous at $x = \frac{\pi}{2}$ and $x = \frac{3\pi}{2}$.
(b) No, $\int_0^b \tan x\,dx \to \infty$ as $b \to \frac{\pi}{2}^-$.

22. (a) No, $f(x) = \frac{x+1}{x^2-1}$ is discontinuous at $x = 1$.
(b) No, $\int_b^2 \frac{x+1}{x^2-1}\,dx \to \infty$ as $b \to 1^+$.

23. (a) No, $f(x) = \frac{\sin x}{x}$ is discontinuous at $x = 0$.
(b) ≈ 2.55. Area is finite. $\frac{\sin x}{x}$ is bounded with only one discontinuity.

24. (a) No, $f(x) = \frac{1-\cos x}{x^2}$ is discontinuous at $x = 0$.
(b) ≈ 2.08. See 23(b).

25. $\frac{5}{6}$
26. 3
27. π
28. $\sqrt{3} - \frac{\pi}{3}$
29. ≈ 3.802
30. ≈ 1.427
31. ≈ 0.914
32. ≈ 8.886
33. $x \approx 0.699$
34. ≈ 0.883
35. $-\frac{3}{2}$
36. $\frac{\sin 2 \cos 2 - 2}{2} \approx -1.189$
37. $\sqrt{1+x^2}$
38. $-\frac{1}{x}$
39. $\frac{\sin x}{2\sqrt{x}}$
40. $2 \cos (2x)$
41. $3x^2 \cos (2x^3) - 2x \cos (2x^2)$
42. $-\sin x \cos^2 x - \sin^2 x \cos x$
43. (d)
44. (c)
45. (b)
46. (a)
47. $x = a$
48. $f(x) = 2x - 2$
49. $L(x) = 2 + 10x$
50. 1
51. $\int_0^{\pi/k} \sin kx\,dx = \frac{2}{k}$

52. (a) $\frac{125}{6}$ **(b)** $\frac{25}{4}$

53. (a) 0
(b) H is increasing on [0,6] where $H'(x) = f(x) > 0$.
(c) H is concave up on (9,12) where $H''(x) = f'(x) > 0$.
(d) $H(12) = \int_0^{12} f(t)\,dt > 0$ because there is more area above the x-axis than below for $y = f(x)$.
(e) $x = 6$ since $H'(6) = f(6) = 0$ and $H''(6) = f'(6) < 0$.
(f) $x = 0$ since $H(x) > 0$ on (0, 12].

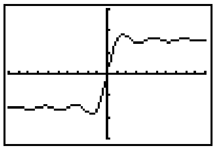
54. (a) $s'(5) = f(5) = 2$ units/sec
(b) $s''(5) = f'(5) < 0$
(c) $s(3) = \int_0^3 f(x)\,dx = \frac{1}{2}(9) = \frac{9}{2}$ units
(d) $s'(6) = f(6) = 0$, $s''(6) = f'(6) < 0$ so s has its largest value at $t = 6$ sec.
(e) $s''(t) = f'(t) = 0$ at $t = 4$ sec and $t = 7$ sec.
(f) Particle is moving toward on (6, 9) and away on (0, 6) since $s'(6) = f(t) > 0$ on (0, 6) and then $s'(6) = f(t) < 0$ on (6, 9).
(g) The positive side

55. (a) $s'(3) = f(3) = 0$ **(b)** $s''(3) = f'(3) > 0$
(c) $s(3) = \int_0^3 f(x)\,dx = -\frac{1}{2}(3)(6) = -9$ units
(d) $s(t) = 0$ at $t = 6$ sec because $\int_0^6 f(x)\,dx = 0$
(e) $s''(t) = f'(t) = 0$ at $t = 7$ sec
(f) $0 < t < 3$: $s < 0$, $s' < 0 \Rightarrow$ away
$3 < t < 6$: $s < 0$, $s' > 0 \Rightarrow$ toward
$t > 6$: $s > 0$, $s' > 0 \Rightarrow$ away
(g) The positive side

56. (a) $f(t)$ is even, so $\int_0^x f(t)\,dt = \int_{-x}^0 f(t)\,dt$, so $-\int_0^x f(t)\,dt = -\int_{-x}^0 f(t)\,dt = \int_0^{-x} f(t)\,dt$.
(b) 0 **(c)** $k\pi$, $k = \pm 1, \pm 2, \ldots$

56. continued

(d)



[−20, 20] by [−3, 3]

57. (a) \$9 **(b)** \$10

58. \$4500

59. (a) 300 drums **(b)** \$6.00

60. (a) True, $h'(x) = f(x) \Rightarrow h''(x) = f'(x)$

(b) True, h and h' are both differentiable.

(c) $h'(1) = f(1) = 0$

(d) True, $h''(1) = f'(1) < 0$ and $h'(1) = 0$

(e) False, $h''(1) = f'(1) < 0$

(f) False, $h''(1) = f'(1) \neq 0$

(g) True, $\frac{dh}{dx} = f(x) = 0$ at $x = 1$ and $h'(x) = f(x)$ is a decreasing function.

61. Using area, $\int_0^x f(t)\,dt = -\int_{-x}^0 f(t)\,dt = \int_0^{-x} f(t)\,dt$

62. Using area, $\int_0^x f(t)\,dt = \int_{-x}^0 f(t)\,dt = -\int_0^{-x} f(t)\,dt$

63. f odd $\Rightarrow \int_0^x f(t)\,dt$ is even, but $\frac{d}{dx}\int_0^x f(t)\,dt = f(x)$ so f is the derivative of an even function. Similarly for f even.

64. $x \approx 1.0648397$. $\lim_{t\to\infty} \frac{\sin t}{t} = 0$. Not enough area between x-axis and $\frac{\sin x}{x}$ for $x > 1.0648397$ to get $\int_0^x \frac{\sin t}{t}\,dt$ back to 1.

or: Si(x) doesn't decrease enough (for $x > 1.0648397$) to get back to 1.

5.5 Trapezoidal Rule (pp. 289–297)

Quick Review 5.5

1. Concave down **2.** Concave up
3. Concave down **4.** Concave down
5. Concave up **6.** Concave down
7. Concave up **8.** Concave up
9. Concave down **10.** Concave down

Section 5.5 Exercises

1. (a) 2 **(b)** Exact
(c) 2

2. (a) 2.75 **(b)** Over
(c) $\frac{8}{3}$

3. (a) 4.25 **(b)** Over
(c) 4

4. (a) 0.697 **(b)** Over
(c) $\ln 2 \approx 0.693$

5. (a) 5.146 **(b)** Under
(c) $\frac{16}{3}$

6. (a) 1.896 **(b)** Under
(c) 2

7. 15,990 ft^3

8. (a) 26,360,000 ft^3 **(b)** 988

9. 0.9785 mi

10. (a) 12 **(b)** 12, $|E_S| = 0$

(c) $f^{(4)}(x) = 0$ for $f(x) = x^3 - 2x$, so $M_{f^{(4)}} = 0$.

(d) Simpson's Rule will always give the exact value for cubic polynomials.

11. The average of the 13 discrete temperatures gives equal weight to the low values at the end.

12. (b) We are approximating the area under the temperature graph. Doubling the endpoints increases the error in the first and last trapezoids.

13. $S_{50} \approx 3.1379$, $S_{100} \approx 3.14029$

14. $S_{50} \approx 1.08943$, $S_{100} \approx 1.08943$

15. $S_{50} = 1.3706$, $S_{100} = 1.3706$ using $a = 0.0001$ as lower limit

$S_{50} = 1.37076$, $S_{100} = 1.37076$ using $a = 0.000000001$ as lower limit

16. $S_{50} \approx 0.82812$, $S_{100} \approx 0.82812$

17. (a) $T_{10} = 1.983523538$, $T_{100} = 1.999835504$
$T_{1000} = 1.999998355$

(b)

n	$\lvert E_T \rvert$
10	$0.016476462 = 1.6476462 \times 10^{-2}$
100	1.64496×10^{-4}
1000	1.645×10^{-6}

(c) $\lvert E_{T_{10n}} \rvert \approx 10^{-2} \times \lvert E_{T_n} \rvert$

(d) $\lvert E_{T_n} \rvert \le \frac{\pi^3 M}{12n^2}$. $\lvert E_{T_{10n}} \rvert \le \frac{\pi^3 M}{12(10n)^2}$
$= \frac{\pi^3 M}{12n^2} \times 10^{-2}$

18. (a) $S_{10} = 2.000109517$
$S_{100} = 2.000000011$
$S_{1000} = 2$

(b)

n	$\lvert E_S \rvert$
10	1.09517×10^{-4}
100	1.1×10^{-8}
1000	0

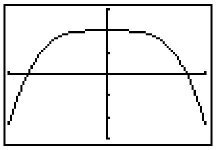
18. continued

(c) $\left|E_{S_{10n}}\right| \approx 10^{-4} \times \left|E_{S_n}\right|$

(d) $\left|E_{S_n}\right| \le \dfrac{M(\pi)^5}{180n^4}$. $\left|E_{S_{10n}}\right| \le \dfrac{M(\pi)^5}{180(10n)^4}$
$= \dfrac{M(\pi)^5}{180n^4} \times 10^{-4}$

19. (a) $f''(x) = 2\cos(x^2) - 4x^2 \sin(x^2)$

(b)



$[-1, 1]$ by $[-3, 3]$

(c) The graph shows that $-3 \le f''(x) \le 2$ for $-1 \le x \le 1$.

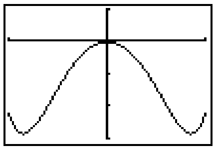
(d) $\left|E_T\right| \le \dfrac{1-(-1)}{12}(h^2)(3) = \dfrac{h^2}{2}$

(e) $\left|E_T\right| \le \dfrac{h^2}{2} \le \dfrac{0.1^2}{2} < 0.1$

(f) $n \ge 20$

20. (a) $-48x^3 \cos(x^2) + 16x^4 \sin(x^2) - 12\sin(x^2)$

(b)



$[-1, 1]$ by $[-30, 10]$

(c) The graph shows that $-30 \le f^{(4)}(x) \le 0$, for $-1 < x < 1$.

(d) $\left|E_S\right| \le \dfrac{1-(-1)}{180}(h^4)(30) = \dfrac{h^4}{3}$

(e) $\left|E_S\right| \le \dfrac{h^4}{3} \le \dfrac{0.4^4}{3} < 0.01$

(f) $n \ge 5$

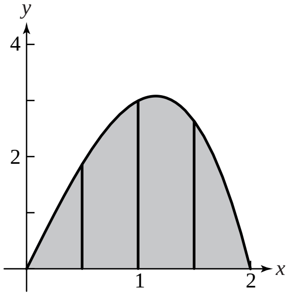
21. 466.67 in^2 **22.** 10.63 ft

23. Each quantity is equal to
$\dfrac{h}{2}(y_0 + 2y_1 + 2y_2 + \cdots + 2y_{n-1} + y_n)$.

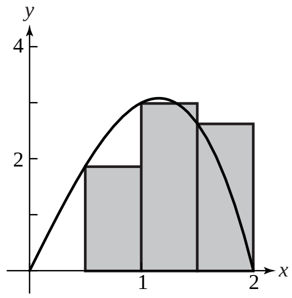
24. Each quantity is equal to
$\dfrac{h}{3}(y_0 + 4y_1 + 2y_2 + 4y_3 + \cdots + 2y_{2n-2} + 4y_{2n-1} + y_{2n})$.

Chapter 5 Review Exercises (pp. 298–301)

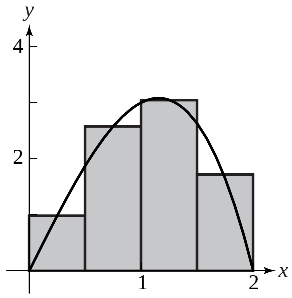
1.



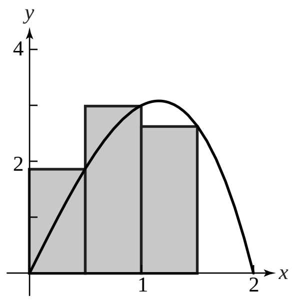
2. 3.75



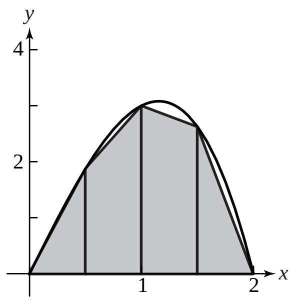
3. 4.125



4. 3.75



5. 3.75



6. 4

7.

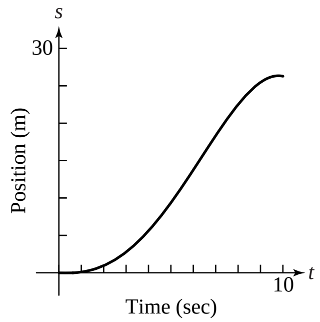
n	LRAM$_n$	MRAM$_n$	RRAM$_n$
10	1.78204	1.60321	1.46204
20	1.69262	1.60785	1.53262
30	1.66419	1.60873	1.55752
50	1.64195	1.60918	1.57795
100	1.62557	1.60937	1.59357
1000	1.61104	1.60944	1.60784

8. ln 5

9. (a) True **(b)** True
(c) False

10. (a) $V = \lim_{n\to\infty} \sum_{i=1}^{n} \pi \sin^2 (m_i)\, \Delta x$
(b) 4.9348

11. (a) 26.5 m
(b)



12. (a) $\int_0^{10} x^3\, dx$ **(b)** $\int_0^{10} x \sin x\, dx$
(c) $\int_0^{10} x(3x-2)^2\, dx$ **(d)** $\int_0^{10} (1+x^2)^{-1}\, dx$
(e) $\int_0^{10} \pi\left(9 - \sin^2 \frac{\pi x}{10}\right) dx$

13. 10 **14.** 2
15. 20 **16.** 42
17. $\frac{\sqrt{2}}{2}$ **18.** 16
19. 3 **20.** 2
21. 2 **22.** 1
23. $\sqrt{3}$ **24.** 1
25. 8 **26.** 2
27. −1 **28.** 0
29. 2 ln 3 **30.** π
31. 40 **32.** 64π

33. (a) Upper = 4.392 L; lower = 4.008 L **(b)** 4.2 L

34. (a) Lower = 87.15 ft; upper = 103.05 ft **(b)** 95.1 ft

35. One possible answer:
The *dx* is important because it corresponds to actual physical quantity Δx in a Riemann sum. Without the Δx, our integral approximations would be way off.

36. $\frac{16}{3}$

37. $1 \le \sqrt{1 + \sin^2 x} \le \sqrt{2}$

38. (a) $\frac{4}{3}$ **(b)** $\frac{2}{3}a^{3/2}$

39. $\sqrt{2 + \cos^3 x}$ **40.** $14x\sqrt{2 + \cos^3 (7x^2)}$

41. $\frac{-6}{3 + x^4}$ **42.** $\frac{2}{4x^2+1} - \frac{1}{x^2+1}$

43. \$230

44. $av(I) = 4800$ cases;
average holding cost = \$192 per day

45. $x \approx 1.63052$ or $x \approx -3.09131$

46. (a) True **(b)** True
(c) True **(d)** False
(e) True **(f)** False
(g) True

47. $F(1) - F(0)$ **48.** $y = \int_5^x \frac{\sin t}{t}\, dt + 3$

49. Use the fact that $y' = 2x + \frac{1}{x}$.

50. (b);
$\frac{dy}{dx} = 2x \to y = x^2 + c.\ y(1) = 4 \to c = 3$

51. (a) ≈2.42 gal **(b)** ≈24.83 mpg

52. (a) 6,144 ft
(b) 4,296 ft **(c)** B

53. (a) $h(y_1 + y_3) + 2(2hy_2) = h(y_1 + 4y_2 + y_3)$
(b) Each expression is equal to
$\frac{1}{3}[h(y_0 + 4y_1 + y_2) + h(y_2 + 4y_3 + y_4) + \cdots + h(y_{2n-2} + 4y_{2n-1} + y_{2n}).]$

54. (a) 0 **(b)** −1
(c) $-\pi$ **(d)** $x = 1$
(e) $y = 2x + 2 - \pi$ **(f)** $x = -1, x = 2$
(g) $[-2\pi, 0]$

55. (a) NINT($e^{-x^2/2}$, x, −10, 10) ≈ 2.506628275
NINT($e^{-x^2/2}$, x, −20, 20) ≈ 2.506628275
(b) The area is $\sqrt{2\pi}$.

56. ≈1500 yd^3

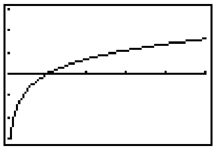
57. (a) $(V^2)_{av} = \frac{(V_{max})^2}{2}$ **(b)** ≈339 volts
$V_{rms} = \frac{V_{max}}{\sqrt{2}}$

Chapter 6
Differential Equations and Mathematical Modeling

6.1 Antiderivatives and Slope Fields (pp. 303–315)

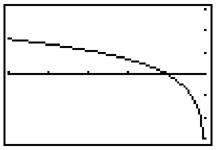
Quick Review 6.1

1. \$106.00 **2.** \$106.14
3. \$106.17 **4.** \$106.18
5. $3 \cos 3x$ **6.** $\frac{5}{2} \sec^2 \frac{5}{2}x$
7. $2Ce^{2x}$ **8.** $\frac{1}{x+2}$

9. 

[0.01, 5] by [−3, 3]

The graphs appear to be the same.

10. 

[−5, −0.01] by [−3, 3]

The graphs appear to be the same.

Section 6.1 Exercises

1. $\frac{x^3}{3} - x^2 + x + C$ **2.** $x^{-3} + C$

3. $\frac{x^3}{3} - \frac{8}{3}x^{3/2} + C$ **4.** $8x - \csc x + C$

5. $\frac{e^{4x}}{4} + C$ **6.** $\ln|x + 3| + C$

7. $\frac{x^6}{6} - 3x^2 + 3x + C$ **8.** $\frac{x^{-2}}{2} + \frac{x^2}{2} - x + C$

9. $2e^{t/2} + \frac{5}{t} + C$ **10.** $t^{4/3} + C$

11. $\frac{x^4}{4} + \frac{1}{2x^2} + C$ **12.** $\frac{3}{4}x^{4/3} + \frac{3}{2}x^{2/3} + C$

13. $x^{1/3} + C$

14. $-3\cos x + \frac{\cos 3x}{3} + C$

15. $\sin\left(\frac{\pi}{2}x\right) + C$ **16.** $2\sec t + C$

17. $2\ln|x - 1| + \ln|x| + C$

18. $\ln|x - 2| - \frac{\cos 5x}{5} + \frac{e^{-2x}}{2} + C$

19. $\tan 5r + C$ **20.** $-\frac{\cot 7t}{7} + C$

21. $\frac{x}{2} + \frac{\sin 2x}{4} + C$ **22.** $\frac{x}{2} - \frac{\sin 2x}{4} + C$

23. $\tan\theta - \theta + C$ **24.** $-\cot t - t + C$

25. (a) Graph (b)

(b) The slope is always positive, so (a) and (c) can be ruled out.

26. (a) Graph (b)

(b) The solution should have positive slope when x is negative, zero slope when x is zero and negative slope when x is positive since slope $= \frac{dy}{dx} = -x$. Graphs (a) and (c) don't show this slope pattern.

27. $y = x^2 - x - 2$ **28.** $y = \frac{x^2}{2} - \frac{1}{x} - \frac{1}{2}$

29. $y = \tan x - 2$ **30.** $y = 3x^{1/3} - 2$

31. $y = 3x^3 - 2x^2 + 5x + 10$

32. $y = \sin x - \cos x$

33. $y = -2e^{-t} + 1$ **34.** $y = \ln|x| - 3$

35. $y = -\sin\theta + \theta - 3$

36. $y = -x^3 + x^2 + 4x + 1$

37. $y = \frac{1}{2}\ln|t| + \frac{5}{4}t^2 - \frac{1}{4}$

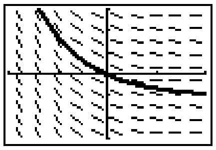
38. $y = \sin\theta + \cos\theta - \frac{\theta^3}{3} - 2\theta - 4$

39. $s = 4.9t^2 + 5t + 10$

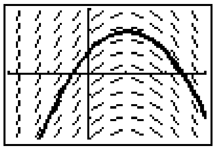
40. $s = -\frac{1}{\pi}(1 + \cos \pi t)$

41. $s = 16t^2 + 20t$

42. $s = -\cos t - t + 2$

43. 

[−2, 2] by [−3, 3]

44. 

[−2, 3] by [−3, 3]

45–48. The derivative of the right hand side of the equation is equal to the integrand on the left hand side.

49. (a) $y = \frac{x^2}{2} + \frac{1}{x} + \frac{1}{2}, x > 0$

(b) $y = \frac{x^2}{2} + \frac{1}{x} + \frac{3}{2}, x < 0$

(c) $y' = \begin{cases} x - \frac{1}{x^2}, & x < 0 \\ x - \frac{1}{x^2}, & x > 0 \end{cases}$

(d) $C_1 = \frac{3}{2}, C_2 = \frac{1}{2}$

(e) $C_1 = \frac{1}{2}, C_2 = -\frac{7}{2}$

50. $r(x) = x^3 - 3x^2 + 12x$

51. $c(x) = x^3 - 6x^2 + 15x + 400$

52. (a) $x^2e^x + C$ **(b)** $x\sin x + C$

(c) $-x^2e^x + C$ **(d)** $-x\sin x + C$

(e) $x^2e^x + x\sin x + C$

(f) $x^2e^x - x\sin x + C$

(g) $\frac{x^2}{2} + x^2e^x + C$ **(h)** $x\sin x - 4x + C$

53. (a) $s = \frac{-kt^2}{2} + 88t$ **(b)** $t = \frac{88}{k}$

(c) $k = 16$ ft/sec^2

54. ≈ 21.5 ft/sec^2 **55.** ≈ 1.240 sec

56. $\frac{d^2s}{dt^2} = a, s(0) = s_0, v(0) = v_0$

57. $h = \left[-\frac{125}{48\pi}t + 10^{5/2}\right]^{2/5}$

$V = \frac{4\pi}{75}\left[-\frac{125}{48\pi}t + 10^{5/2}\right]^{6/5}$

58. **(a)** $y = 500e^{0.0475t}$

(b) $t = \dfrac{\ln 2}{0.0475} \approx 14.6$ yr

59. **(a)** $y = 1200e^{0.0625t}$

(b) $t = \dfrac{\ln 3}{0.0625} \approx 17.6$ yr

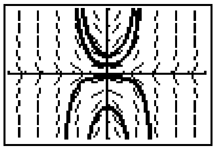
60. **(a)** $\int_0^x t^2 \cos t\, dt + C$ **(b)** $\int_0^x t^2 \cos t\, dt + 1$

61. **(a)** $\int_0^x te^t\, dt + C$ **(b)** $\int_0^x te^t\, dt + 1$

62. **(a)** $y = x^3 + 1$

(b) Only one function satisfies the differential equation and the initial conditions.

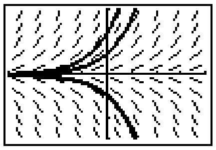
63.



$[-6, 6]$ by $[-4, 4]$

The concavity of each solution curve indicates the sign of y''.

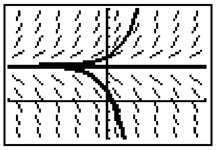
64.



$[-4, 4]$ by $[-3, 3]$

The concavity of each solution curve indicates the sign of y''.

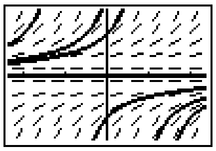
65.



$[-3, 3]$ by $[-4, 10]$

The concavity of each solution curve indicates the sign of y''.

66.



$[-2.35, 2.35]$ by $[-1.55, 1.55]$

The concavity of each solution curve indicates the sign of y''.

67. **(a)** $\dfrac{d}{dx}(\ln x + C) = \dfrac{1}{x}$ for $x > 0$

(b) $\dfrac{d}{dx}(\ln(-x) + C) = \dfrac{1}{x}$ for $x < 0$

(c) $\dfrac{d}{dx}\ln|x| = \dfrac{1}{x}$ for all x except 0.

(d) $\dfrac{dy}{dx} = \dfrac{1}{x}$ for all x except 0.

6.2 Integration by Substitution (pp. 315–323)

Quick Review 6.2

1. $\dfrac{32}{5}$ **2.** $\dfrac{16}{3}$

3. 3^x **4.** 3^x

5. $4(x^3 - 2x^2 + 3)^3(3x^2 - 4x)$

6. $8 \sin(4x - 5)\cos(4x - 5)$

7. $-\tan x$ **8.** $\cot x$

9. $\sec x$ **10.** $-\csc x$

Section 6.2 Exercises

1. $-\dfrac{1}{3}\cos 3x + C$ **2.** $\dfrac{1}{4}\sin(2x^2) + C$

3. $\dfrac{1}{2}\sec 2x + C$ **4.** $(7x - 2)^4 + C$

5. $\dfrac{1}{3}\tan^{-1}\left(\dfrac{x}{3}\right) + C$ **6.** $-6\sqrt{1 - r^3} + C$

7. $\dfrac{2}{3}\left(1 - \cos\dfrac{t}{2}\right)^3 + C$ **8.** $\dfrac{2}{3}(y^4 + 4y^2 + 1)^3 + C$

9. $\dfrac{1}{1 - x} + C$ **10.** $\tan(x + 2) + C$

11. $\dfrac{2}{3}(\tan x)^{3/2} + C$ **12.** $\sec\left(\theta + \dfrac{\pi}{2}\right) + C$

13. $\ln(\ln 6)$ **14.** $\dfrac{2}{3}$

15. $\dfrac{1}{3}\sin(3z + 4) + C$ **16.** $-\dfrac{2}{3}(\cot x)^{3/2} + C$

17. $\dfrac{1}{7}(\ln x)^7 + C$ **18.** $\dfrac{1}{4}\tan^8\left(\dfrac{x}{2}\right) + C$

19. $\dfrac{3}{4}\sin(s^{4/3} - 8) + C$ **20.** $-\dfrac{1}{3}\cot(3x) + C$

21. $\dfrac{1}{2}\sec(2t + 1) + C$ **22.** $\dfrac{-6}{2 + \sin t} + C$

23. 0 **24.** $\ln 9 - \ln 2 \approx 1.504$

25. $\dfrac{1}{2}\ln 5 \approx 0.805$ **26.** 2π

27. $\dfrac{1}{3}\ln|\sec(3x)| + C = -\dfrac{1}{3}\ln|\cos(3x)| + C$

28. $\dfrac{2}{5}\sqrt{5x + 8} + C$

29. $\ln|\sec x + \tan x| + C$

30. $-\ln|\csc x + \cot x| + C$

31. $\dfrac{14}{3}$ **32.** $\dfrac{1}{3}$

33. $-\dfrac{1}{2}$ **34.** 0

35. $\dfrac{10}{3}$ **36.** 0

37. $2\sqrt{3}$ **38.** $\dfrac{3}{4}$

39. $y = Ce^{(1/2)x^2 + 2x} - 5$

40. $y = \left[\tan^{-1}\left(\dfrac{x^2}{4} + C\right)\right]^2$

41. $y = -\ln\left(C - e^{\sin x}\right)$ **42.** $y = \ln(e^x + C)$

43. $y = \frac{1}{x^2 + 3}$

44. $y = (\ln x)^4$

45. (a) $\frac{d}{dx}\left(\frac{2}{3}(x + 1)^{3/2} + C\right) = \sqrt{x + 1}$

(b) Because $\frac{dy_1}{dx} = \sqrt{x + 1}$ and $\frac{dy_2}{dx} = \sqrt{x + 1}$

(c) $4\frac{2}{3}$

(d) $C = y_1 - y_2$

$= \int_0^x \sqrt{x + 1}\, dx - \int_3^x \sqrt{x + 1}\, dx$

$= \int_0^x \sqrt{x + 1}\, dx + \int_x^3 \sqrt{x + 1}\, dx$

$= \int_0^3 \sqrt{x + 1}\, dx$

46. (a) $\frac{d}{dx}[F(x) + C]$ should equal $f(x)$.

(b) The slope field should help you visualize the solution curve $y = F(x)$.

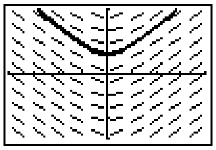
(c) The graphs of $y_1 = F(x)$ and $y_2 = \int_0^x f(t)\, dt$ should differ only by a vertical shift, C.

(d) A table of values for $y_1 - y_2$ should show C.

(e) The graph of NDER of $F(x)$ and $f(x)$ should be the same.

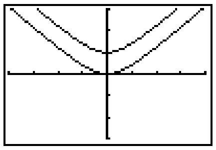
(f) a) $\frac{d}{dx}\left(\sqrt{x^2 + 1} + C\right) = \frac{x}{\sqrt{x^2 + 1}}$

b)



[−4, 4] by [−3, 3]

c)

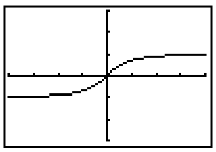


[−4, 4] by [−3, 3]

d)

x	$y_1 - y_2$
0	1
1	1
2	1
3	1
4	1

e)



[−4, 4] by [−3, 3]

47. (a) $\frac{1}{2}\sqrt{10} - \frac{3}{2} \approx 0.081$

(b) $\frac{1}{2}\sqrt{10} - \frac{3}{2} \approx 0.081$

48. (a) $\frac{1}{2}$ **(b)** $\frac{1}{2}$

49. Show $\frac{dy}{dx} = \tan x$ and $y(3) = 5$.

50. (a) $F_1(\theta) = -\frac{1}{4}\cot^2 2\theta$

(b) $F_2(\theta) = -\frac{1}{4}\csc^2 2\theta$

(c) $F_1'(\theta) = F_2'(\theta) = \csc^2 2\theta \cot 2\theta$

(d) $\frac{1}{4}$

51. (a) $\sin^2 x + C$

(b) $-\cos^2 x + C$

(c) $-\frac{1}{2}\cos 2x + C$

(d) The derivative of each expression is $2 \sin x \cos x$.

6.3 Integration by Parts (pp. 323–329)

Quick Review 6.3

1. $2x^3 \cos 2x + 3x^2 \sin 2x$
2. $\frac{3e^{2x}}{3x + 1} + 2e^{2x} \ln (3x+1)$
3. $\frac{2}{1 + 4x^2}$
4. $\frac{1}{\sqrt{1 - (x + 3)^2}}$
5. $x = \frac{1}{3}\tan y$
6. $x = \cos y - 1$
7. $\frac{2}{\pi}$
8. $y = \frac{1}{2}e^{2x} + C$
9. $y = \frac{1}{2}x^2 - \cos x + 3$
10. $\frac{d}{dx}\left[\frac{1}{2}e^x(\sin x - \cos x)\right] = e^x \sin x$

Section 6.3 Exercises

1. $-x \cos x + \sin x + C$
2. $2x \cos x + (x^2 - 2) \sin x + C$
3. $\frac{1}{2}y^2 \ln y - \frac{1}{4}y^2 + C$
4. $y \tan^{-1} y - \frac{1}{2}\ln (1 + y^2) + C$
5. $x \tan x + \ln |\cos x| + C$
6. $\theta \sin^{-1} \theta + \sqrt{1 - \theta^2} + C$
7. $(2 - t^2) \cos t + 2t \sin t + C$
8. $-t \cot t + \ln |\sin t| + C$
9. $\frac{1}{4}x^4 \ln x - \frac{1}{16}x^4 + C$
10. $(-x^4 - 4x^3 - 12x^2 - 24x - 24)e^{-x} + C$
11. $(x^2 - 7x + 7)e^x + C$
12. $\left(-\frac{x^3}{2} - \frac{3x^2}{4} - \frac{3x}{4} - \frac{3}{8}\right)e^{-2x} + C$
13. $\frac{1}{2}e^y(\sin y - \cos y) + C$
14. $\frac{1}{2}e^{-y}(\sin y - \cos y) + C$

15. $\frac{\pi^2}{8} - \frac{1}{2} \approx 0.734$

16. $\frac{3}{4} - \frac{3\pi^2}{16} \approx -1.101$

17. $\frac{1}{13}[e^6(2\cos 9 + 3\sin 9) - e^{-4}(2\cos 6 - 3\sin 6)]$
≈ -18.186

18. $-\frac{e^{-4}}{4}(\cos 4 + \sin 4) + \frac{e^6}{4}(\cos 6 - \sin 6)$
≈ 125.03

19. $y = \left(\frac{x^2}{4} - \frac{x}{8} + \frac{1}{32}\right)e^{4x} + C$

20. $y = \frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + C$

21. $y = \frac{\theta^2}{2}\sec^{-1}\theta - \frac{1}{2}\sqrt{\theta^2 - 1} + C$

22. $y = \theta \sec\theta - \ln|\sec\theta + \tan\theta| + C$

23. **(a)** π **(b)** 3π
(c) 4π

24. ≈ 0.726

25. $\frac{1 - e^{-2\pi}}{2\pi} \approx 0.159$

26. **(a)** $(x - 1)e^x + C$
(b) $(x^2 - 2x + 2)e^x + C$
(c) $(x^3 - 3x^2 + 6x - 6)e^x + C$
(d) $\left[x^n - \frac{d(x^n)}{dx} + \frac{d^2(x^n)}{dx^2} - \cdots + (-1)^n\frac{d^n(x^n)}{dx^n}\right]e^x$
$+ C$ or

$[x^n - nx^{n-1} + n(n-1)x^{n-2} - \cdots$
$+ (-1)^{n-1}(n - 1)!x + (-1)^n(n!)]e^x + C$

(e) Use mathematical induction or argue based on tabular integration.

27. $-2(\sqrt{x}\cos\sqrt{x} - \sin\sqrt{x}) + C$

28. $\frac{2(\sqrt{3x + 9} - 1)e^{\sqrt{3x+9}}}{3} + C$

29. $\frac{(x^6 - 3x^4 + 6x^2 - 6)e^{x^2}}{2} + C$

30. $\frac{r}{2}[\sin(\ln r) - \cos(\ln r)] + C$

31. $u = x^n,\ dv = \cos x\, dx$

32. $u = x^n,\ dv = \sin x\, dx$

33. $u = x^n,\ dv = e^{ax}\, dx$

34. $u = (\ln x)^n,\ dv = dx$

35. **(a)** Let $y = f^{-1}(x)$. Then $x = f(y)$, so $dx = f'(y)\, dy$. Substitute directly.
(b) $u = y,\ dv = f'(y)\, dy$

36. $u = f^{-1}(x),\ dv = dx$

37. **(a)** $\int \sin^{-1} x\, dx = x\sin^{-1} x + \cos(\sin^{-1} x) + C$
(b) $\int \sin^{-1} x\, dx = x\sin^{-1} x + \sqrt{1 - x^2} + C$
(c) $\cos(\sin^{-1} x) = \sqrt{1 - x^2}$

38. **(a)** $\int \tan^{-1} x\, dx$
$= x\tan^{-1} x + \ln|\cos(\tan^{-1} x)| + C$
(b) $\int \tan^{-1} x\, dx = x\tan^{-1} x - \frac{1}{2}\ln(1 + x^2) + C$
(c) $\ln|\cos(\tan^{-1} x)| = \ln\left(\frac{1}{\sqrt{1 + x^2}}\right)$
$= -\frac{1}{2}\ln(1 + x^2)$

39. **(a)** $\int \cos^{-1} x\, dx = x\cos^{-1} x - \sin(\cos^{-1} x) + C$
(b) $\int \cos^{-1} x\, dx = x\cos^{-1} x - \sqrt{1 - x^2} + C$
(c) $\sin(\cos^{-1} x) = \sqrt{1 - x^2}$

40. **(a)** $\int \log_2 x\, dx = x\log_2 x - \left(\frac{1}{\ln 2}\right)2^{\log_2 x} + C$
(b) $\int \log_2 x\, dx = x\log_2 x - \left(\frac{1}{\ln 2}\right)x + C$
(c) $2^{\log_2 x} = x$

6.4 Exponential Growth and Decay (pp. 330–341)

Quick Review 6.4

1. $a = e^b$

2. $c = \ln(d)$

3. $x = e^2 - 3$

4. $x = \frac{1}{2}\ln 6$

5. $x = \frac{\ln 2.5}{\ln 0.85} \approx -5.638$

6. $k = \frac{\ln 2}{\ln 3 - \ln 2} \approx 1.710$

7. $t = \frac{\ln 10}{\ln 1.1} \approx 24.159$

8. $t = \frac{1}{2}\ln 4 = \ln 2$

9. $y = -1 + e^{2x+3}$

10. $y = -2 \pm e^{3t-1}$

Section 6.4 Exercises

Most of the numerical answers in this section are approximations.

1. $y(t) = 100e^{1.5t}$

2. $y(t) = 200e^{-0.5t}$

3. $y(t) = 50e^{(0.2\ln 2)t}$

4. $y(t) = 60e^{-(0.1\ln 2)t}$

5. 8.06 yr doubling time; \$13,197.14 in 30 yr

6. 4.62% rate; \$8000 in 30 yr

7. \$600 initially; 13.2 yr doubling time

8. 7.2% rate; 9.63 yr doubling time

9. **(a)** 14.94 yr **(b)** 14.62 yr
(c) 14.68 yr **(d)** 14.59 yr

10. (a) 8.74 yr **(b)** 8.43 yr
(c) 8.49 yr **(d)** 8.40 yr

11. (a) 2.8×10^{14} bacteria
(b) The bacteria reproduce fast enough that even if many are destroyed there are enough left to make the person sick.

12. 1250 bacteria

13. 0.585 days

14. (a) 138.6 days **(b)** 599 days

15. $y \approx 2e^{0.4581t}$ **16.** $y \approx 1.1e^{-0.3344t}$

17. $y = y_0e^{-kt} = y_0e^{-k(3/k)} = y_0e^{-3} < 0.05y_0$

18. 5°F

19. (a) 17.53 minutes longer
(b) 13.26 minutes

20. (a) 53.45° above room temperature
(b) 23.79° above room temperature
(c) 232.5 min or 3.9 hours

21. 6658 years

22. (a) 12,571 B.C.
(b) 12,101 B.C.
(c) 13,070 B.C.

23. (a) 168.5 meters
(b) 41.13 seconds

24. (a) 7780 meters
(b) 31.65 minutes

25. 585.4 kg

26. 16.09 years

27. (a) $p = 1013e^{-0.121h}$
(b) 2.383 millibars
(c) 0.977 km

28. (a) 54.88 grams

29. (a) $V = V_0e^{-t/40}$
(b) 92.1 seconds

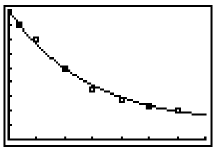
30. (a) $A(t) = A_0e^t$;
It grows by a factor of e each year.
(b) $\ln 3 \approx 1.1$ yr
(c) $(e - 1)$ times your initial amount, or $\approx$ 172% increase.

31. (b) $\lim_{t\to\infty} s(t) = \frac{v_0m}{k}$

32. (a) $\frac{\ln 90}{100} = 0.045$ or 4.5%
(b) $\frac{\ln 131}{100} = 0.049$ or 4.9%

33. $s(t) = 1.32(1 - e^{-0.606t})$

34. (a) $T - T_s = 79.47(0.932^t)$
(b) $T = 10 + 79.47(0.932^t)$

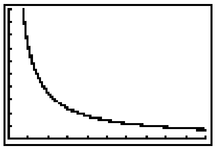


[0, 35] by [0, 90]

(c) 52.5 seconds
(d) 89.47°C

35. (b) $T = T_s$ is a horizontal asymptote.

36. (a) $2y_0 = y_0e^{rt} \Rightarrow t = \frac{\ln 2}{r}$
(b)



[0, 0.1] by [0, 100]

(c) $\ln 2 \approx 0.69$, so the doubling time is $\frac{0.69}{r}$ which is almost the same as the rules.
(d) $\frac{70}{5} = 14$ years or $\frac{72}{5} = 14.4$ years
(e) $\frac{108}{100r}$ (108 has more factors than 110.)

37. (a)

x	$\left(1 + \frac{1}{x}\right)^x$
10	2.5937
100	2.7048
1000	2.7169
10,000	2.7181
100,000	2.7183

$e \approx 2.7183$

(b) $r = 2$

x	$\left(1 + \frac{2}{x}\right)^x$
10	6.1917
100	7.2446
1000	7.3743
10,000	7.3876
100,000	7.3889

$e^2 \approx 7.389$

$r = 0.5$

x	$\left(1 + \frac{0.5}{x}\right)^x$
10	1.6289
100	1.6467
1000	1.6485
10,000	1.6487
100,000	1.6487

$e^{0.5} \approx 1.6487$

(c) As we compound more times the increment of time between compounding approaches 0. Continuous compounding is based on an instantaneous rate of change which is a limit of average rates as the increment in time approaches 0.

38. (b) $\sqrt{\frac{mg}{k}}$
(c) 179 ft/sec $\approx$ 122 mi/hr

6.5 Population Growth (pp. 342–349)

Quick Review 6.5

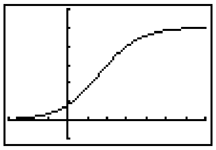
1. All real numbers

2. $\lim_{x \to +\infty} f(x) = 50$; $\lim_{x \to -\infty} f(x) = 0$

3. $y = 0$ and $y = 50$

4. All real numbers

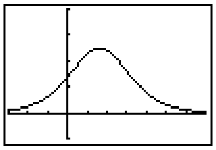
5.



$[-30, 70]$ by $[-10, 60]$

no zeros

6.

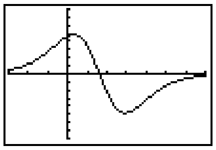


$[-30, 70]$ by $[-0.5, 2]$

(a) $(-\infty, \infty)$

(b) None

7.



$[-30, 70]$ by $[-0.08, 0.08]$

(a) $(-\infty, 10 \ln 5) \approx (-\infty, 16.094)$

(b) $(10 \ln 5, \infty) \approx (16.094, \infty)$

8. $(10 \ln 5, 25) \approx (16.094, 25)$

9. $A = 3, B = -2$

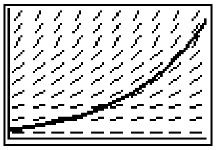
10. $A = -2, B = 4$

Section 6.5 Exercises

1. (a) $\frac{dP}{dt} = 0.025P$

(b) $P(t) = 75{,}000e^{0.025t}$

(c)

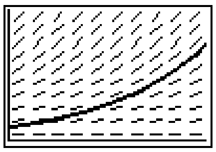


$[0, 100]$ by $[0, 1{,}000{,}000]$

2. (a) $\frac{dP}{dt} = 0.019P$

(b) $P(t) = 110{,}000e^{0.019t}$

(c)

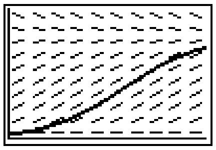


$[0, 100]$ by $[0, 1{,}000{,}000]$

3. (a) $\frac{dP}{dt} = 0.00025P(200 - P)$

(b) $P(t) = \frac{200}{1 + 19e^{-0.05t}}$

(c)

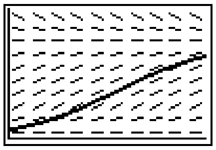


$[0, 100]$ by $[0, 250]$

4. (a) $\frac{dP}{dt} = \frac{1}{7500}P(150 - P)$

(b) $P(t) = \frac{150}{1 + 9e^{-0.02t}}$

(c)



$[0, 200]$ by $[0, 200]$

5. -30%

6. 7.5%

7. $k = 0.04$; $M = 100$

8. $k = 0.04$; $M = 500$

9. (d)

10. (b)

11. (c)

12. (a)

13. (a) $k = 0.7$; $M = 1000$

(b) $P(0) \approx 8$; Initially there are 8 rabbits.

14. (a) $k = 1$; $M = 200$

(b) $P(0) \approx 1$; Initially 1 student has the measles.

15. (a) 0.875% **(b)** 275,980,017

16. (a) About 10.32 yrs

(b) About 44.4 yrs

17. (a) $P(t) = \frac{150}{1 + 24e^{-0.225t}}$

(b) About 17.21 weeks; 21.28 weeks

18. (a) $P(t) \approx \frac{250}{1 + 7.9286e^{-0.1t}}$

(b) 83 yrs to reach $249.5 \approx 250$

19. (a) $y = y_0 e^{-0.00001t}$

(b) $\approx$10,536 yrs

(c) $\approx$81.9%

20. $\approx$13.8 yrs

21. (a) $x = 11{,}000e^{0.1t} - 10{,}000$

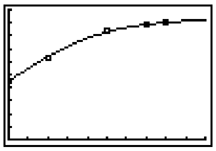
(b) $\approx$ 23 yrs

22. (a) $p(x) = 20.09e^{1-0.01x}$

(b) $p(10) \approx \$49.41$ $\quad p(90) \approx \$22.20$

(c) $r'(x) = 20.09e^{1-0.01x}(1 - 0.01x)$, so $r'(x) > 0$ for $x < 100$ and $r'(x) < 0$ for $x > 100$.

23. (a) $y = \frac{18.70}{1 + 1.075e^{-0.0422x}}$

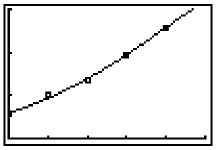


$[0, 100]$ by $[0, 20]$

(b) 18.7 million

(c) $x \approx 1.7$ (year 1912); population ≈ 9.35 million

24. (a) $y = \dfrac{24.76}{1 + 7.195e^{-0.0513x}}$



[0, 50] by [0, 15]

(b) 24.76 million

(c) $x \approx 38.44$ (year 1988); population ≈ 12.38 million

25. Separate variables and rewrite $\dfrac{M}{P(M-P)}$ as $\dfrac{1}{P} - \dfrac{1}{M-P}$ in order to integrate.

26. (a) $y = \dfrac{16.90}{1 + 5.132e^{-0.0666x}}$

(b) 16.9 million

27. $y = e^{\sin x} - 1$

28. $y = 3 + 2e^{-2x}$

29. $y = \sqrt{x^2 + 4}$

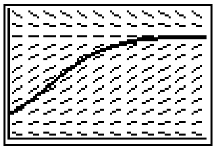
30. $y = e^{(2/3)x^{(3/2)}}$

31. (a) $\dfrac{dP}{dt}$ has the same sign as $(M - P)(P - m)$.

(b) $P(t) = \dfrac{1200Ae^{11kt/12} + 100}{1 + Ae^{11kt/12}}$

(c) $P(t) = \dfrac{300(8e^{11kt/12} + 3)}{9 + 2e^{11kt/12}}$

(d)



[0, 75] by [0, 1500]

(e) $P(t) = \dfrac{AMe^{(M-m)kt/M} + m}{1 + Ae^{(M-m)kt/M}}$ where $A = \dfrac{P(0) - m}{M - P(0)}$

32. (a) Use the Fundamental Theorem of Calculus.

(b) $p(9) \approx 109.65$. This is the price of \$100 item after 9 years during which the inflation rate is $\dfrac{0.04}{1+t}$ per year.

(c) \$143.33

(d) \$168.54

33. (a) $P(t) = \dfrac{P_0}{1 - kP_0t}$

(b) Vertical asymptote at $t = \dfrac{1}{kP_0}$

6.6 Numerical Methods (pp. 350–356)

Quick Review 6.6

1. 9

2. $L(x) = 9x - 16$

3. 2

4. $L(x) = 2x - \dfrac{\pi}{2} + 1$

5. 0.4875

6. $y = 0.4875x + 0.9$

7. (a) 0.001762 **(b)** 0.061%

8. (a) 0.006976 **(b)** 0.236%

9. (a) 0.042361 **(b)** 1.351%

10. (a) 0.047321 **(b)** 1.783%

Section 6.6 Exercises

1. $f'(x) = 1 - 2e^{-x} = x - f(x)$ and $f(0) = 1$.

2. $f'(x) = 1 + e^{-x} = x - f(x)$ and $f(0) = -2$.

3. $f'(x) = \dfrac{1}{5}(2e^{2x} - 2\cos x + \sin x) = 2f(x) + \sin x$ and $f(0) = 0$.

4. $f'(x) = e^x - 2e^{2x} = f(x) - e^{2x} + 1$ and $f(0) = -1$.

5. $y = 2e^x - 1$

6. $y = -e^{(-x^2/2)+2} + 1$

7. $y = 2e^{x^2+2x}$

8. $y = -\dfrac{1}{x^2 + x + 1}$

9.

x	y (Euler)	y (exact)	Error
0	0	0	0
0.1	0	0.0053	0.0053
0.2	0.0100	0.0229	0.0129
0.3	0.0318	0.0551	0.0233
0.4	0.0678	0.1051	0.0374
0.5	0.1203	0.1764	0.0561
0.6	0.1923	0.2731	0.0808
0.7	0.2872	0.4004	0.1132
0.8	0.4090	0.5643	0.1553
0.9	0.5626	0.7723	0.2097
1.0	0.7534	1.0332	0.2797

10.

x	y (Euler)	y (exact)	Error
0	−2	−2	0
0.1	−1.8000	−1.8048	0.0048
0.2	−1.6100	−1.6187	0.0087
0.3	−1.4290	−1.4408	0.0118
0.4	−1.2561	−1.2703	0.0142
0.5	−1.0905	−1.1065	0.0160
0.6	−0.9314	−0.9488	0.0174
0.7	−0.7783	−0.7966	0.0183
0.8	−0.6305	−0.6493	0.0189
0.9	−0.4874	−0.5066	0.0191
1.0	−0.3487	−0.3679	0.0192

11.

x	y (improved Euler)	y (exact)	Error
−2	2	2	0
−1.9	1.6560	1.6539	0.0021
−1.8	1.3983	1.3954	0.0030
−1.7	1.2042	1.2010	0.0032
−1.6	1.0578	1.0546	0.0032
−1.5	0.9478	0.9447	0.0031
−1.4	0.8663	0.8634	0.0029
−1.3	0.8077	0.8050	0.0027
−1.2	0.7683	0.7658	0.0025
−1.1	0.7456	0.7415	0.0024
−1.0	0.7381	0.7358	0.0023
−0.9	0.7455	0.7432	0.0023
−0.8	0.7682	0.7658	0.0024
−0.7	0.8075	0.8050	0.0024
−0.6	0.8659	0.8634	0.0025
−0.5	0.9473	0.9447	0.0026
−0.4	1.0572	1.0546	0.0026
−0.3	1.2036	1.2010	0.0026
−0.2	1.3976	1.3954	0.0022
−0.1	1.6553	1.6540	0.0014
0	1.9996	2	0.0004

12.

x	y (improved Euler)	y (exact)	Error
−2	0	0	0
−1.9	−0.2140	−0.2153	0.0013
−1.8	−0.4593	−0.4623	0.0029
−1.7	−0.7371	−0.7419	0.0049
−1.6	−1.0473	−1.0544	0.0071
−1.5	−1.3892	−1.3989	0.0097
−1.4	−1.7607	−1.7732	0.0125
−1.3	−2.1585	−2.1740	0.0155
−1.2	−2.5780	−2.5966	0.0186
−1.1	−3.0131	−3.0350	0.0219
−1.0	−3.4565	−3.4817	0.0252
−0.9	−3.9000	−3.9283	0.0284
−0.8	−4.3341	−4.3656	0.0315
−0.7	−4.7491	−4.7834	0.0344
−0.6	−5.1348	−5.1719	0.0370
−0.5	−5.4815	−5.5208	0.0394
−0.4	−5.7796	−5.8210	0.0413
−0.3	−6.0210	−6.0639	0.0430
−0.2	−6.1986	−6.2427	0.0441
−0.1	−6.3073	−6.3522	0.0449
0	−6.3438	−6.3891	0.0452

13.

x	y (Euler)	y (improved Euler)	y (exact)	Error (Euler)	Error (improved Euler)
0	1	1	1	0	0
0.1	0.9000	0.9100	0.9097	0.0097	0.0003
0.2	0.8200	0.8381	0.8375	0.0175	0.0006
0.3	0.7580	0.7824	0.7816	0.0236	0.0008
0.4	0.7122	0.7416	0.7406	0.0284	0.0010
0.5	0.6810	0.7142	0.7131	0.0321	0.0011
0.6	0.6629	0.6988	0.6976	0.0347	0.0012
0.7	0.6566	0.6944	0.6932	0.0366	0.0012
0.8	0.6609	0.7000	0.6987	0.0377	0.0013
0.9	0.6748	0.7145	0.7131	0.0383	0.0013
1.0	0.6974	0.7371	0.7358	0.0384	0.0013
1.1	0.7276	0.7671	0.7657	0.0381	0.0013
1.2	0.7649	0.8037	0.8024	0.0375	0.0013
1.3	0.8084	0.8463	0.8451	0.0367	0.0013
1.4	0.8575	0.8944	0.8932	0.0357	0.0012
1.5	0.9118	0.9475	0.9463	0.0345	0.0012
1.6	0.9706	1.0050	1.0038	0.0332	0.0012
1.7	1.0335	1.0665	1.0654	0.0318	0.0011
1.8	1.1002	1.1317	1.1306	0.0304	0.0011
1.9	1.1702	1.2002	1.1991	0.0290	0.0010
2.0	1.2432	1.2716	1.2707	0.0275	0.0010

14.

x	y (Euler)	y (improved Euler)	y (exact)	error (Euler)	Error (improved Euler)
0	−1	−1	−1	0	0
0.1	−1.1000	−1.1161	−1.1162	0.0162	0.0002
0.2	−1.2321	−1.2700	−1.2704	0.0383	0.0004
0.3	−1.4045	−1.4715	−1.4723	0.0677	0.0007
0.4	−1.6272	−1.7325	−1.7337	0.1065	0.0012
0.5	−1.9125	−2.0678	−2.0696	0.1571	0.0018
0.6	−2.2756	−2.4954	−2.4980	0.2224	0.0026
0.7	−2.7351	−3.0378	−3.0414	0.3063	0.0037
0.8	−3.3142	−3.7224	−3.7275	0.4133	0.0050
0.9	−4.0409	−4.5832	−4.5900	0.5492	0.0068
1.0	−4.9499	−5.6616	−5.6708	0.7209	0.0092
1.1	−6.0838	−7.0087	−7.0208	0.9370	0.0121
1.2	−7.4947	−8.6872	−8.7031	1.2084	0.0159
1.3	−9.2465	−10.7738	−10.7944	1.5480	0.0206
1.4	−11.4175	−13.3628	−13.3894	1.9719	0.0267
1.5	−14.1037	−16.5696	−16.6038	2.5001	0.0342
1.6	−17.4227	−20.5358	−20.5795	3.1568	0.0437
1.7	−21.5182	−25.4345	−25.4902	3.9720	0.0556
1.8	−26.5664	−31.4781	−31.5486	4.9822	0.0705
1.9	−32.7829	−38.9262	−39.0153	6.2324	0.0891
2.0	−40.4313	−48.0970	−48.2091	7.7778	0.1121

15. (a) $y = \dfrac{-1}{x^2 - 2x + 2}$, $y(3) = -0.2$

(b) -0.1851, error ≈ 0.0149

(c) -0.1929, error ≈ 0.0071

(d) -0.1965, error ≈ 0.0035

16. (a) $y = 2e^x + 1$, $y(1) \approx 6.4366$

(b) 5.9766, error ≈ 0.4599

(c) 6.1875, error ≈ 0.2491

(d) 6.3066, error ≈ 0.1300

17. (a) -0.2024, error ≈ 0.0024

(b) -0.2005, error ≈ 0.0005

(c) -0.2001, error ≈ 0.0001

(d) As the step size decreases, the accuracy of the method increases and so the error decreases.

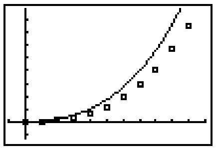
18. (a) 6.4054, error ≈ 0.0311

(b) 6.4282, error ≈ 0.0084

(c) 6.4344, error ≈ 0.0022

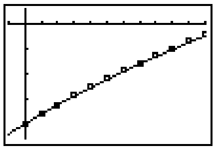
(d) As the step size decreases, the accuracy of the method increases and the error decreases.

19.



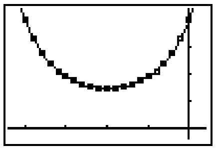
[−0.1, 1.1] by [−0.13, 0.88]

20.



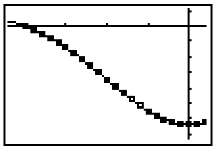
[−0.1, 1.1] by [−2.3, 0.3]

21.



[−2.2, 0.2] by [−0.2, 2.2]

22.



[−2.2, 0.2] by [−7.3, 1.1]

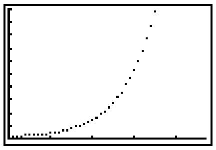
23.

x	y (Euler)	y (exact)	Error
0	1	1.0	0
−0.1	0.9000	0.9097	0.0097
−0.2	0.8200	0.8375	0.0175
−0.3	0.7580	0.7816	0.0236
−0.4	0.7122	0.7406	0.0284
−0.5	0.6810	0.7131	0.0321
−0.6	0.6629	0.6976	0.0347
−0.7	0.6566	0.6932	0.0366
−0.8	0.6609	0.6987	0.0377
−0.9	0.6748	0.7131	0.0383
−1.0	0.6974	0.7358	0.0384

24.

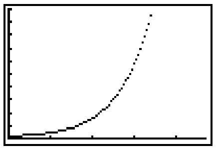
x	y (Improved Euler)	y (exact)	Error
0	1	1.0	0
−0.1	0.9100	0.9097	0.0003
−0.2	0.8381	0.8375	0.0006
−0.3	0.7824	0.7816	0.0008
−0.4	0.7416	0.7406	0.0010
−0.5	0.7142	0.7131	0.0011
−0.6	0.6988	0.6976	0.0012
−0.7	0.6944	0.6932	0.0012
−0.8	0.7000	0.6987	0.0013
−0.9	0.7145	0.7131	0.0013
−1.0	0.7371	0.7358	0.0013

25. (a)



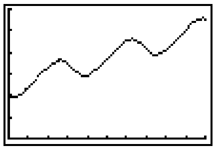
[0, 4.7] by [0, 100]

(b)



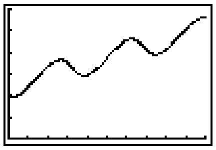
[0, 4.7] by [0, 100]

26. (a)



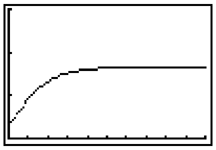
[0, 10] by [0, 6]

(b)



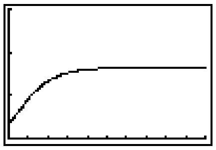
[0, 10] by [0, 6]

27. (a)



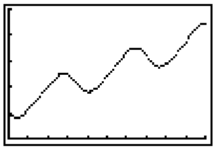
[0, 10] by [0, 3]

(b)



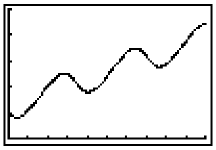
[0, 10] by [0, 3]

28. (a)



[0, 10] by [0, 5]

(b)



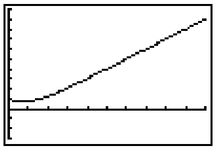
[0, 10] by [0, 5]

29. 2.6533, e **30.** ≈ 19.8845, e^3

31.

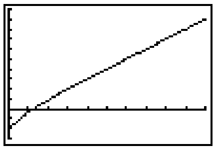
x	y (Runge-Kutta)	y (exact)	error
0	1	1	0
0.1	1.2103	1.2103	0.0000002
0.2	1.4428	1.4428	0.0000004
0.3	1.6997	1.6997	0.0000006
0.4	1.9836	1.9836	0.0000009
0.5	2.2974	2.2974	0.0000013
0.6	2.6442	2.6442	0.0000017
0.7	3.0275	3.0275	0.0000022
0.8	3.4511	3.4511	0.0000027
0.9	3.9192	3.9192	0.0000034
1.0	4.4366	4.4366	0.0000042

32. (a)



[0, 10] by [−3, 10]

(b)



[0, 10] by [−3, 10]

Chapter 6 Review Exercises (pp. 358–361)

1. $\sqrt{3}$ **2.** 2

3. 8 **4.** 0

5. 2 **6.** $\frac{147}{8}$

7. $e - 1$ **8.** $\frac{2}{3}$

9. $-\ln|2 - \sin x| + C$ **10.** $\frac{1}{2}(3x + 4)^{2/3} + C$

11. $\frac{1}{2}\ln(t^2 + 5) + C$ **12.** $-\sec\frac{1}{\theta} + C$

13. $-\ln|\cos(\ln y)| + C$

14. $\ln|\sec(e^x) + \tan(e^x)| + C$

15. $\ln|\ln x| + C$ **16.** $\frac{-2}{\sqrt{t}} + C$

17. $x^3 \sin x + 3x^2 \cos x - 6x \sin x - 6\cos x + C$

18. $\frac{x^5 \ln x}{5} - \frac{x^5}{25} + C$

19. $\left(\frac{3\sin x}{10} - \frac{\cos x}{10}\right)e^{3x} + C$

20. $\left(-\frac{x^2}{3} - \frac{2x}{9} - \frac{2}{27}\right)e^{-3x} + C$

21. $y = \frac{x^3}{6} + \frac{x^2}{2} + x + 1$

22. $y = \frac{x^3}{3} + 2x - \frac{1}{x} - \frac{1}{3}$

23. $y = \ln(t + 4) + 2$

24. $y = -\frac{1}{2}\csc 2\theta + \frac{3}{2}$

25. $y = \frac{x^3}{3} + \ln x - x + \frac{2}{3}$

26. $r = \sin t - \frac{t^2}{2} - 2t - 1$

27. $y = 4e^x - 2$

28. $y = 2e^{x^2+x} - 1$

29. $-1 + \sqrt{x} + C$ or $\sqrt{x} + C$

30. $\frac{x^2}{2} + 1 - \sqrt{x} + C$ or $\frac{x^2}{2} - \sqrt{x} + C$

31. $-2\sqrt{x} - x + C$

32. $2 - 3x + C$ or $-3x + C$

33. (b)

34. (d)

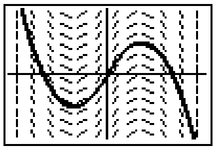
35. **iv**, since the given graph looks like $y = x^2$, which satisfies $\frac{dy}{dx} = 2x$ and $y(1) = 1$.

36. Yes, $y = x$ is a solution.

37. (a) $v = 2t + 3t^2 + 4$

(b) 6 m

38.



[−10, 10] by [−10, 10]

39.

x	y
0	0
0.1	0.1000
0.2	0.2095
0.3	0.3285
0.4	0.4568
0.5	0.5946
0.6	0.7418
0.7	0.8986
0.8	1.0649
0.9	1.2411
1.0	1.4273
1.1	1.6241
1.2	1.8319
1.3	2.0513
1.4	2.2832
1.5	2.5285
1.6	2.7884
1.7	3.0643
1.8	3.3579
1.9	3.6709
2.0	4.0057

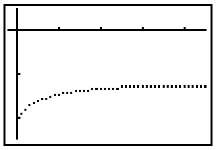
40.

x	y
−3	1
−2.9	0.6680
−2.8	0.2599
−2.7	−0.2294
−2.6	−0.8011
−2.5	−1.4509
−2.4	−2.1687
−2.3	−2.9374
−2.2	−3.7333
−2.1	−4.5268
−2.0	−5.2840
−1.9	−5.9686
−1.8	−6.5456
−1.7	−6.9831
−1.6	−7.2562
−1.5	−7.3488
−1.4	−7.2553
−1.3	−6.9813
−1.2	−6.5430
−1.1	−5.9655
−1.0	−5.2805

41. 0.9063

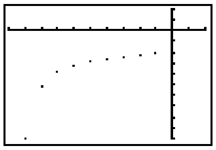
42. 4.4974

43. (a)



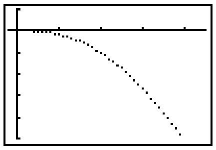
[−0.2, 4.5] by [−2.5, 0.5]

(b)



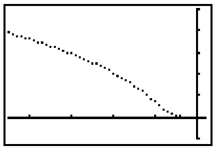
[−1, 0.2] by [−10, 2]

44. (a)



[−0.2, 4.5] by [−5, 1]

(b)



[−4.5, 0.2] by [−1, 5]

45. (a) $k \approx 0.262059$
(b) About 3.81593 years

46. About 92 minutes

47. −3°C

48. About 41.2 years

49. About 18,935 years old

50. About 5.3%

51. About 59.8 ft

52. (a) $y = c + (y_0 - c)e^{-(kA/V)t}$
(b) c

53. (a) $k = 1$; carrying capacity $= 150$
(b) ≈ 2; Initially there were 2 infected students.
(c) About 6 days

54. Use the Fundamental Theorem of Calculus to obtain $y' = \sin(x^2) + 3x^2 + 1$. Then differentiate again and also verify the initial conditions.

55. $P = \dfrac{800}{1 + 15e^{-0.002t}}$

56. Method 1—Compare graph of $y_1 = x^2 \ln x$ with $y_2 = \text{NDER}\left(\dfrac{x^3 \ln x}{3} - \dfrac{x^3}{9}\right)$.

Method 2—Compare graph of $y_1 = \text{NINT}(x^2 \ln x)$ with $y_2 = \dfrac{x^3 \ln x}{3} - \dfrac{x^3}{9}$.

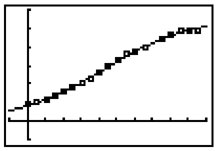
57. (a) About 11.3 years
(b) About 11 years

58. (a) $\dfrac{d}{dx}\displaystyle\int_0^x u(t)\,dt = u(x)$

$\dfrac{d}{dx}\displaystyle\int_3^x u(t)\,dt = u(x)$

(b) $C = \displaystyle\int_0^3 u(t)\,dt$

59. (a) $y = \dfrac{56.0716}{1 + 5.894e^{-0.0205x}}$

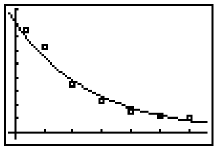


[−20, 200] by [−10, 60]

(b) $\approx$56.0716 million

(c) $\approx$1887, $\approx$28.0 million

60. (a) $T = 79.961(0.9273)^t$



[−1, 33] by [−5, 90]

(b) About 9.2 sec
(c) About 79.96°C

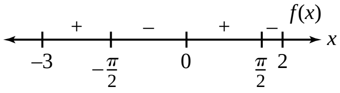
61. $s = 0.97(1 - e^{-0.8866t})$

Chapter 7
Applications of Definite Integrals

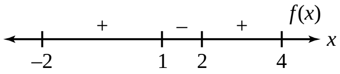
7.1 Integral as Net Change (pp. 363–374)

Quick Review 7.1

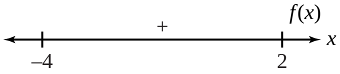
1. Changes sign at $-\frac{\pi}{2}, 0, \frac{\pi}{2}$



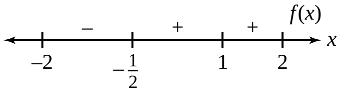
2. Changes sign at 1, 2



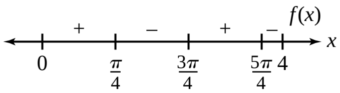
3. Always positive



4. Changes sign at $-\frac{1}{2}$

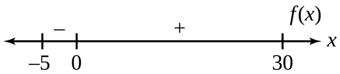


5. Changes sign at $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}$

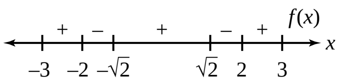


6. Always positive

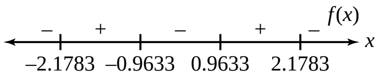
7. Changes sign at 0



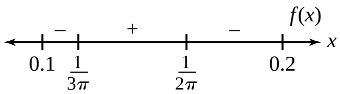
8. Changes sign at $-2, -\sqrt{2}, \sqrt{2}, 2$



9. Changes sign at $0.9633 + k\pi$
$2.1783 + k\pi$
where k is an integer



10. Changes sign at $\frac{1}{3\pi}, \frac{1}{2\pi}$



Section 7.1 Exercises

1. (a) Right: $0 \le t < \frac{\pi}{2}, \frac{3\pi}{2} < t \le 2\pi$
Left: $\frac{\pi}{2} < t < \frac{3\pi}{2}$
Stopped: $t = \frac{\pi}{2}, \frac{3\pi}{2}$
(b) 0 **(c)** 20

2. (a) Right: $0 < t < \frac{\pi}{3}$
Left: $\frac{\pi}{3} < t \le \frac{\pi}{2}$
Stopped: $t = 0, \frac{\pi}{3}$
(b) 2 **(c)** 6

3. (a) Right: $0 \le t < 5$
Left: $5 < t \le 10$
Stopped: $t = 5$
(b) 0 **(c)** 245

4. (a) Right: $0 \le t < 1$
Left: $1 < t < 2$
Stopped: $t = 1, 2$
(b) 4 **(c)** 6

5. (a) Right: $0 < t < \frac{\pi}{2}, \frac{3\pi}{2} < t < 2\pi$
Left: $\frac{\pi}{2} < t < \pi, \pi < t < \frac{3\pi}{2}$
Stopped: $t = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$
(b) 0 **(c)** $\frac{20}{3}$

6. (a) Right: $0 \le t < 4$
Left: never
Stopped: $t = 4$
(b) $\frac{16}{3}$ **(c)** $\frac{16}{3}$

7. (a) Right: $0 \le t < \frac{\pi}{2}, \frac{3\pi}{2} < t \le 2\pi$
Left: $\frac{\pi}{2} < t < \frac{3\pi}{2}$
Stopped: $t = \frac{\pi}{2}, \frac{3\pi}{2}$
(b) 0 **(c)** $2e - \frac{2}{e} \approx 4.7$

8. (a) Right: $0 < t \le 3$
Left: never
Stopped: $t = 0$
(b) $\frac{\ln 10}{2} \approx 1.15$ **(c)** $\frac{\ln 10}{2} \approx 1.15$

9. (a) 63 mph **(b)** 344.52 feet

10. (a) ≈ -1.44952 meters
(b) ≈ 1.91411 meters

11. (a) -6 ft/sec **(b)** 5.625 sec
(c) 0 **(d)** 253.125 feet

12. -23 cm **13.** 33 cm

14. a: 11
b: 16
c: -8

15. $t = a$
16. $t = c$
17. (a) 6 **(b)** 4 meters
18. (a) 2 **(b)** 4 meters
19. (a) 5 **(b)** 7 meters
20. (a) -2.5 **(b)** 19.5 meters
21. $\approx$332.965 billion barrels
22. 93.6 kilowatt-hours
23. (a) 2 miles
(b) $2\pi r\Delta r$
(c) Population = Population density $\times$ Area
(d) $\approx$83,776
24. (a) $2\pi r\Delta r$
(b) $8(10 - r^2)\,\frac{\text{in.}}{\text{sec}} \cdot (2\pi r)\Delta r \text{ in}^2 = \text{flow in } \frac{\text{in}^3}{\text{sec}}$
(c) $396\pi \frac{\text{in}^3}{\text{sec}}$ or $\approx 1244.07 \frac{\text{in}^3}{\text{sec}}$
25. One possible answer:
Plot the speeds vs. time. Connect the points and find the area under the line graph. The definite integral also gives the area under the curve.
26. (a) 797.5 thousand
(b) $B(x) = 1.6x^2 + 2.3x + 5.0$
(c) $\approx$904.02 thousand
(d) The answer in (a) corresponds to the area of left hand rectangles. These rectangles lie under the curve $B(x)$. The answer in (c) corresponds to the area under the curve. This area is greater than the area of the rectangles.
27. (a) $\approx$798.97 thousand
(b) The answer in (a) corresponds to the area of midpoint rectangles. Part of each rectangle is above the curve and part is below.
28. 1156.5
29. (a) 18 N **(b)** 81 N $\cdot$ cm
30. (a) 1250 inch-pounds
(b) 3750 inch-pounds
31. 0.04875
32. 40 thousandths or 0.040
33. (a, b) Take $dm = \delta\, dA$ as m_k and letting $dA \to 0$, $k \to \infty$ in the center of mass equations.
34. $\bar{x} = 0, \bar{y} = \frac{12}{5}$
35. $\bar{x} = \frac{4}{3}, \bar{y} = 0$

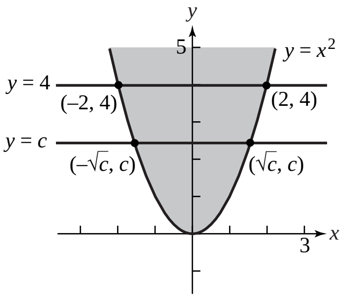
7.2 Areas in the Plane (pp. 374–382)

Quick Review 7.2

1. 2
2. $\frac{1}{2}(e^2 - 1) \approx 3.195$
3. 2
4. 4
5. $\frac{9\pi}{2}$
6. $(6, 12); (-1, 5)$
7. $(0, 1)$
8. $(0, 0); (\pi, 0)$
9. $(-1, -1); (0, 0); (1, 1)$
10. $(-0.9286, -0.8008); (0, 0); (0.9286, 0.8008)$

Section 7.2 Exercises

1. $\frac{\pi}{2}$
2. $\frac{4\pi}{3}$
3. $\frac{1}{12}$
4. $\frac{4}{3}$
5. $\frac{128}{15}$
6. $\frac{22}{15}$
7. $\frac{5}{6}$
8. $\frac{5}{6}$
9. 16
10. $8\frac{1}{6}$
11. $10\frac{2}{3}$
12. $10\frac{2}{3}$
13. 4
14. 8
15. $\frac{2}{3}a^3$
16. $1\frac{2}{3}$ (3 points of intersection)
17. $21\frac{1}{3}$
18. $4\frac{1}{2}$
19. $30\frac{3}{8}$
20. 4
21. $\frac{8}{3}$
22. $6\frac{14}{15}$
23. 8
24. 4
25. $6\sqrt{3}$
26. $\frac{4}{3} - \frac{4}{\pi} \approx 0.0601$
27. $\frac{4 - \pi}{\pi} \approx 0.273$
28. $\frac{\pi}{2}$
29. $4 - \pi \approx 0.858$
30. 2
31. $\frac{1}{2}$
32. 1
33. $\sqrt{2} - 1 \approx 0.414$
34. $\frac{32}{3}$
35. (a) $(-\sqrt{c}, c); (\sqrt{c}, c)$



(b) $\int_0^c \sqrt{y}\, dy = \int_c^4 \sqrt{y}\, dy \Rightarrow c = 2^{4/3}$
(c) $\int_0^{\sqrt{c}} (c - x^2)\, dx$
$= (4 - c)\sqrt{c} + \int_{\sqrt{c}}^2 (4 - x^2)\, dx \Rightarrow c = 2^{4/3}$
36. $\frac{11}{3}$
37. $\frac{3}{4}$
38. 4
39. Neither; both are zero
40. Sometimes; If $f(x) \geq g(x)$ on (a, b), then true.
41. $\ln 4 - \frac{1}{2} \approx 0.886$
42. ≈ 0.4303

43. $k \approx 1.8269$

44. (a) $y = \pm b\sqrt{1 - \frac{x^2}{a^2}}$

(b) $2\int_{-a}^{a} b\sqrt{1 - \frac{x^2}{a^2}}\, dx$

(c) Answers may vary.

(d, e) $ab\pi$

45. Since $f(x) - g(x)$ is the same for each region where $f(x)$ and $g(x)$ represent the upper and lower edges, area $= \int_a^b [f(x) - g(x)]\, dx$ will be the same for each.

46. $m - \ln(m) - 1$

7.3 Volumes (pp. 383–394)

Quick Review 7.3

1. x^2 **2.** $\frac{x^2}{2}$

3. $\frac{\pi x^2}{2}$ **4.** $\frac{\pi x^2}{8}$

5. $\frac{\sqrt{3}}{4}x^2$ **6.** $\frac{x^2}{2}$

7. $\frac{x^2}{4}$ **8.** $\frac{\sqrt{15}}{4}x^2$

9. $6x^2$ **10.** $\frac{3\sqrt{3}}{2}x^2$

Section 7.3 Exercises

1. (a) $\pi(1 - x^2)$ **(b)** $4(1 - x^2)$

(c) $2(1 - x^2)$ **(d)** $\sqrt{3}(1 - x^2)$

2. (a) πx **(b)** $4x$

(c) $2x$ **(d)** $\sqrt{3}x$

3. 16 **4.** $\frac{16}{15}\pi$

5. $\frac{16}{3}$ **6.** $\frac{8}{3}$

7. (a) $2\sqrt{3}$ **(b)** 8

8. (a) $\pi\sqrt{3} - \frac{\pi^2}{6}$ **(b)** $4\sqrt{3} - \frac{2}{3}\pi$

9. 8π **10.** $\frac{8}{3}$

11. (a) s^2h **(b)** s^2h

12. The volumes are equal by Cavalieri's Theorem.

13. $\frac{2}{3}\pi$ **14.** 6π

15. $4 - \pi$ **16.** $\frac{\pi^2}{16}$

17. $\frac{32\pi}{5}$ **18.** $\frac{128\pi}{7}$

19. 36π **20.** $\frac{\pi}{30}$

21. $\frac{2}{3}\pi$ **22.** π

23. $\frac{117\pi}{5}$ **24.** $\frac{108\pi}{5}$

25. $\pi^2 - 2\pi$ **26.** 8π

27. 2.301 **28.** $\pi(3\pi - 8)$

29. 2π **30.** 4π

31. $\frac{4}{3}\pi$ **32.** $\frac{2}{3}\pi$

33. 8π **34.** $\frac{2\pi}{15}$

35. (a) 8π **(b)** $\frac{32\pi}{5}$

(c) $\frac{8\pi}{3}$ **(d)** $\frac{224\pi}{15}$

36. (a) $\frac{2}{3}\pi$ **(b)** $\frac{8\pi}{3}$

37. (a) $\frac{16\pi}{15}$ **(b)** $\frac{56\pi}{15}$

(c) $\frac{64\pi}{15}$

38. (a) $\frac{\pi}{3}bh^2$ **(b)** $\frac{\pi}{3}b^2h$

39. 8π **40.** $\frac{5\pi}{6}$

41. $\frac{128\pi}{5}$ **42.** $\frac{7\pi}{15}$

43. (a) $\frac{6\pi}{5}$ **(b)** $\frac{4\pi}{5}$

(c) 2π **(d)** 2π

44. (a) $\frac{8\pi}{3}$ **(b)** $\frac{8\pi}{5}$

(c) 8π **(d)** 4π

45. (a) $\frac{512\pi}{21}$ **(b)** $\frac{832\pi}{21}$

46. (a) $\frac{\pi}{6}$ **(b)** $\frac{\pi}{6}$

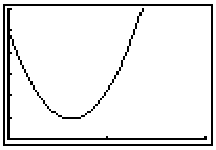
47. (a) $\frac{11\pi}{48}$ **(b)** $\frac{11\pi}{48}$

48. 4π

49. (a) $\frac{36\pi}{5}$ cm^3 **(b)** 192.3 g

50. (a) $\frac{2}{\pi}, \frac{\pi^2 - 8}{2}$ **(b)** 0

(c) $V = \frac{\pi(2c^2\pi - 8c + \pi)}{2}$



[0, 2] by [0, 6]

Volume $\to \infty$

51. (a) 2.3, 1.6, 1.5, 2.1, 3.2, 4.8, 7.0, 9.3, 10.7, 10.7, 9.3, 6.4, 3.2

(b) $\frac{1}{4\pi}\int_0^6 C(y)^2\, dy$

(c) $\approx 34.7\ \text{in}^3$

52. **(a)** 25π **(b)** $\frac{3}{8\pi}$

53. **(a)** $\frac{32\pi}{3}$

(b) The answer is independent of r.

54. Partition the appropriate interval on the axis of revolution and measure the radius $r(x)$ of the shadow region at these points. Then use an approximation such as the trapezoidal rule to estimate the integral $\int_a^b \pi r^2(x)\,dx$.

55. 5

56. For a tiny horizontal slice,

slant height $= \Delta s = \sqrt{(\Delta x)^2 + (\Delta y)^2}$

$= \sqrt{1 + (g'(y))^2}\,\Delta y$. So the surface area is approximated by the Riemann sum

$\sum_{k=1}^{n} 2\pi\, g(y_k)\sqrt{1 + (g'(y))^2}\,\Delta y$. The limit of that is the integral.

57. ≈ 13.614 **58.** ≈ 0.638

59. ≈ 16.110 **60.** ≈ 2.999

61. ≈ 53.226 **62.** ≈ 44.877

63. ≈ 6.283 **64.** ≈ 51.313

65. Hemisphere cross sectional area:

$\pi(\sqrt{R^2 - h^2})^2 = A_1$

Right circular cylinder with cone removed cross sectional area: $\pi R^2 - \pi h^2 = A_2$

Since $A_1 = A_2$, the two volumes are equal by Cavalieri's theorem. Thus,

volume of hemisphere

$= \text{volume of cylinder} - \text{volume of cone}$

$= \pi R^3 - \frac{1}{3}\pi R^3 = \frac{2}{3}\pi R^3.$

66. $2a^2 b\pi^2$

67. **(a)** $\frac{\pi h^2(3a - h)}{3}$

(b) $\frac{1}{120\pi}$ m/sec

68. **(a)** A cross section has radius $r = \sqrt{a^2 - x^2}$ and area $A(x) = \pi r^2 = \pi(a^2 - x^2)$.

$V = \int_{-a}^{a} \pi(a^2 - x^2) = \frac{4}{3}\pi a^3$

(b) A cross section has radius $x = r\left(1 - \frac{y}{h}\right)$ and area $A(y) = \pi x^2 = \pi r^2\left(1 - \frac{2y}{h} + \frac{y^2}{h^2}\right)$.

$V = \int_0^h \pi r^2\left(1 - \frac{2y}{h} + \frac{y^2}{h}\right) dy = \frac{1}{3}\pi r^2 h$

7.4 Lengths of Curves (pp. 395–401)

Quick Review 7.4

1. $x + 1$ **2.** $\frac{2 - x}{2}$

3. $\sec x$ **4.** $\frac{x^2 + 4}{4x}$

5. $\sqrt{2}\cos x$ **6.** 4

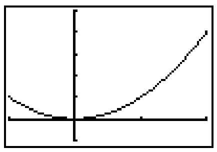
7. 0 **8.** -3

9. 2 **10.** $k\pi$, k any integer

Section 7.4 Exercises

1. **(a)** $\int_{-1}^{2} \sqrt{1 + 4x^2}\,dx$

(b)

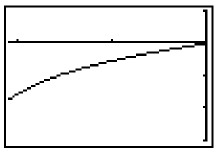


$[-1, 2]$ by $[-1, 5]$

(c) ≈ 6.126

2. **(a)** $\int_{-\pi/3}^{0} \sqrt{1 + \sec^4 x}\,dx$

(b)

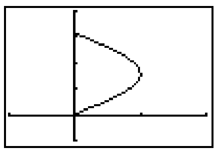


$\left[-\frac{\pi}{3}, 0\right]$ by $[-3, 1]$

(c) ≈ 2.057

3. **(a)** $\int_0^{\pi} \sqrt{1 + \cos^2 y}\,dy$

(b)

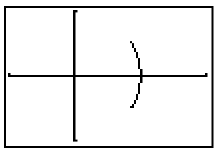


$[-1, 2]$ by $[-1, 4]$

(c) ≈ 3.820

4. **(a)** $\int_{-1/2}^{1/2} \sqrt{1 + \frac{y^2}{1 - y^2}}\,dy$

(b)

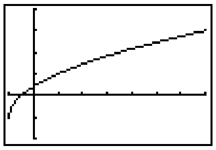


$[-1, 2]$ by $[-1, 1]$

(c) ≈ 1.047

5. (a) $\int_{-1}^{7} \sqrt{1 + \frac{1}{2x+2}}\, dx$

(b)

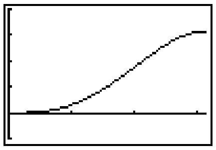


$[-1, 7]$ by $[-2, 4]$

(c) ≈ 9.294

6. (a) $\int_{0}^{\pi} \sqrt{1 + x^2 \sin^2 x}\, dx$

(b)

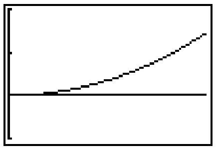


$[0, \pi]$ by $[-1, 4]$.

(c) ≈ 4.698

7. (a) $\int_{0}^{\pi/6} \sqrt{1 + \tan^2 x}\, dx$

(b)

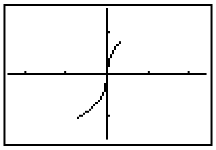


$\left[0, \frac{\pi}{6}\right]$ by $[-0.1, 0.2]$

(c) ≈ 0.549

8. (a) $\int_{-\pi/3}^{\pi/4} \sec y\, dy$

(b)

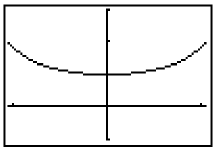


$[-2.4, 2.4]$ by $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

(c) ≈ 2.198

9. (a) $\int_{-\pi/3}^{\pi/3} \sqrt{1 + \sec^2 x \tan^2 x}\, dx$

(b)

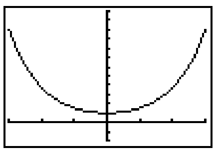


$\left[-\frac{\pi}{3}, \frac{\pi}{3}\right]$ by $[-1, 3]$

(c) ≈ 3.139

10. (a) $\int_{-3}^{3} \sqrt{1 + \left(\frac{e^x - e^{-x}}{2}\right)^2}\, dx$

(b)



$[-3, 3]$ by $[-2, 12]$

(c) ≈ 20.036

11. 12

12. $\frac{80\sqrt{10} - 8}{27}$

13. $\frac{53}{6}$

14. $\frac{123}{32}$

15. $\frac{17}{12}$

16. $\frac{53}{6}$

17. 2

18. $\frac{7\sqrt{3}}{3}$

19. (a) $y = \sqrt{x}$

(b) Only one. We know the derivative of the function and the value of the function at one value of x.

20. (a) $y = \frac{1}{1 - x}$

(b) Only one. We know the derivative of the function and the value of the function at one value of x.

21. 1

22. 6

23. ≈ 21.07 inches

24. \$38,422

25. $\approx(-19.909, 8.410)$

26. ≈ 3.6142

27. ≈ 2.1089

28. ≈ 13.132

29. ≈ 1.623

30. ≈ 16.647

31. Because the limit of the sum $\sum \Delta x_k$ as the norm of the partition goes to zero will always be the length $(b - a)$ of the interval (a, b).

32. No. Consider the curve $y = \frac{1}{3} \sin\left(\frac{1}{x}\right) + 0.5$ for $0 < x < 1$.

33. (a) The fin is the hypotenuse of a right triangle with leg lengths Δx_k and $\left.\frac{df}{dx}\right|_{x = x_{k-1}} \Delta x_k = f'(x_{k-1})\, \Delta x_k$.

(b) $\lim_{n\to\infty} \sum_{k=1}^{n} \sqrt{(\Delta x_k)^2 + (f'(x_{k-1})\, \Delta x_k)^2}$

$= \lim_{n\to\infty} \sum_{k=1}^{n} \Delta x_k \sqrt{1 + (f'(x_{k-1}))^2}$

$= \int_{a}^{b} \sqrt{1 + (f'(x))^2}\, dx$

34. Yes. Any curve of the form $y = \pm x + c$, c a constant.

7.5 Applications from Science and Statistics (pp. 401–411)

Quick Review 7.5

1. a. $1 - \frac{1}{e}$ **b.** ≈ 0.632

2. a. $e - 1$ **b.** ≈ 1.718

3. a. $\frac{\sqrt{2}}{2}$ **b.** ≈ 0.707

4. 15

5. a. $\frac{1}{3} \ln\left(\frac{9}{2}\right)$ **b.** ≈ 0.501

6. $\int_0^7 2\pi(x+2)\sin x\,dx$ **7.** $\int_0^7 (1-x^2)(2\pi x)\,dx$

8. $\int_0^7 \pi\cos^2 x\,dx$ **9.** $\int_0^7 \pi\left(\frac{y}{2}\right)^2(10-y)\,dy$

10. $\int_0^7 \frac{\sqrt{3}}{4}\sin^2 x\,dx$

Section 7.5 Exercises

1. ≈4.4670 J **2.** ≈3.8473 J
3. 9 J **4.** ≈19.5804 J
5. 4900 J **6.** 5880 J
7. 1944 ft-lb
8. **(a)** 200 lb/in. **(b)** 400 in.-lb
(c) 8 in.
9. **(a)** 7238 lb/in.
(b) ≈905 in.-lb and ≈2714 in.-lb
10. **(a)** 300 lb **(b)** 18.75 in.-lb
11. 780 J
12. **(b)** −37,968.75 in.-lb
13. 1123.2 lb **14.** 7987.2 lb
15. 3705 lb **16.** ≈1506.1 lb
17. **(a)** 1,497,600 ft-lb **(b)** ≈100 min
(d) 1,494,240 ft-lb, ≈100 min
1,500,000 ft-lb, 100 min
18. ≈7,238,229 ft-lb
19. Through valve: ≈84,687.3 ft-lb
Over the rim: ≈98,801.8 ft-lb
Through a hose attached to a valve in the bottom is faster, because it takes more time to do more work.
20. ≈91.3244 in.-oz
21. ≈53,482.5 ft-lb **22.** ≈34,582.65 ft-lb
23. ≈967,611 ft-lb, yes **24.** ≈31 hr
25. **(a)** ≈209.73 lb
(b) ≈838.93 lb; the fluid force doubles
26. ≈4.2 lb
27. **(a)** 0.5 (50%) **(b)** ≈0.24 (24%)
(c) ≈0.0036 (0.36%)
(d) 0 if we assume a continuous distribution; ≈0.071; 7.1% between 59.5 in. and 60.5 in.
28. **(a)** ≈0.34 (34%) **(b)** 6.1
29. Integration is a good approximation to the area.
30. The proportion of lightbulbs that last between 100 and 800 hours.
31. 5.1446×10^{10} J
32. **(a)** 1.15×10^{-28} J
(b) $\approx 7.6667 \times 10^{-29}$ J

33. $F = m\frac{dv}{dt} = mv\frac{dv}{dx}$, so $W = \int_{x_1}^{x_2} F(x)\,dx$

$$= \int_{x_1}^{x_2} mv\frac{dv}{dx}\,dx = \int_{v_1}^{v_2} mv\,dv = \frac{1}{2}mv_2{}^2 - \frac{1}{2}mv_1{}^2$$

34. 50 ft-lb **35.** ≈85.1 ft-lb
36. 122.5 ft-lb **37.** ≈64.6 ft-lb
38. ≈109.7 ft-lb **39.** ≈110.6 ft-lb
40. 4.5 ft

Chapter 7 Review Exercises (pp. 413–415)

1. ≈10.417 ft **2.** ≈31.361 gal
3. ≈1464 **4.** 14 g
5. 14,400 **6.** 1
7. $\frac{9}{2}$ **8.** $\frac{1}{6}$
9. 18 **10.** 30.375
11. ≈0.0155 **12.** 4
13. ≈8.9023 **14.** ≈2.1043
15. $2\sqrt{3} - \frac{2}{3}\pi \approx 1.370$ **16.** $2\sqrt{3} + \frac{4}{3}\pi \approx 7.653$
17. ≈1.2956 **18.** ≈5.7312
19. 4π **20.** 2π
21. **(a)** $\frac{32\pi}{3}$ **(b)** $\frac{128\pi}{15}$
(c) $\frac{64\pi}{5}$ **(d)** $\frac{32\pi}{3}$
22. **(a)** 4π **(b)** πk^2
(c) $\frac{1}{\pi}$
23. $88\pi \approx 276$ in^3 **24.** $\frac{\pi^2}{4}$
25. $\pi(2 - \ln 3)$
26. $\frac{28\pi}{3}$ ft$^3 \approx 29.3215$ ft^3
27. ≈19.4942 **28.** ≈5.2454
29. 2.296 sec **30.** (a) is true
31. 39
32. **(a)** 4000 J **(b)** 640 J
(c) 4640 J
33. 22,800,000 ft-lb **34.** 12 J, ≈213.3 J
35. No, the work going uphill is positive, but the work going downhill is negative.
36. ≈113.097 in.-lb **37.** ≈426.67 lbs
38. base ≈ 6.6385 lb, front and back: 5.7726 lb, sides ≈ 9.4835 lb
39. ≈14.4
40. ≈0.2051 (20.5%)
41. Answers will vary.
42. **(a)** ≈0.6827 (68.27%)
(b) ≈0.9545 (95.45%)
(c) ≈0.9973 (99.73%)
43. The probability that the variable has some value in the range of all possible values is 1.
44. π **45.** 3π
46. $2\pi^2$ **47.** $\frac{16\pi}{3}$
48. ≈9.7717
49. **(a)** $y = 5 - \frac{5}{4}x^2$ **(b)** ≈335.1032 in^3
50. $f(x) = \frac{x^2 - 2\ln x + 3}{4}$
51. ≈3.84 **52.** ≈5.02

Chapter 8
L'Hôpital's Rule, Improper Integrals, and Partial Fractions

8.1 L'Hôpital's Rule (pp. 417–425)

Quick Review 8.1

1. 1.1052 **2.** 2.7183
3. 1 **4.** ∞
5. 2 **6.** 2
7. 3 **8.** 1
9. $y = \frac{\sin h}{h}$ **10.** $y = (1 + h)^{1/h}$

Section 8.1 Exercises

1. $\frac{1}{4}$ **2.** 5
3. 1 **4.** $\frac{5}{7}$
5. $\frac{3}{11}$ **6.** $\frac{1}{2}$
7. e^2 **8.** 0
9. (a)

x	10	10^2	10^3	10^4	10^5
$f(x)$	1.1513	0.2303	0.0354	0.0046	0.00058

Estimated limit = 0

(b) Note $\ln x^5 = 5 \ln x$.

$$\lim_{x\to\infty} \frac{5 \ln x}{x} = \lim_{x\to\infty} \frac{5/x}{1} = \frac{0}{1} = 0$$

10. (a)

x	10^0	10^{-1}	10^{-2}	10^{-3}	10^{-4}
$f(x)$	0.1585	0.1666	0.1667	0.1667	0.1667

Estimated limit = $\frac{1}{6}$

(b) $$\lim_{x\to 0^+} \frac{x - \sin x}{x^3} = \lim_{x\to 0^+} \frac{1 - \cos x}{3x^2} = \lim_{x\to 0^+} \frac{\sin x}{6x} = \lim_{x\to 0^+} \frac{\cos x}{6} = \frac{1}{6}$$

11. $\frac{3}{4}$ **12.** 0
13. 1 **14.** $-\frac{2}{3}$
15. 0 **16.** $\frac{1}{4}$
17. -1 **18.** $\frac{1}{\pi + 1}$
19. ln 2 **20.** $\frac{\ln 3}{\ln 2}$
21. 1 **22.** 1
23. 0 **24.** 1
25. 1 **26.** ln 2
27. 0 **28.** ∞
29. e^2 **30.** 1
31. 0 **32.** $\frac{7}{11}$
33. 1 **34.** $e^{1/2}$
35. 1 **36.** 1
37. e **38.** e
39. 1 **40.** 1
41. e^{-1} **42.** ln 2

43. (a) L'Hôpital's Rule does not help because applying l'Hôpital's Rule to this quotient essentially "inverts" the problem by interchanging the numerator and denominator. It is still essentially the same problem and one is no closer to a solution. Applying l'Hôpital's Rule a second time returns to the original problem.
(b, c) 3

44. (a) L'Hôpital's Rule does not help because applying l'Hôpital's Rule to this quotient essentially "inverts" the problem by interchanging the numerator and denominator. It is still essentially the same problem and one is no closer to a solution. Applying l'Hôpital's Rule a second time returns to the original problem.
(b, c) 1

45. Possible answers:
(a) $f(x) = 7(x - 3)$, $g(x) = x - 3$
(b) $f(x) = (x - 3)^2$, $g(x) = x - 3$
(c) $f(x) = x - 3$, $g(x) = (x - 3)^3$

46. Possible answers:
(a) $f(x) = 3x + 1$, $g(x) = x$
(b) $f(x) = x + 1$, $g(x) = x^2$
(c) $f(x) = x^2$, $g(x) = x + 1$

47. $c = \frac{27}{10}$, because this is the limit of $f(x)$ as x approaches 0.

48. The limit of $f(x)$ as $x \to 0$ is 1. Therefore, f has a removable discontinuity at $x = 0$, and the definition of f should be extended by defining $f(0) = 1$.

49. (a) $\ln\left(1 + \frac{r}{k}\right)^{kt} = kt \ln\left(1 + \frac{r}{k}\right)$. And, as $k \to \infty$,

$$\lim_{k\to\infty} kt \ln\left(1 + \frac{r}{k}\right) = \lim_{k\to\infty} \frac{t \ln\left(1 + \frac{r}{k}\right)}{\frac{1}{k}}$$

$$= \lim_{k\to\infty} \frac{t\left(\frac{-r}{k^2}\right)\left(1 + \frac{r}{k}\right)^{-1}}{\frac{-1}{k^2}}.$$

$$= \lim_{k\to\infty} \frac{rt}{1 + \frac{r}{k}} = rt.$$

Hence, $\lim_{k\to\infty} A_0\left(1 + \frac{r}{k}\right)^{kt} = A_0 e^{rt}$.

49. continued

(b) Part (a) shows that as the number of compoundings per year increases toward infinity, the limit of interest compounded k times per year is interest compounded continuously.

50. (a) For $x \neq 0$, $\dfrac{f'(x)}{g'(x)} = \dfrac{1}{1} = 1$ and $\dfrac{f(x)}{g(x)} = \dfrac{x+2}{x+1} \to 2$ as $x \to 0$.

(b) This does not contradict l'Hôpital's Rule since $\lim_{x\to 0} f(x) = 2$ and $\lim_{x\to 0} g(x) = 1$.

51. (a) 1

(b) $\dfrac{\pi}{2}$

(c) π

52. (a) 0

(b) L'Hôpital's Rule cannot be applied to $\dfrac{\sin x}{1+2x}$ because the denominator has limit 1.

53. (a) $(-\infty, -1) \cup (0, \infty)$

(b) ∞

(c) e

54. (a) Because the difference in the numerator is so small compared to the values being subtracted, any calculator or computer with limited precision will give the incorrect result that $1 - \cos x^6$ is 0 for even moderately small values of x. For example, at $x = 0.1$, $\cos x^6 \approx 0.9999999999995$ (13 places), so on a 10-place calculator, $\cos x^6 = 1$ and $1 - \cos x^6 = 0$.

(b) Same as (a)

(c) $\dfrac{1}{2}$

(d) The graph and/or table on a grapher show the value of the function to be 0 for x-values moderately close to 0, but the limit is 1/2. The calculator is giving unreliable information because there is significant round-off error in computing values of this function on a limited precision device.

55. (a) $c = \dfrac{1}{3}$

(b) $c = \dfrac{\pi}{4}$

56. (a) $\ln f(x)^{g(x)} = g(x) \ln f(x)$.

$\lim_{x\to c} (g(x) \ln f(x)) = \left(\lim_{x\to c} g(x)\right)\left(\lim_{x\to c} \ln f(x)\right) = (\infty)(-\infty) = -\infty$

Therefore, $\lim_{x\to c} f(x)^{g(x)} = 0$.

(b) $\lim_{x\to c} (g(x) \ln f(x)) = (-\infty)(-\infty) = \infty$

Therefore, $\lim_{x\to c} f(x)^{g(x)} = \infty$.

8.2 Relative Rates of Growth (pp. 425–433)

Quick Review 8.2

1. 0
2. ∞
3. ∞
4. 0
5. $-3x^4$
6. $2x^2$
7. $\lim_{x\to\infty} \dfrac{f(x)}{g(x)} = \lim_{x\to\infty} \left(1 + \dfrac{\ln x}{x}\right) = 1 + 0 = 1$
8. $\lim_{x\to\infty} \dfrac{f(x)}{g(x)} = \lim_{x\to\infty} \sqrt{1 + \dfrac{5}{4x}} = 1$
9. (a) Local minimum at (0, 1)
Local maximum at $\approx$(2, 1.541)
(b) [0, 2]
(c) $(-\infty, 0]$ and $[2, \infty)$
10. f doesn't have an absolute maximum value. The values are always less than 2 and the values get arbitrarily close to 2 near $x = 0$, but the function is undefined at $x = 0$.

Section 8.2 Exercises

1. Slower
2. Slower
3. Faster
4. Slower
5. Same rate
6. Slower
7. Slower
8. Faster
9. Slower
10. Same rate
11. Same rate
12. Faster
13. Slower
14. Same rate
15. Slower
16. Faster
17. Same rate
18. Same rate
19. Slower
20. Slower
21. Faster
22. Same rate
23. $e^{x/2}, e^x, (\ln x)^x, x^x$
24. $(\ln 2)^x, x^2, 2^x, e^x$
25. $\lim_{x\to\infty} \dfrac{f_2(x)}{f_1(x)} = \sqrt{10}$ and $\lim_{x\to\infty} \dfrac{f_3(x)}{f_1(x)} = 1$, so f_2 and f_3 also grow at the same rate.
26. $\lim_{x\to\infty} \dfrac{f_2(x)}{f_1(x)} = 1$ and $\lim_{x\to\infty} \dfrac{f_3(x)}{f_1(x)} = 1$, so f_2 and f_3 also grow at the same rate.
27. $\lim_{x\to\infty} \dfrac{f_2(x)}{f_1(x)} = 1$ and $\lim_{x\to\infty} \dfrac{f_3(x)}{f_1(x)} = 1$, so f_2 and f_3 also grow at the same rate.
28. $\lim_{x\to\infty} \dfrac{f_2(x)}{f_1(x)} = 1$ and $\lim_{x\to\infty} \dfrac{f_3(x)}{f_1(x)} = 2$, so f_2 and f_3 also grow at the same rate.
29. (a) False
(b) False
(c) True
(d) True
(e) True
(f) True
(g) False
(h) True

30. (a) True **(b)** True
(c) False **(d)** True
(e) True **(f)** True
(g) True **(h)** False

31. $g = o(f)$ **32.** $f = o(g)$

33. f and g grow at the same rate.

34. f and g grow at the same rate.

35. (a) The n^{th} derivative of x^n is $n!$, which is a constant. Therefore n applications of l'Hôpital's Rule give $\lim_{x\to\infty} \frac{e^x}{x^n} = \cdots = \lim_{x\to\infty} \frac{e^x}{n!} = \infty$.

(b) In this case, n applications of l'Hôpital's Rule give $\lim_{x\to\infty} \frac{a^x}{x^n} = \cdots = \lim_{x\to\infty} \frac{a^x(\ln a)^n}{n!} = \infty$

36. (a) $\lim_{x\to\infty} \frac{e^x}{a_n x^n + \cdots + a_0} = \lim_{x\to\infty} \frac{e^x}{a_n n!} = \infty$

(b) $\lim_{x\to\infty} \frac{a^x}{a_n x^n + \cdots + a_0} = \lim_{x\to\infty} \frac{(\ln a)^n a^x}{a_n n!} = \infty$

37. (a) $\lim_{x\to\infty} \frac{\ln x}{x^{1/n}} = \lim_{x\to\infty} \frac{\frac{1}{x}}{\frac{x^{(1/n)-1}}{n}} = \lim_{x\to\infty} \frac{n}{x^{1/n}} = 0$

(b) $\lim_{x\to\infty} \frac{\ln x}{x^a} = \lim_{x\to\infty} \frac{\frac{1}{x}}{ax^{a-1}} = \lim_{x\to\infty} \frac{1}{ax^a} = 0$

38. Let $p(x)$ be any nonconstant polynomial. Then $p'(x)$ is either a nonzero constant or a polynomial of degree at least one. Therefore,

$$\lim_{x\to\infty} \frac{\ln x}{p(x)} = \lim_{x\to\infty} \frac{\frac{1}{x}}{p'(x)} = \lim_{x\to\infty} \frac{1}{xp'(x)} = 0$$

and ln grows slower than $p(x)$.

39. The one which is $O(n \log_2 n)$ is likely the most efficient, because of the three given functions, it grows the most slowly as $n \to \infty$.

40. (a) 1,000,000
(b) 20

41. This is the case because if $\lim_{x\to\infty} \frac{f(x)}{g(x)} = L$ where L is a nonzero finite real number, then for sufficiently large x, it must be the case that $\frac{f(x)}{g(x)} < L + 1 \le M$, for some integer M. Similarly for $g = O(f)$.

42. (a) The limit will be the ratio of the leading coefficients of the polynomials.
(b) The limit will be the same as in (a).

43. (a) x^5 grows faster than x^2.
(b) They grow at the same rate.
(c) $m > n$
(d) $m = n$
(e) $m > n$ (or, degree of $g >$ degree of f)
(f) $m = n$ (or, degree of $g =$ degree of f)

44. (a) $f = o(g)$ as $x \to a$ if $\lim_{x\to a} \frac{f(x)}{g(x)} = 0$.
Suppose that f and g are both positive in some open interval containing a. Then $f = O(g)$ as $x \to a$ if there is a positive integer M for which $\frac{f(x)}{g(x)} \le M$ for x sufficiently close to a.

(b) $\frac{|E_s|}{h^4} \le (b-a)\frac{M}{180} \le \text{int}\left[(b-a)\frac{M}{180}\right] + 1$, for all values of h, where M is a bound for the absolute value of $f^{(4)}$.

(c) $\frac{|E_T|}{h^2} \le (b-a)\frac{M}{12} \le \text{int}\left[(b-a)\frac{M}{12}\right] + 1$, for all values of h, where M is a bound for the absolute value of f''.

45. (a) and (b) both follow from the fact that if f and g are negative, then
$\lim_{x\to\infty} \frac{|f(x)|}{|g(x)|} = \lim_{x\to\infty} \frac{-f(x)}{-g(x)} = \lim_{x\to\infty} \frac{f(x)}{g(x)}$

46. (a) and (b) both follow from the fact that
$\lim_{x\to-\infty} \frac{f(x)}{g(x)} = \lim_{x\to\infty} \frac{f(-x)}{g(-x)}$

8.3 Improper Integrals (pp. 433–444)

Quick Review 8.3

1. ln 2 **2.** 0

3. $\frac{1}{2}\tan^{-1}\frac{x}{2} + C$ **4.** $-\frac{1}{3}x^{-3} + C$

5. $(-3, 3)$ **6.** $(1, \infty)$

7. Because $-1 \le \cos x \le 1$ for all x

8. Because $\sqrt{x^2 - 1} < \sqrt{x^2} = x$ for $x > 1$

9. $\lim_{x\to\infty} \frac{4e^x - 5}{3e^x + 7} = \frac{4}{3}$ **10.** $\lim_{x\to\infty} \frac{\sqrt{2x-1}}{\sqrt{x+3}} = \sqrt{2}$

Section 8.3 Exercises

1. (a) Because of an infinite limit of integration
(b) Converges
(c) $\frac{\pi}{2}$

2. (a) Because the integrand has an infinite discontinuity at $x = 0$
(b) Converges
(c) 2

3. (a) Because the integrand has an infinite discontinuity at $x = 0$
(b) Converges
(c) $-\frac{9}{2}$

4. (a) Because of two infinite limits of integration
(b) Converges
(c) 0

5. (a) Because the integrand has an infinite discontinuity at $x = 0$
(b) Diverges
(c) No value

6. (a) Because the integrand has an infinite discontinuity at $x = 0$
(b) Diverges
(c) No value

7. 1000
8. 6
9. 4
10. 1000
11. $\frac{\pi}{2}$
12. $\frac{3\pi}{4}$
13. $\ln 3$
14. $3 \ln 2$
15. $\sqrt{3}$
16. $\frac{\pi}{2} + 2$
17. π
18. $\frac{\pi}{2}$
19. $\frac{\pi}{3}$
20. $\ln 2$
21. $2\pi^2$
22. 6
23. -1
24. 1
25. 2
26. $-\frac{1}{4}$
27. Diverges
28. Converges
29. Converges
30. Converges
31. Converges
32. Diverges
33. Converges
34. Diverges
35. Diverges
36. Converges
37. Converges
38. Diverges
39. Converges
40. Diverges
41. Diverges
42. Converges
43. Converges
44. Converges
45. Converges
46. Diverges
47. 1
48. ∞

49. (a) The integral in Example 1 gives the area of the region.
(b) ∞
(c) π
(d) Gabriel's horn has finite volume so it could only hold a finite amount of paint, but it has infinite surface area so it would require an infinite amount of paint to cover itself.

50. (a) Increasing on $(-\infty, 0]$; decreasing on $[0, \infty)$; local maximum at $\left(0, \frac{1}{\sqrt{2\pi}}\right)$
(b) $n = 1$: integral ≈ 0.683
$n = 2$: integral ≈ 0.954
$n = 3$: integral ≈ 0.997
(c) We can make $\int_{-b}^{b} f(x)\,dx$ as close to 1 as we want by choosing $b > 1$ large enough. Also we can make $\int_{b}^{\infty} f(x)\,dx$ and $\int_{-\infty}^{-b} f(x)\,dx$ as small as we want by choosing b large enough.

51. (a) For $x \geq 6$, $x^2 \geq 6x$, and therefore, $e^{-x^2} \leq e^{-6x}$. The inequality for the integrals follows. The value of the second integral is $\frac{e^{-36}}{6}$, which is less than 4×10^{-17}.
(b) The error in the estimate is the integral over the interval $[6, \infty)$, and we have shown that it is bounded by 4×10^{-17} in part (a).
(c) 0.13940279264 (This agrees with Figure 8.16.)
(d) $\int_0^{\infty} e^{-x^2}\,dx = \int_0^{3} e^{-x^2}\,dx + \int_3^{\infty} e^{-x^2}\,dx$. The error in the approximation is $\int_3^{\infty} e^{-x^2}\,dx \leq \int_3^{\infty} e^{-3x}\,dx < 0.000042$.

52. (a) Since f is an even function, the substitution $u = -x$ gives $\int_{-\infty}^{0} f(x)\,dx = \int_0^{\infty} f(u)\,du$.
(b) Since f is an odd function, the substitution $u = -x$ gives $\int_{-\infty}^{0} f(x)\,dx = -\int_0^{\infty} f(u)\,du$.

53. (a) It is divergent because as $x \to \infty$, $\lim_{x\to\infty} \ln(x^2 + 1) = \infty$.
(b) *Both* the integral over $[0, \infty)$ and the integral over $(-\infty, 0]$ must converge in order for the integral over $(-\infty, \infty)$ to converge.
(c) Since this is an odd function, the integral over any interval of the form $[-b, b]$ equals 0. Therefore the limit as $b \to \infty$ is 0.
(d) Because the determination of convergence is not made using the method in part (c). In order for the integral to converge, there must be finite areas in both directions (toward ∞ and toward $-\infty$). In this case, there are infinite areas in both directions, but when one computes the integral over an interval $[-b, b]$, there is cancellation which gives 0 as the result.

54. 6

55. From the properties of integrals, for any $b > a$, $\int_a^b f(x) \leq \int_a^b g(x)\,dx$.
If the infinite integral of g converges, then taking the limit in the above inequality as $b \to \infty$ shows that the infinite integral of f is bounded above by the infinite integral of g. Therefore, the infinite integral of f must be finite and it converges.
If the infinite integral of f diverges, it must grow to infinity. So taking the limit in the above inequality as $b \to \infty$ shows that the infinite integral of g must also diverge to infinity.

56. (a) $n = 0$: integral $= 1$
$n = 1$: integral $= 1$
$n = 2$: integral $= 2$

(b) Integration by parts gives
$\int x^n e^{-x}\,dx = -x^n e^{-x} + n\int x^{n-1} e^{-x}\,dx + C.$
Since the term $(-x^n e^{-x})$ has value 0 at $x = 0$ and has limit equal to 0 as $x \to \infty$, when the above equation is evaluated "from 0 to infinity", it gives $f(n + 1) = n\,f(n)$.

(c) This follows from the formula

$f(n + 1) = n\,f(n)$ by an induction argument.

In fact, it follows that $\int_0^{\infty} x^n e^{-x}\,dx = n!$.

57. (a) Although the values oscillate a bit, they appear to be approaching a limit of approximately 1.57.

(b) Yes, it converges.

58. (a) $\int_{-\infty}^{1} \frac{dx}{1+x^2} = \frac{3\pi}{4}, \int_1^{\infty} \frac{dx}{1+x^2} = \frac{\pi}{4}$
$\int_{-\infty}^{\infty} \frac{dx}{1+x^2} = \frac{3\pi}{4} + \frac{\pi}{4} = \pi$

(b) $\int_{-\infty}^{c} f(x)\,dx = \int_{-\infty}^{0} f(x)\,dx + \int_0^c f(x)\,dx$
$\int_c^{\infty} f(x)\,dx = \int_c^0 f(x)\,dx + \int_0^{\infty} f(x)\,dx$

Thus,

$$\int_{-\infty}^{c} f(x)\,dx + \int_c^{\infty} f(x)\,dx$$
$$= \int_{-\infty}^{0} f(x)\,dx + \int_0^c f(x)\,dx + \int_c^0 f(x)\,dx + \int_0^{\infty} f(x)\,dx$$
$$= \int_{-\infty}^{0} f(x)\,dx + \int_0^{\infty} f(x)\,dx,$$

because

$$\int_0^c f(x)\,dx + \int_c^0 f(x)\,dx = 0.$$

8.4 Partial Fractions and Integral Tables (pp. 444–453)

Quick Review 8.4

1. $A = -2, B = 1$
2. $A = -1, B = 2, C = 3$
3. $2x + 1 + \frac{x-3}{x^2 - 3x - 4}$
4. $2 + \frac{3x-4}{x^2 + 4x + 5}$
5. $(x - 3)(x^2 + 1)$
6. $(y - 2)(y + 2)(y - 1)(y + 1)$
7. $-\frac{x+13}{x^2 + x - 6}$
8. $\frac{-x^2 + 12x - 15}{(x+5)(x^2 - 4x + 5)}$
9. $-\frac{2t^3 + 5t^2 + 5t + 9}{(t^2 + 2)(t^2 + 1)}$
10. $\frac{2x^2 - 7x + 6}{(x-1)^3}$

Section 8.4 Exercises

1. $\frac{2}{x-1} + \frac{3}{x-2}$
2. $\frac{2}{x-1} + \frac{4}{(x-1)^2}$
3. $\frac{2}{t-1} - \frac{2}{t} - \frac{1}{t^2}$
4. $\frac{2}{5(s+2)} + \frac{4}{15(s-3)} - \frac{2}{3s}$
5. $1 - \frac{12}{x-2} + \frac{17}{x-3}$
6. $y + \frac{-4y+1}{y^2+4}$
7. $\frac{1}{2}\ln|x+1| - \frac{1}{2}\ln|x-1| + C$
8. $\frac{1}{2}\ln|x| - \frac{1}{2}\ln|x+2| + C$
9. $\frac{1}{4}\ln|y+1| + \frac{3}{4}\ln|y-3| + C$
10. $4\ln|y| - 3\ln|y+1| + C$
11. $\frac{1}{6}\ln|t+2| + \frac{1}{3}\ln|t-1| - \frac{1}{2}\ln|t| + C$
12. $\frac{1}{16}\ln|t+2| + \frac{5}{16}\ln|t-2| - \frac{3}{8}\ln|t| + C$
13. $\frac{s^2}{2} - 2\ln(s^2+4) + C$
14. $\frac{s^3}{3} - s + \ln(s^2+1) + \tan^{-1} s + C$
15. $5x - \frac{5}{2}\ln(x^2+x+1) - \frac{5}{\sqrt{3}}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + C$
16. $\frac{1}{4}\ln|x-1| + \frac{3}{4}\ln|x+1| + \frac{1}{2(x+1)} + C$
17. $\frac{1}{4}\ln|x+1| - \frac{1}{4(x+1)} - \frac{1}{4}\ln|x-1| - \frac{1}{4(x-1)} + C$
18. $\frac{5}{7}\ln|x-1| + \frac{2}{7}\ln|x+6| + C$
19. $2\tan^{-1}(r-1) + C$
20. $3\tan^{-1}(r-2) + C$
21. $\ln|x^2+x+1| - \ln|x-1| + C$
22. $3\ln|x+1| - \ln(x^2-x+1) + C$
23. $\frac{1}{x^2+4} + \frac{3}{2}\tan^{-1}\frac{x}{2} + C$
24. $2\tan^{-1}x + \frac{1}{2}\ln(x^2+1) + \frac{1}{2(x^2+1)} + C$
25. $1 - \ln 2$
26. $2 - \tan^{-1} 2$
27. $\ln|y-1| - \ln|y| = e^x - 1 - \ln 2$
28. $y = \frac{1}{\cos\theta + 1} - 1$
29. $y = \ln|x-2| - \ln|x-1| + \ln 2$

30. $\ln|s+1| = \ln|t| - \ln|t+2| + \ln 6$ or $|s+1| = 6\left|\dfrac{t}{t+2}\right|$

31. (a) Use $5 + 4x - x^2 = 9 - (x-2)^2$, substitute $u = x - 2$, then use Formula 18 with $x = u$ and $a = 3$.

The integral is $\dfrac{1}{6}\ln\left|\dfrac{x+1}{x-5}\right| + C$.

(b) Rewrite as $\dfrac{1}{2a}\Big(\ln|x+a| - \ln|x-a|\Big)$. Differentiating gives $\dfrac{1}{2a}\left(\dfrac{1}{x+a} - \dfrac{1}{x-a}\right)$ which equals $\dfrac{1}{a^2 - x^2}$.

32. (a) Use $x^2 - 2x + 2 = (x-1)^2 + 1$, substitute $u = x - 1$, then use Formula 17 with $x = u$ and $a = 1$.

The integral is
$\dfrac{x-1}{2(x^2-2x+2)} + \dfrac{1}{2}\tan^{-1}(x-1) + C$.

(b) The derivative is
$$\frac{1}{2a^2}\left[\frac{(1)(a^2+x^2)-(x)(2x)}{(a^2+x^2)^2}\right] + \frac{1}{2a^3}\left[\frac{1/a}{1+\left(\frac{x}{a}\right)^2}\right]$$

which equals $\dfrac{1}{(a^2+x^2)^2}$.

33. $6\pi \ln 5$

34. $\dfrac{4\pi \ln 2}{3}$

35. $\ln\left|\sqrt{9+y^2} + y\right| + C$

36. $\dfrac{25}{2}\sin^{-1}\left(\dfrac{t}{5}\right) + \dfrac{t\sqrt{25-t^2}}{2} + C$

37. $\dfrac{1}{2}\ln\left|2x + \sqrt{4x^2 - 49}\right| + C$

38. $-\dfrac{\sqrt{x^2+1}}{x} + C$

39. $-\dfrac{x^2\sqrt{1-x^2}}{3} - \dfrac{2}{3}\sqrt{1-x^2} + C$

40. $\sec^{-1}x + \dfrac{\sqrt{x^2-1}}{x^2} + C$

41. $\sqrt{16-z^2} - 4\ln\left|\dfrac{4}{z} + \dfrac{\sqrt{16-z^2}}{z}\right| + C$

42. $\dfrac{-2\sqrt{4-w^2}}{w} + C$

43. $y = \ln\left|\dfrac{x}{3} + \dfrac{\sqrt{x^2-9}}{3}\right|$

44. $y = \dfrac{x}{\sqrt{x^2+1}} + 1$

45. $\dfrac{3\pi}{4} \approx 2.356$

46. $\pi\left(\dfrac{\pi}{2} + 1\right) \approx 8.076$

47. (a) $x(t) = \dfrac{1000e^{4t}}{e^{4t} + 499}$ or $x(t) = \dfrac{1000}{1 + 499e^{-4t}}$

(b) After $t = \dfrac{\ln 499}{4} \approx 1.553$ days

(c) Since $\dfrac{dx}{dt} = kx(1000 - x)$, $\dfrac{dx}{dt}$ will have its maximum value where $x(1000 - x)$ is greatest, which is at $x = 500$.

48. $\ln 3 - \dfrac{1}{2}$

49. (a) This can be seen geometrically in the figure.

(b) Using part (a), substitute $z = \dfrac{\sin x}{1 + \cos x}$ and then obtain a trigonometric identity.

Or, use the trigonometric identity $\dfrac{1 - \tan^2\theta}{1 + \tan^2\theta} = \cos 2\theta$ with $\theta = \dfrac{x}{2}$.

(c) Using part (a), substitute $z = \dfrac{\sin x}{1 + \cos x}$ and then obtain a trigonometric identity.

Or, use the trigonometric identity $\dfrac{2\tan\theta}{1 + \tan^2\theta} = \sin 2\theta$ with $\theta = \dfrac{x}{2}$.

(d)
$$dz = \left(\sec^2\frac{x}{2}\right)\frac{1}{2}\,dx = \left(1 + \tan^2\frac{x}{2}\right)\frac{1}{2}\,dx = \left(1 + z^2\right)\frac{1}{2}\,dx,$$ then solve for dx.

50. $-\dfrac{2}{1 + \tan\frac{x}{2}} + C$

51. $-\dfrac{1}{\tan\frac{x}{2}} + C$

52. $\dfrac{2}{1 - \tan\frac{\theta}{2}} + C$

53. $\ln\left|1 + \tan\dfrac{t}{2}\right| + C$

Chapter 8 Review Exercises (pp. 454–455)

1. The limit doesn't exist. **2.** $\dfrac{3}{5}$

3. 2 **4.** $\dfrac{1}{e}$

5. 1 **6.** e^3

7. 0 **8.** -1

9. $-\dfrac{1}{2}$ **10.** 1

11. 1 **12.** ∞

13. ∞ **14.** 0

15. Same rate, because $\lim_{x\to\infty} \frac{f(x)}{g(x)} = \frac{1}{5}$

16. Same rate, because $\lim_{x\to\infty} \frac{f(x)}{g(x)} = \frac{\ln 3}{\ln 2}$

17. Same rate, because $\lim_{x\to\infty} \frac{f(x)}{g(x)} = 1$

18. Faster, because $\lim_{x\to\infty} \frac{f(x)}{g(x)} = \infty$

19. Faster, because $\lim_{x\to\infty} \frac{f(x)}{g(x)} = \infty$

20. Same rate, because $\lim_{x\to\infty} \frac{f(x)}{g(x)} = 1$

21. Slower, because $\lim_{x\to\infty} \frac{f(x)}{g(x)} = 0$

22. Slower, because $\lim_{x\to\infty} \frac{f(x)}{g(x)} = 0$

23. Same rate, because $\lim_{x\to\infty} \frac{f(x)}{g(x)} = \frac{1}{2}$

24. Slower, because $\lim_{x\to\infty} \frac{f(x)}{g(x)} = 0$

25. Same rate, because $\lim_{x\to\infty} \frac{f(x)}{g(x)} = 1$

26. Faster, because $\lim_{x\to\infty} \frac{f(x)}{g(x)} = \infty$

27. (a) $\lim_{x\to 0} f(x) = \ln 2$

(b) Define $f(0) = \ln 2$

28. (a) $\lim_{x\to 0^+} f(x) = 0$

(b) Define $f(0) = 0$

29. True, $\lim_{x\to\infty} \frac{\frac{1}{x^2} + \frac{1}{x^4}}{\frac{1}{x^2}} = 1$

30. False, $\lim_{x\to\infty} \frac{\frac{1}{x^2} + \frac{1}{x^4}}{\frac{1}{x^4}} = \infty$

31. False, $\lim_{x\to\infty} \frac{x}{x + \ln x} = 1$

32. True, $\lim_{x\to\infty} \frac{\ln(\ln x)}{\ln x} = 0$

33. True, $\lim_{x\to\infty} \frac{\tan^{-1} x}{1} = \frac{\pi}{2}$

34. True, $\lim_{x\to\infty} \frac{\frac{1}{x^4}}{\frac{1}{x^2} + \frac{1}{x^4}} = 0$

35. True, $\lim_{x\to\infty} \frac{\frac{1}{x^4}}{\frac{1}{x^2} + \frac{1}{x^4}} = 0$

36. True, $\lim_{x\to\infty} \frac{\ln x}{x + 1} = 0$

37. True, $\lim_{x\to\infty} \frac{\ln 2x}{\ln x} = 1$

38. True, $\lim_{x\to\infty} \frac{\sec^{-1} x}{1} = \frac{\pi}{2}$

39. $\frac{\pi}{2}$

40. -1

41. 6

42. 0

43. $\ln 3$

44. $\ln \frac{3}{4} + 1$

45. 2

46. $-\frac{1}{9}$

47. Diverges

48. π

49. Diverges, by the limit comparison test, comparing with $\frac{1}{\theta}$ or directly from the antiderivative.

50. Converges; directly from the antiderivative, value $\frac{1}{2}$.

51. Diverges; by the direct comparison test with $\frac{1}{z}$ or directly from the antiderivative.

52. Converges; by the direct comparison test with e^{-t}.

53. Converges; by the direct comparison test with e^{-x} on $[0, \infty)$ and with e^x on $(-\infty, 0]$ or directly from the antiderivative, value $\frac{\pi}{2}$.

54. Diverges; the problem is near $x = 0$. Compare with $\frac{1}{4x^2}$ there. For $0 < x \le 1$, $1 + e^x < 4$ and $\frac{1}{x^2(1 + e^x)} \ge \frac{1}{4x^2}$.

55. $9 \ln |x - 4| - 7 \ln |x - 3| + C$

56. $\ln |x| - \ln |x + 2| + \frac{2}{x} - \frac{2}{x^2} + C$

57. $4 \ln |t| - \frac{1}{2} \ln (t^2 + 1) + 4 \tan^{-1} t + C$

58. $\frac{1}{2} \tan^{-1} t - \frac{1}{2\sqrt{3}} \tan^{-1} \frac{t}{\sqrt{3}} + C$

59. $x + \ln |x - 1| - \ln |x| + C$

60. $\frac{x^2}{2} + \frac{3}{2} \ln |x + 1| - \frac{9}{2} \ln |x + 3| + C$

61. $y = \frac{500}{1 + 24e^{-x}}$

62. $y = \tan\left(\ln |x + 1| + \tan^{-1} \frac{\pi}{4}\right)$

63. $\ln \left|\sqrt{1 + 9y^2} + 3y\right| + C$

64. $\frac{1}{6} \sin^{-1} 3t + \frac{1}{2} t\sqrt{1 - 9t^2} + C$

65. $\ln (5x + \sqrt{25x^2 - 9}) + C$

66. $\frac{4x}{\sqrt{1 - x^2}} - 4 \sin^{-1} x + C$

67. 2π

68. 1

69. (a) $x = a - \frac{1}{kt + \frac{1}{a}}$

(b) $x = \frac{ab(e^{akt} - e^{bkt})}{ae^{akt} - be^{bkt}}$

Chapter 9
Infinite Series

9.1 Power Series (pp. 457–468)

Quick Review 9.1

1. $\frac{4}{3}, 1, \frac{4}{5}, \frac{2}{3}, \frac{1}{8}$ **2.** $-1, \frac{1}{2}, -\frac{1}{3}, \frac{1}{4}, \frac{1}{30}$

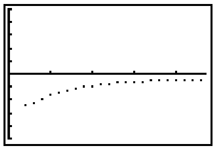
3. (a) 3 **(b)** 39,366

(c) $a_n = 2(3^{n-1})$

4. (a) $-\frac{1}{2}$ **(b)** $-\frac{1}{64}$

(c) $a_n = 8\left(-\frac{1}{2}\right)^{n-1} = 8(-0.5)^{n-1}$

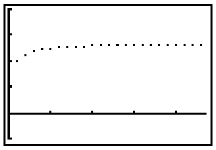
5. (a)



[0, 25] by [−0.5, 0.5]

(b) 0

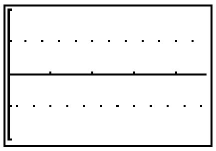
6. (a)



[0, 23.5] by [−1, 4]

(b) e

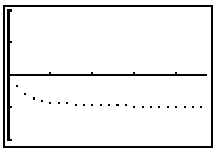
7. (a)



[0, 23.5] by [−2, 2]

(b) The limit does not exist.

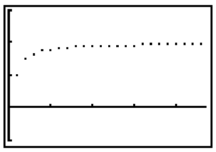
8. (a)



[0, 23.5] by [−2, 2]

(b) −1

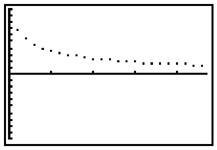
9. (a)



[0, 23,5] by [−1, 3]

(b) 2

10. (a)



[0, 23.5] by [−1, 1]

(b) 0

Section 9.1 Exercises

1. (a) $* = n^2$ **(b)** $* = (n+1)^2$

(c) $* = 3$

2. (a) $\left(\frac{1}{3}\right)^n$ **(b)** $\frac{(-1)^{n-1}}{n}$

(c) $\frac{5}{10^n}$

3. Different **4.** Same

5. Same **6.** Different

7. Converges; sum = 3 **8.** Diverges

9. Converges; sum = $\frac{15}{4}$ **10.** Diverges

11. Diverges **12.** Converges; sum = $\frac{30}{11}$

13. Converges; sum = $2 - \sqrt{2}$

14. Diverges

15. Converges; sum = $\frac{\pi}{\pi - e}$

16. Converges; sum = 1

17. Interval: $-\frac{1}{2} < x < \frac{1}{2}$; function: $f(x) = \frac{1}{1-2x}$

18. Interval: $-2 < x < 0$; function: $f(x) = \frac{1}{x+2}$

19. Interval: $1 < x < 5$; function: $f(x) = \frac{2}{x-1}$

20. Interval: $-1 < x < 3$; function: $f(x) = \frac{6}{3-x}$

21. Converges for all values of x except odd integer multiples of $\frac{\pi}{2}$; function: $f(x) = \frac{1}{1 - \sin x}$

22. Converges for $-\frac{\pi}{4} + k\pi < x < \frac{\pi}{4} + k\pi$, k any integer; function: $f(x) = \frac{1}{1 - \tan x}$

23. (a) The partial sums tend toward infinity.

(b) The partial sums are alternately 1 and 0.

(c) The partial sums alternate between positive and negative while their magnitude increases toward infinity.

24. This is a geometric series with $r = \frac{e^\pi}{\pi^e}$, which is greater than one.

25. $x = \frac{19}{20}$

26. One possible answer:
For any real number $a \neq 0$, use

$\frac{a}{2} + \frac{a}{4} + \frac{a}{8} + \frac{a}{16} + \frac{a}{32} + \cdots$.

To get 0, use $1 - \frac{1}{2} - \frac{1}{4} - \frac{1}{8} - \frac{1}{16} - \frac{1}{32} - \cdots$.

27. Assuming the series begins at $n = 1$:
(a) $\sum_{n=1}^{\infty} 2\left(\frac{3}{5}\right)^{n-1}$ **(b)** $\sum_{n=1}^{\infty} \frac{13}{2}\left(-\frac{3}{10}\right)^{n-1}$

28. Let $a = \frac{21}{100}$ and $r = \frac{1}{100}$, giving
$(0.21) + (0.21)(0.01) + (0.21)(0.01)^2 + (0.21)(0.01)^3 + \cdots$
The sum is $\frac{7}{33}$.

29. Let $a = \frac{234}{1000}$ and $r = \frac{1}{1000}$, giving
$(0.234) + (0.234)(0.001) + (0.234)(0.001)^2 + (0.234)(0.001)^3 + \cdots$
The sum is $\frac{26}{111}$.

30. $\frac{7}{9}$

31. $\frac{d}{9}$

32. $\frac{1}{15}$

33. $\frac{157}{111}$

34. $\frac{41,333}{33,300}$

35. $\frac{22}{7}$

36. 16 meters

37. ≈ 7.113 seconds

38. 8 m^2

39. $\frac{\pi}{2}$

40. **(a)** $S - rS = a - ar^n$
(b) Just factor and divide by $1 - r$.

41. For $r \neq 1$, the result follows from:
If $|r| < 1$, $r^n \to 0$ as $n \to \infty$, and
if $|r| > 1$ or $r = -1$, r^n has no finite limit as $n \to \infty$.
When $r = 1$, the nth partial sum is na, which goes to $\pm\infty$.

42. Series: $1 - 3x + 9x^2 - \cdots + (-3x)^n + \cdots$
Interval: $-\frac{1}{3} < x < \frac{1}{3}$

43. Series: $x + 2x^2 + 4x^3 + \cdots + 2^{n-1}x^n + \cdots$
Interval: $-\frac{1}{2} < x < \frac{1}{2}$

44. Series: $3 + 3x^3 + 3x^6 + \cdots + 3x^{3n} + \cdots$
Interval: $-1 < x < 1$

45. Series: $1 - (x-4) + (x-4)^2 - (x-4)^3 + \cdots + (-1)^n(x-4)^n + \cdots$
Interval: $3 < x < 5$

46. Series: $\frac{1}{4} - \frac{1}{4}(x-1) + \frac{1}{4}(x-1)^2 - \frac{1}{4}(x-1)^3 + \cdots + \frac{1}{4}(-1)^n(x-1)^n + \cdots$
Interval: $0 < x < 2$

47. One possible series:
$1 + (x-1) + (x-1)^2 + \cdots + (x-1)^n + \cdots$
Interval: $0 < x < 2$

48. $b = \ln\left(\frac{8}{9}\right)$

49. **(a)** 2
(b) $t > -\frac{1}{2}$
(c) $t > 9$

50. **(a)** First 4 terms: $4 - 4t^2 + 4t^4 - 4t^6$
General term: $(-1)^n(4t^{2n})$
(b) First 4 terms: $4x - \frac{4}{3}x^3 + \frac{4}{5}x^5 - \frac{4}{7}x^7$
General term: $(-1)^n\left(\frac{4}{2n+1}\right)x^{2n+1}$
(c) $-1 < t < 1$
(d) $x = \pm 1$

51. $x - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - \cdots + \frac{(-1)^{n-1}(x-1)^n}{n} + \cdots$

52. Series: $2 + 6x + 12x^2 + (n+2)(n+1)x^n + \cdots$
Interval: $-1 < x < 1$

53. **(a)** No, because if you differentiate it again, you would have the original series for f, but by Theorem 1, that would have to converge for $-2 < x < 2$, which contradicts the assumption that the original series converges only for $-1 < x < 1$.
(b) No, because if you integrate it again, you would have the original series for f, but by Theorem 2, that would have to converge for $-2 < x < 2$, which contradicts the assumption that the original series converges only for $-1 < x < 1$.

54. Let $L = \lim_{n\to\infty} a_n$. Then by definition of convergence, for $\frac{\epsilon}{2}$ there corresponds an N such that for all m and n,
$n, m > N \Rightarrow |a_m - L| < \frac{\epsilon}{2}$ and $|a_n - L| < \frac{\epsilon}{2}$.
Now,
$|a_m - a_n| = |a_m - L + L - a_n|$
$\leq |a_m - L| + |a_n - L| < \frac{\epsilon}{2} + \frac{\epsilon}{2} = \epsilon$
whenever $m > N$ and $n > N$.

55. Given an $\epsilon > 0$, by definition of convergence there corresponds an N such that for all $n < N$,
$|L_1 - a_n| < \epsilon$ and $|L_2 - a_n| < \epsilon$.
Now $|L_2 - L_1| =$
$|L_2 - a_n + a_n - L_1| \leq |L_2 - a_n| + |a_n - L_1|$
$< \epsilon + \epsilon = 2\epsilon$.
$|L_2 - L_1| < 2\epsilon$ says that the difference between two fixed values is smaller than any positive number 2ϵ. The only nonnegative number smaller than every positive number is 0, so $|L_2 - L_1| = 0$ or $L_1 = L_2$.

56. Consider the two subsequences $a_{k(n)}$ and $a_{i(n)}$, where $\lim_{n\to\infty} a_{k(n)} = L_1$, $\lim_{n\to\infty} a_{i(n)} = L_2$, and $L_1 \neq L_2$. Given an $\epsilon > 0$ there corresponds an N_1 such that for $k(n) > N_1$, $|a_{k(n)} - L_1| < \epsilon$, and an N_2 such that for $i(n) > N_2$, $|a_{i(n)} - L_2| < \epsilon$. Assume a_n converges. Let $N = \max\{N_1, N_2\}$. Then for $n > N$, we have that $|a_n - L_1| < \epsilon$ and $|a_n - L_2| < \epsilon$ for infintely many n. This implies that $\lim_{n\to\infty} a_n = L_1$ and $\lim_{n\to\infty} a_n = L_2$ where $L_1 \neq L_2$. Since the limit of a sequence is unique (by Exercise 55), a_n does not converge and hence diverges.

57. (a) $\lim_{n\to\infty} \frac{3n+1}{n+1} = 3$

(b) The line $y = 3$ is a horizontal asymptote of the graph of the function $f(x) = \frac{3x+1}{x+1}$, which means $\lim_{x\to\infty} f(x) = 3$. Because $f(n) = a_n$ for all positive integers n, it follows that $\lim_{n\to\infty} a_n$ must also be 3.

9.2 Taylor Series (pp. 469–479)

Quick Review 9.2

1. $2^n e^{2x}$

2. $(-1)^n n!(x-1)^{-(n+1)}$

3. $3^x(\ln 3)^n$

4. $(-1)^{n-1}(n-1)!\,x^{-n}$

5. $n!$

6. $\frac{x^{n-1}}{(n-1)!}$

7. $\frac{2^n(x-a)^{n-1}}{(n-1)!}$

8. $\frac{(-1)^n x^{2n}}{(2n)!}$

9. $\frac{(x+a)^{2n-1}}{(2n-1)!}$

10. $-\frac{(1-x)^{n-1}}{(n-1)!}$

Section 9.2 Exercises

1. $2x - \frac{4x^3}{3} + \frac{4x^5}{15} - \cdots + (-1)^n \frac{(2x)^{2n+1}}{(2n+1)!} + \cdots$
converges for all real x

2. $-x - \frac{x^2}{2} - \frac{x^3}{3} - \cdots - \frac{x^n}{n} - \cdots$
converges for $-1 \le x < 1$

3. $x^2 - \frac{x^6}{3} + \frac{x^{10}}{5} - \cdots + (-1)^n \frac{x^{4n+2}}{2n+1} + \cdots$
converges for $-1 \le x \le 1$

4. $7x + 7x^2 + \frac{7x^3}{2!} + \cdots + \frac{7x^{n+1}}{n!} + \cdots$
converges for all real x

5. $(\cos 2) - (\sin 2)x - \frac{(\cos 2)\,x^2}{2} + \cdots + \frac{(-1)^A B x^n}{n!} + \cdots$, where $A = \text{int}\left(\frac{n+1}{2}\right)$, and B is cos 2 if n is even and sin 2 if n is odd. Or, the general term may be written as $\left[\frac{1}{n!}\cos\left(2 + \frac{n\pi}{2}\right)\right]x^n$. The series converges for all real x.

6. $x^2 - \frac{x^4}{2} + \frac{x^6}{24} - \cdots + (-1)^n \frac{x^{2n+1}}{(2n)!} + \cdots$
converges for all real x

7. $x + x^4 + x^7 + \cdots + x^{3n+1} + \cdots$
converges for $-1 < x < 1$

8. $1 - 2x + 2x^2 - \cdots + (-1)^n \frac{2^n x^n}{n!} + \cdots$
converges for all real x

9. $P_0(x) = \frac{1}{2}$

$P_1(x) = \frac{1}{2} - \frac{x-2}{4}$

$P_2(x) = \frac{1}{2} - \frac{x-2}{4} + \frac{(x-2)^2}{8}$

$P_3(x) = \frac{1}{2} - \frac{x-2}{4} + \frac{(x-2)^2}{8} - \frac{(x-2)^3}{16}$

10. $P_0(x) = \frac{\sqrt{2}}{2}$

$P_1(x) = \frac{\sqrt{2}}{2} + \left(\frac{\sqrt{2}}{2}\right)\left(x - \frac{\pi}{4}\right)$

$P_2(x) = \frac{\sqrt{2}}{2} + \left(\frac{\sqrt{2}}{2}\right)\left(x - \frac{\pi}{4}\right) - \left(\frac{\sqrt{2}}{4}\right)\left(x - \frac{\pi}{4}\right)^2$

$P_3(x) = \frac{\sqrt{2}}{2} + \left(\frac{\sqrt{2}}{2}\right)\left(x - \frac{\pi}{4}\right) - \left(\frac{\sqrt{2}}{4}\right)\left(x - \frac{\pi}{4}\right)^2 - \left(\frac{\sqrt{2}}{12}\right)\left(x - \frac{\pi}{4}\right)^3$

11. $P_0(x) = \frac{\sqrt{2}}{2}$

$P_1(x) = \frac{\sqrt{2}}{2} - \left(\frac{\sqrt{2}}{2}\right)\left(x - \frac{\pi}{4}\right)$

$P_2(x) = \frac{\sqrt{2}}{2} - \left(\frac{\sqrt{2}}{2}\right)\left(x - \frac{\pi}{4}\right) - \left(\frac{\sqrt{2}}{4}\right)\left(x - \frac{\pi}{4}\right)^2$

$P_3(x) = \frac{\sqrt{2}}{2} - \left(\frac{\sqrt{2}}{2}\right)\left(x - \frac{\pi}{4}\right) - \left(\frac{\sqrt{2}}{4}\right)\left(x - \frac{\pi}{4}\right)^2 + \left(\frac{\sqrt{2}}{12}\right)\left(x - \frac{\pi}{4}\right)^3$

12. $P_0(x) = 2$

$P_1(x) = 2 + \frac{x-4}{4}$

$P_2(x) = 2 + \frac{x-4}{4} - \frac{(x-4)^2}{64}$

$P_3(x) = 2 + \frac{x-4}{4} - \frac{(x-4)^2}{64} + \frac{(x-4)^3}{512}$

13. **(a)** $4 - 2x + x^3$
(b) $3 + (x-1) + 3(x-1)^2 + (x-1)^3$

14. **(a)** $-8 + 3x + x^2 + 2x^3$
(b) $-2 + 11(x-1) + 7(x-1)^2 + 2(x-1)^3$

15. **(a)** 0
(b) $1 + 4(x-1) + 6(x-1)^2 + 4(x-1)^3$

16. **(a)** $P_3(x) = 4 + 5x - 4x^2 + x^3$
$f(0.2) \approx P_3(0.2) = 4.848$
(b) For $f', P_2(x) = 5 - 8x + 3x^2$
$f'(0.2) \approx P_2(0.2) = 3.52$

17. **(a)** $P_3(x) = 4 - (x-1) + \frac{3}{2}(x-1)^2 + \frac{1}{3}(x-1)^3$
$f(1.2) \approx P_3(1.2) \approx 3.863$
(b) For f', $P_2(x) = -1 + 3(x-1) + (x-1)^2$
$f'(1.2) \approx P_2(1.2) = -0.36$

18. **(a)** $f'(0) = \frac{1}{2}, f^{(10)}(0) = \frac{1}{11}$
(b) $x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots + \frac{x^{n+1}}{(n+1)!} + \cdots$
(c) $e^x - 1$

19. **(a)** $1 + \frac{x}{2} + \frac{x^2}{8} + \cdots + \frac{x^n}{2^n \cdot n!} + \cdots$
(b) $1 + \frac{x}{2!} + \frac{x^2}{3!} + \cdots + \frac{x^n}{(n+1)!} + \cdots$
(c) $g'(1) = 1$ and from the series,

$$g'(1) = \frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \cdots + \frac{n}{(n+1)!} + \cdots = \sum_{n=1}^{\infty} \frac{n}{(n+1)!}$$

20. **(a)** $2 + 2t^2 + 2t^4 + 2t^6 + \cdots + 2t^{2n} + \cdots$
(b) $2x + \frac{2x^3}{3} + \frac{2x^5}{5} + \frac{2x^7}{7} + \cdots + \frac{2x^{2n+1}}{2n+1} + \cdots$

21. **(a)** $1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16}$
(b) $1 + \frac{x^2}{2} - \frac{x^4}{8} + \frac{x^6}{16}$
(c) $5 + x + \frac{x^3}{6} - \frac{x^5}{40}$

22. **(a)** $1 + 3x + \frac{9x^2}{2} + \frac{9x^3}{2} + \cdots + \frac{3^n x^n}{n!} + \cdots$
(b) $f(x) = e^{3x}$ **(c)** $3e^3$

23. 27 terms (or, up to and including the 52nd degree term)

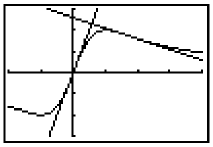
24. One possible answer: Because the end behavior of a polynomial must be unbounded and sin x is not unbounded.
Another: Because sin x has an infinite number of local extrema, but a polynomial can only have a finite number.

25. (1) sin x is odd and cos x is even
(2) sin 0 = 0 and cos 0 = 1

26. $\frac{81}{40}$ **27.** $\frac{1}{24}$

28. The linearization is the first order Taylor polynomial.

29. **(a)**



[−2, 4] by [−3, 3]

(b) $f''(a)$ must be 0 because of the inflection point, so the second degree term in the Taylor series of f at $x = a$ is zero.

30. When $x = 1$: $\frac{\pi}{4}$
When $x = -1$: $-\frac{\pi}{4}$

31. **(a)** $1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \cdots + \frac{(-1)^n x^{2n}}{(2n+1)!} + \cdots$
(b) Because f is undefined at $x = 0$.
(c) $k = 1$

32. $\frac{x}{(x-1)^2}$

33. **(a)** Differentiate 3 times.
(b) Differentiate k times and let $x = 0$.
(c) $\frac{m(m-1)(m-2)\cdots(m-k+1)}{k!}$
(d) $f(0) = 1, f'(0) = m$, and we're done by part (c).

34. Because $f(x) = (1+x)^m$ is a polynomial of degree m.

9.3 Taylor's Theorem (pp. 480–487)

Quick Review 9.3

1. 2 **2.** 7
3. 1 **4.** $\frac{1}{2}$
5. 7 **6.** Yes
7. No **8.** Yes
9. Yes **10.** No

Section 9.3 Exercises

1. $1 - 2x + 2x^2 - \frac{4}{3}x^3 + \frac{2}{3}x^4$; $f(0.2) \approx 0.6704$

2. $1 - \frac{\pi^2}{8}x^2 + \frac{\pi^4}{384}x^4$; $f(0.2) \approx 0.9511$

3. $-5x + \frac{5}{6}x^3$; $f(0.2) \approx -0.9933$

4. $x^2 - \frac{x^4}{2}$; $f(0.2) \approx 0.0392$

5. $1 + 2x + 3x^2 + 4x^3 + 5x^4$; $f(0.2) \approx 1.56$

6. $x + x^2 + \frac{x^3}{2!} + \frac{x^4}{3!} + \cdots + \frac{x^{n+1}}{n!} + \cdots$

7. $\frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \cdots + (-1)^n \frac{x^{2n+5}}{(2n+5)!} + \cdots$

8. $1 - x^2 + \frac{x^4}{3} - \cdots + (-1)^n \frac{2^{2n-1}x^{2n}}{(2n)!} + \cdots$

9. $x^2 - \frac{x^4}{3} + \frac{2x^6}{45} - \cdots + (-1)^n \frac{2^{2n+1}x^{2n+2}}{(2n+2)!} + \cdots$

10. $x^2 + 2x^3 + 4x^4 + \cdots + 2^n x^{n+2} + \cdots$

11. Using the theorem, $-0.56 < x < 0.56$
Graphically, $-0.57 < x < 0.57$

12. $|\text{Error}| < 0.0026$ (approximately)
$1 - \frac{x^2}{2}$ is too small.

13. $|\text{Error}| < 1.67 \times 10^{-10}$
$x < \sin x$ for negative values of x.

14. $\approx 1.27 \times 10^{-5}$

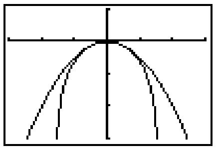
15. $|\text{Error}| < 1.842 \times 10^{-4}$

16. $\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots + \frac{x^{2n+1}}{(2n+1)!} + \cdots$
$\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots + \frac{x^{2n}}{(2n)!} + \cdots$

17. All of the derivatives of $\cosh x$ are either $\cosh x$ or $\sinh x$. For any real x, $\cosh x$ and $\sinh x$ are both bounded by $e^{|x|}$. So for any real x, let $M = e^{|x|}$ and $r = 1$ in the Remainder Estimation Theorem. It follows that the series converges to $\cosh x$ for all real values of x.

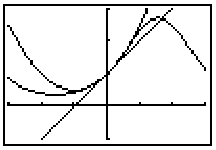
18. $\frac{f(b) - f(a)}{b - a} = f'(c)$ can be rewritten as $f(b) = f(a) + f'(c)(b - a)$. But $f(a)$ is the zeroth order Taylor polynomial for f at $x = a$, and letting $b = x$, $f'(c)(b - a)$ is the remainder from Taylor's Theorem.

19. **(a)** 0
(b) $-\frac{x^2}{2}$
(c) The graphs of the linear and quadratic approximations fit the graph of the function near $x = 0$.



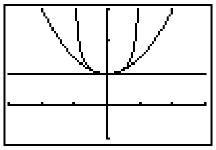
[−3, 3] by [−3, 1]

20. **(a)** $1 + x$
(b) $1 + x + \frac{x^2}{2}$
(c) The graphs of the linear and quadratic approximations fit the graph of the function near $x = 0$.



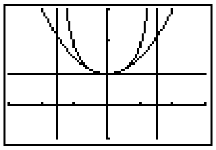
[−3, 3] by [−1, 3]

21. **(a)** 1
(b) $1 + \frac{x^2}{2}$
(c) The graphs of the linear and quadratic approximations fit the graph of the function near $x = 0$.



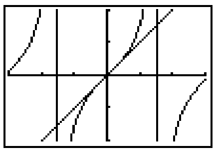
[−3, 3] by [−1, 3]

22. **(a)** 1
(b) $1 + \frac{x^2}{2}$
(c) The graphs of the linear and quadratic approximations fit the graph of the function near $x = 0$.



[−3, 3] by [−1, 3]

23. **(a)** x
(b) x
(c) The graphs of the linear and quadratic approximations fit the graph of the function near $x = 0$.



[−3, 3] by [−2, 2]

24. $P_2(x) = 1 + kx + \frac{k(k-1)x^2}{2}$.
Error is less than $\frac{1}{100}$ for $0 \le x < 0.01^{1/3} \approx 0.215$.

25. $|\text{Error}| < 4.61 \times 10^{-6}$, by Remainder Estimation Theorem (actual maximum error is $\approx 4.251 \times 10^{-6}$)

26. $P_3(x) = 1 + x + x^2 + x^3$.
$|\text{Error}| < 1.70 \times 10^{-4}$, by Remainder Estimation Theorem (actual maximum error is $\approx 1.11 \times 10^{-4}$)

27. (a) No

(b) Yes. $2 + x - \frac{x^3}{3} + \frac{x^5}{10} - \cdots + \frac{(-1)^{n+1}x^{2n-1}}{[(2n-1)(n-1)!]} + \cdots$

(c) For all real values of x. This is assured by Theorem 2 of Section 9.1, because the series for e^{-x^2} converges for all real values of x.

28. (a) $-x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \cdots - \frac{x^n}{n} - \cdots$

(b) $2x + \frac{2x^3}{3} + \frac{2x^5}{5} + \cdots + \frac{2x^{2n+1}}{2n+1} + \cdots$

29. (a) $\tan x$ **(b)** $\sec x$

30. (a) $x^2 - \frac{x^4}{3} + \frac{2x^6}{45} - \frac{x^8}{315} + \frac{2x^{10}}{14{,}175} - \cdots$

(b) $2x - \frac{4x^3}{3} + \frac{4x^5}{15} - \frac{8x^7}{315} + \cdots$

(c) part (b) $= 2x - \frac{(2x)^3}{3!} + \frac{(2x)^5}{5!} - \frac{(2x)^7}{7!} + \cdots$
$= \sin 2x$

31. (a) It works.

(b) Let $P = \pi + x$ where x is the error in the original estimate. Then

$$P + \sin P = (\pi + x) + \sin(\pi + x) = \pi + x - \sin x$$

But by the Remainder Theorem, $|x - \sin x| < \frac{|x|^3}{6}$. Therefore, the difference between the new estimate $P + \sin P$ and π is less than $\frac{|x|^3}{6}$.

32. (a) $\frac{1}{2}(e^{i\theta} + e^{-i\theta})$

$$= \frac{(\cos\theta + i\sin\theta) + (\cos(-\theta) + i\sin(-\theta))}{2}$$
$$= \frac{(\cos\theta + i\sin\theta) + (\cos\theta - i\sin\theta)}{2}$$
$$= \frac{2\cos\theta}{2} = \cos\theta$$

(b) $\frac{e^{i\theta} - e^{-i\theta}}{2i}$

$$= \frac{(\cos\theta + i\sin\theta) - (\cos(-\theta) + i\sin(-\theta))}{2i}$$
$$= \frac{(\cos\theta + i\sin\theta) - (\cos\theta - i\sin\theta)}{2i}$$
$$= \frac{2i\sin\theta}{2i} = \sin\theta$$

33. The derivative is

$$(ae^{ax})(\cos bx + i\sin bx) + (e^{ax})(-b\sin bx + ib\cos bx)$$
$$= a[e^{ax}(\cos bx + i\sin bx)] + ib[e^{ax}(\cos bx + i\sin bx)]$$
$$= (a + ib)e^{(a+ib)x}.$$

34. (a) The derivative of the right-hand side is

$$\frac{a - bi}{a^2 + b^2}(a + bi)e^{(a+bi)x} = \frac{a^2 - (bi)^2}{a^2 + b^2}e^{(a+bi)x} = \frac{a^2 + b^2}{a^2 + b^2}e^{(a+bi)x} = e^{(a+bi)x},$$

which confirms the antiderivative formula.

(b) $\int e^{ax}\cos bx\,dx + i\int e^{ax}\sin bx\,dx$

$$= \int e^{(a+bi)x}\,dx$$
$$= \frac{a - bi}{a^2 + b^2}e^{(a+bi)x}$$
$$= \frac{a - bi}{a^2 + b^2}e^{ax}(\cos bx + i\sin bx)$$
$$= \left(\frac{e^{ax}}{a^2 + b^2}\right)(a\cos bx + b\sin bx - bi\cos bx + ai\sin bx)$$
$$= \left(\frac{e^{ax}}{a^2 + b^2}\right)[(a\cos bx + b\sin bx) + i(a\sin bx - b\cos bx)]$$

Separating the real and imaginary parts gives

$$\int e^{ax}\cos bx\,dx = \frac{e^{ax}}{a^2 + b^2}(a\cos bx + b\sin bx)$$

and

$$\int e^{ax}\sin bx\,dx = \frac{e^{ax}}{a^2 + b^2}(a\sin bx - b\cos bx)$$

9.4 Radius of Convergence (pp. 487–496)

Quick Review 9.4

1. $|x|$
2. $|x - 3|$
3. 0
4. $\frac{x^2}{16}$
5. $\frac{|2x + 1|}{2}$
6. $a_n = n^2$, $b_n = 5n$, $N = 6$
7. $a_n = 5^n$, $b_n = n^5$, $N = 6$
8. $a_n = \sqrt{n}$, $b_n = \ln n$, $N = 1$
9. $a_n = \frac{1}{10^n}$, $b^n = \frac{1}{n!}$, $N = 25$
10. $a_n = \frac{1}{n^2}$, $b_n = n^{-3}$, $N = 2$

Section 9.4 Exercises

1. Diverges (nth-Term Test)
2. Diverges (nth-Term Test, Ratio Test)
3. Converges (Ratio Test)
4. Converges (geometric series)
5. Converges (Ratio Test, Direct Comparison Test)
6. Diverges (nth-Term Test)
7. Converges (Ratio Test)
8. Converges (Ratio Test)
9. Converges (Ratio Test)
10. Diverges (nth-Term Test)
11. Converges (geometric series)
12. Diverges (nth-Term Test, Ratio Test)
13. Diverges (nth-Term Test, Ratio Test)
14. Converges (Ratio Test)
15. Converges (Ratio Test)
16. Converges (Ratio Test)

17. One possible answer:
$\sum\frac{1}{n}$ diverges (see Exploration 1 in this section) even though $\lim_{n\to\infty}\frac{1}{n}=0$.

18. One possible answer:
$a_n = 2^{-n}$ and $b_n = 3^{-n}$

19. 1 **20.** 1

21. $\frac{1}{4}$ **22.** $\frac{1}{3}$

23. 10 **24.** 1

25. 3 **26.** ∞

27. 5 **28.** 4

29. 3 **30.** 0

31. $\frac{1}{2}$ **32.** $\frac{1}{4}$

33. 1 **34.** $\sqrt{2}$

35. Interval: $-1 < x < 3$
Sum: $-\frac{4}{x^2-2x-3}$

36. Interval: $-4 < x < 2$
Sum: $-\frac{9}{x^2+2x-8}$

37. Interval: $0 < x < 16$
Sum: $\frac{2}{4-\sqrt{x}}$

38. Interval: $\frac{1}{e} < x < e$
Sum: $\frac{1}{1-\ln x}$

39. Interval: $-2 < x < 2$
Sum: $\frac{3}{4-x^2}$

40. Interval: $-\infty < x < \infty$
Sum: $\frac{2}{2-\sin x}$

41. Almost, but the Ratio Test won't determine whether there is convergence or divergence at the endpoints of the interval.

42. (a) For $k \le N$, it's obvious that

$$a_1 + \cdots + a_k \le a_1 + \cdots + a_N + \sum_{n=N+1}^{\infty} c_n$$

For all $k > N$,

$$\begin{aligned} a_1 + \cdots + a_k &= a_1 + \cdots + a_N + a_{N+1} + \cdots + a_k \\ &\le a_1 + \cdots + a_N + c_{N+1} + \cdots + c_k \\ &\le a_1 + \cdots + a_N + \sum_{n=N+1}^{\infty} c_n. \end{aligned}$$

(b) Since all of the a_n are nonnegative, the partial sums of the series form a nondecreasing sequence of real numbers. Part (a) shows that the sequence is bounded above, so it must converge to a limit.

43. (a) For $k \le N$, it's obvious that

$$d_1 + \cdots + d_k \le d_1 + \cdots + d_N + \sum_{n=N+1}^{\infty} a_n$$

For all $k > N$,

$$\begin{aligned} d_1 + \cdots + d_k &= d_1 + \cdots + d_N + d_{N+1} + \cdots + d_k \\ &\le d_1 + \cdots + d_N + a_{N+1} + \cdots + a_k \\ &\le d_1 + \cdots + d_N + \sum_{n=N+1}^{\infty} a_n. \end{aligned}$$

(b) If $\sum a_n$ converged, that would imply that $\sum d_n$ was also convergent.

44. Answers will vary.

45. 1 **46.** 3

47. 5 **48.** 1

49. 1 **50.** $-\frac{1}{\ln 2}$

51. $-\frac{\pi}{4}$ **52.** The sum is 6.

9.5 Testing Convergence at Endpoints (pp. 496–508)

Quick Review 9.5

1. Converges, $p > 1$
2. Diverges, limit comparison test with integral of $\frac{1}{x}$
3. Diverges, comparison test with integral of $\frac{1}{x}$
4. Converges, comparison test with integral of $\frac{2}{x^2}$
5. Diverges, limit comparison test with integral of $\frac{1}{\sqrt{x}}$

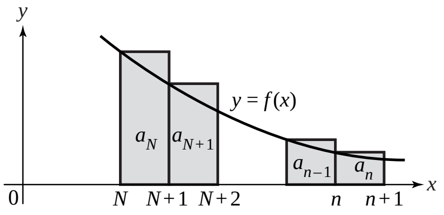
6. Yes
7. Yes
8. No
9. No
10. No

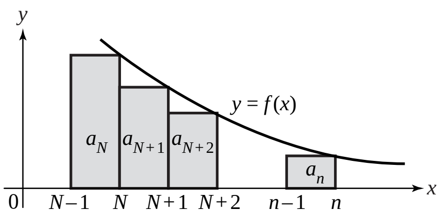
Section 9.5 Exercises

1. Diverges
2. Diverges
3. Diverges
4. Diverges
5. Diverges
6. Converges
7. Diverges
8. Converges
9. Converges
10. Converges
11. Diverges
12. Converges
13. Diverges
14. Converges
15. Diverges
16. Diverges
17. Converges absolutely
18. Converges conditionally
19. Converges absolutely
20. Converges conditionally
21. Diverges
22. Converges absolutely
23. Converges conditionally
24. Converges absolutely
25. Converges conditionally
26. Converges conditionally
27. **(a)** $(-1, 1)$ **(b)** $(-1, 1)$
(c) None
28. **(a)** $(-6, -4)$ **(b)** $(-6, -4)$
(c) None
29. **(a)** $\left(-\frac{1}{2}, 0\right)$ **(b)** $\left(-\frac{1}{2}, 0\right)$
(c) None
30. **(a)** $\left[\frac{1}{3}, 1\right)$ **(b)** $\left(\frac{1}{3}, 1\right)$
(c) At $x = \frac{1}{3}$
31. **(a)** $(-8, 12)$ **(b)** $(-8, 12)$
(c) None
32. **(a)** $(-1, 1)$ **(b)** $(-1, 1)$
(c) None
33. **(a)** $[-3, 3]$ **(b)** $[-3, 3]$
(c) None
34. **(a)** All real numbers **(b)** All real numbers
(c) None
35. **(a)** $(-8, 2)$ **(b)** $(-8, 2)$
(c) None
36. **(a)** $[-4, 4)$ **(b)** $(-4, 4)$
(c) At $x = -4$
37. **(a)** $(-3, 3)$ **(b)** $(-3, 3)$
(c) None
38. **(a)** Only at $x = 4$ **(b)** At $x = 4$
(c) None
39. **(a)** $\left(\frac{1}{2}, \frac{3}{2}\right)$ **(b)** $\left(\frac{1}{2}, \frac{3}{2}\right)$
(c) None
40. **(a)** $\left[1, \frac{3}{2}\right]$ **(b)** $\left[1, \frac{3}{2}\right]$
(c) None
41. **(a)** $[-\pi - 1, -\pi + 1)$
(b) $(-\pi - 1, -\pi + 1)$
(c) At $x = -\pi - 1$
42. **(a)** $\left(\frac{1}{e}, e\right)$ **(b)** $\left(\frac{1}{e}, e\right)$
(c) None
43. $40.554 < \text{sum} < 41.555$

44. Comparing areas in the figures, we have for all $n \geq 1$, $\int_1^{n+1} f(x)\,dx < a_1 + \cdots + a_n < a_1 + \int_1^n f(x)\,dx$
If the integral diverges, it must go to infinity, and the first inequality forces the partial sums of the series to go to infinity as well, so the series is divergent.
If the integral converges, then the second inequality puts an upper bound on the partial sums of the series, and since they are a nondecreasing sequence, they must converge to a finite sum for the series.

45.





Comparing areas in the figures, we have for all $n \geq N$, $\int_N^{n+1} f(x)\,dx < a_N + \cdots + a_n < a_1 + \int_N^n f(x)\,dx$.
If the integral diverges, it must go to infinity, and the first inequality forces the partial sums of the series to go to infinity as well, so the series is divergent.
If the integral converges, then the second inequality puts an upper bound on the partial sums of the series, and since they are a nondecreasing sequence, they must converge to a finite sum for the series.

46. **(a)** Diverges by Limit Comparison Test with $\frac{1}{k^{1/2}}$
(b) Diverges by the nth-Term Test
(c) Converges absolutely by Direct Comparison Test with $\frac{1}{k^2}$
(d) Diverges by the Integral Test

47. Possible answer: $\sum \frac{1}{n \ln n}$
This series diverges by the Integral Test, but its partial sums are roughly ln (ln n), so they are much smaller than the partial sums for the harmonic series, which are about ln n.

48. (a) $a_k = (-1)^{k+1}\left(\frac{2}{k}\right)$
(b) Converges by the Alternating Series Test.
(c) The first few partial sums are:
$S_1 = 2, S_2 = 1, S_3 = \frac{5}{3}, S_4 = \frac{7}{6}, S_5 = \frac{47}{30},$
$S_6 = \frac{37}{30}, S_7 = \frac{319}{210}, S_8 = \frac{533}{420}, S_9 = \frac{1879}{1260}.$
For an alternating series, the sum is between any two adjacent partial sums, so
$1 < S_8 \le \text{sum} \le S_9 < \frac{3}{2}.$

49. (a) Diverges
(b) $S = \sum_{n=1}^{\infty} \frac{3n}{3n^3 + n} = \sum_{n=1}^{\infty} \frac{3}{3n^2 + 1}$ which converges.

50. (a) $x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots + \frac{(-1)^{n+1}x^n}{n} + \cdots$
(b) $-1 < x \le 1$
(c) Error bound $= \frac{\left(\frac{1}{2}\right)^6}{6} < 0.002605$
(d) $\frac{1}{2} \ln(1 + x^2)$

51. Convergent for $-\frac{1}{2} \le x < \frac{1}{2}$.
Use the Ratio Test, Direct Comparison Test, and Alternating Series Test.

52. (a) By Direct Comparison Test with $\frac{1}{n^p}$
(b) Divergent by the Integral Test
(c) Use Direct Comparison Test with $\frac{1}{n \ln n}$ from part (b).

53. Use the Alternating Series Test.
54. Use the Alternating Series Test.
55. (a) It fails to satisfy $u_n \ge u_{n+1}$ for all $n \ge N$.
(b) The sum is $-\frac{1}{2}$.
56. Answers will vary.
57. (a) Converges **(b)** Converges
(c) Converges
58. (a) $(-3, 5)$ **(b)** $[-1, 5)$
(c) $\left(-\frac{1}{2}, \frac{1}{2}\right)$ **(d)** $\left(\frac{1}{e}, e\right)$

Chapter 9 Review Exercises (pp. 509–511)

1. (a) ∞ **(b)** All real numbers
(c) All real numbers **(d)** None
2. (a) 3 **(b)** $[-7, -1)$
(c) $(-7, -1)$ **(d)** At $x = -7$
3. (a) $\frac{3}{2}$ **(b)** $\left(-\frac{1}{2}, \frac{5}{2}\right)$
(c) $\left(-\frac{1}{2}, \frac{5}{2}\right)$ **(d)** None
4. (a) ∞ **(b)** All real numbers
(c) All real numbers **(d)** None
5. (a) $\frac{1}{3}$ **(b)** $\left[0, \frac{2}{3}\right]$
(c) $\left[0, \frac{2}{3}\right]$ **(d)** None
6. (a) 1 **(b)** $(-1, 1)$
(c) $(-1, 1)$ **(d)** None
7. (a) 1 **(b)** $\left(-\frac{3}{2}, \frac{1}{2}\right)$
(c) $\left(-\frac{3}{2}, \frac{1}{2}\right)$ **(d)** None
8. (a) ∞ **(b)** All real numbers
(c) All real numbers **(d)** None
9. (a) 1 **(b)** $[-1, 1)$
(c) $(-1, 1)$ **(d)** At $x = -1$
10. (a) $\frac{1}{e}$ **(b)** $\left[-\frac{1}{e}, \frac{1}{e}\right]$
(c) $\left[-\frac{1}{e}, \frac{1}{e}\right]$ **(d)** None
11. (a) $\sqrt{3}$ **(b)** $(-\sqrt{3}, \sqrt{3})$
(c) $(-\sqrt{3}, \sqrt{3})$ **(d)** None
12. (a) 1 **(b)** $[0, 2]$
(c) $(0, 2)$ **(d)** At $x = 0$ and $x = 2$
13. (a) 0 **(b)** $x = 0$ only
(c) $x = 0$ **(d)** None
14. (a) $\frac{1}{10}$ **(b)** $\left[-\frac{1}{10}, \frac{1}{10}\right)$
(c) $\left(-\frac{1}{10}, \frac{1}{10}\right)$ **(d)** At $x = -\frac{1}{10}$
15. (a) 0 **(b)** $x = 0$ only
(c) $x = 0$ **(d)** None
16. (a) $\sqrt{3}$ **(b)** $(-\sqrt{3}, \sqrt{3})$
(c) $(-\sqrt{3}, \sqrt{3})$ **(d)** None
17. $f(x) = \frac{1}{1 + x}$ evaluated at $x = \frac{1}{4}$. Sum $= \frac{4}{5}$.
18. $f(x) = \ln(1 + x)$ evaluated at $x = \frac{2}{3}$.
Sum $= \ln\left(\frac{5}{3}\right)$.
19. $f(x) = \sin x$ evaluated at $x = \pi$. Sum $= 0$.
20. $f(x) = \cos x$ evaluated at $x = \frac{\pi}{3}$. Sum $= \frac{1}{2}$.
21. $f(x) = e^x$ evaluated at $x = \ln 2$. Sum $= 2$.
22. $f(x) = \tan^{-1} x$ evaluated at $x = \frac{1}{\sqrt{3}}$. Sum $= \frac{\pi}{6}$.
23. $1 + 6x + 36x^2 + \cdots + (6x)^n + \cdots$

24. $1 - x^3 + x^6 - \cdots + (-1)^n x^{3n} + \cdots$
25. $1 - 2x^2 + x^9$
26. $4x + 4x^2 + 4x^3 + \cdots + 4x^{n+1} + \cdots$
27. $\pi x - \frac{(\pi x)^3}{3!} + \frac{(\pi x)^5}{5!} - \cdots + (-1)^n \frac{(\pi x)^{2n+1}}{(2n+1)!} + \cdots$
28. $-\frac{2x}{3} + \frac{4x^3}{81} - \frac{4x^5}{3645} + \cdots + \frac{(-1)^{n+1}}{(2n+1)!}\left(\frac{2x}{3}\right)^{2n+1} + \cdots$
29. $-\frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + \cdots$
30. $1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots + \frac{x^{2n}}{(2n)!} + \cdots$
31. $1 - \frac{5x}{2!} + \frac{(5x)^2}{4!} - \cdots + (-1)^n \frac{(5x)^n}{(2n)!} + \cdots$
32. $1 + \frac{\pi x}{2} + \frac{\pi^2 x^2}{8} + \cdots + \frac{1}{n!}\left(\frac{\pi x}{2}\right)^n + \cdots$
33. $x - x^3 + \frac{x^5}{2!} - \frac{x^7}{3!} + \cdots + (-1)^n \frac{x^{2n+1}}{n!} + \cdots$
34. $3x - \frac{(3x)^3}{3} + \frac{(3x)^5}{5} - \cdots + (-1)^n \frac{(3x)^{2n+1}}{2n+1} + \cdots$
35. $-2x - 2x^2 - \frac{8x^3}{3} - \cdots - \frac{(2x)^n}{n} - \cdots$
36. $-x^2 - \frac{x^3}{2} - \frac{x^4}{3} - \cdots - \frac{x^{n+1}}{n} - \cdots$
37. $1 + (x-2) + (x-2)^2 + (x-2)^3 + \cdots (x-2)^n + \cdots$
38. $2 + 7(x+1) - 5(x+1)^2 + (x+1)^3$
(Finite. General term for $n \geq 4 = 0$)
39. $\frac{1}{3} - \frac{x-3}{9} + \frac{(x-3)^2}{27} - \frac{(x-3)^3}{81} + \cdots + (-1)^n \frac{(x-3)^n}{3^{n+1}} + \cdots$
40. $-(x-\pi) + \frac{(x-\pi)^3}{3!} - \frac{(x-\pi)^5}{5!} + \frac{(x-\pi)^7}{7!} - \cdots + (-1)^{n+1} \frac{(x-\pi)^{2n+1}}{(2n+1)!} + \cdots$
41. Diverges. It is -5 times the harmonic series.
42. Converges conditionally. Alternating Series Test and $p = \frac{1}{2}$.
43. Converges absolutely. Direct Comparison Test with $\frac{1}{n^2}$.
44. Converges absolutely. Ratio Test
45. Converges conditionally. Alternating Series Test and Direct Comparison Test with $\frac{1}{n}$.
46. Converges absolutely. Integral Test
47. Converges absolutely. Ratio Test
48. Converges absolutely. nth-Root Test or Ratio Test
49. Diverges. nth-Term Test for Divergence
50. Converges absolutely. Direct Comparison Test with $\frac{1}{n^{3/2}}$.
51. Converges absolutely. Limit Comparison Test with $\frac{1}{n^2}$.
52. Diverges. nth-Term Test for Divergence
53. $\frac{1}{6}$
54. -1
55. **(a)** $P_3(x) = 1 + 4(x-3) + 3(x-3)^2 + 2(x-3)^3$
$f(3.2) \approx P_3(3.2) = 1.936$
(b) For f': $P_2(x) = 4 + 6(x-3) + 6(x-3)^2$
$f'(2.7) \approx P_2(2.7) = 2.74$
(c) It underestimates the values, since the graph of f is concave up near $x = 3$.
56. **(a)** $f(4) = 7$ and $f'''(4) = -12$
(b) For f': $P_2(x) = -3 + 10(x-4) - 6(x-4)^2$
$f'(4.3) \approx P_2(4.3) = -0.54$
(c) $7(x-4) - \frac{3}{2}(x-4)^2 + \frac{5}{3}(x-4)^3 - \frac{1}{2}(x-4)^4$
(d) No. One would need the entire Taylor series for $f(x)$, and it would have to converge to $f(x)$ at $x = 3$.
57. **(a)** $\frac{5x}{2} - \frac{5x^3}{48} + \frac{x^5}{768} - \cdots + (-1)^n \frac{5}{(2n+1)!}\left(\frac{x}{2}\right)^{2n+1} + \cdots$
(b) All real numbers. Use the Ratio Test.
(c) Note that the absolute value of $f^{(n)}(x)$ is bounded by $\frac{5}{2^n}$ for all x and all $n = 1, 2, 3, \cdots$
So if $-2 < x < 2$, the truncation error using P_n is bounded by $\frac{5}{2^{n+1}} \cdot \frac{2^{n+1}}{(n+1)!} = \frac{5}{(n+1)!}$.
To make this less than 0.1 requires $n \geq 4$.
So, two nonzero terms (up through degree 4) are needed.
58. **(a)** $1 + 2x + 4x^2 + 8x^3 + \cdots + (2x)^n + \cdots$
(b) $\left(-\frac{1}{2}, \frac{1}{2}\right)$. The series for $\frac{1}{1-t}$ is known to converge for $-1 < t < 1$, so by substituting $t = 2x$, we find the resulting series converges for $-1 < 2x < 1$.

58. continued

(c) Possible answer:
$f\left(-\frac{1}{4}\right) = \frac{2}{3}$, so one percent is approximately 0.0067. It takes 7 terms (up through degree 6). This can be found by trial and error.

59. (a) $\frac{1}{e}$ **(b)** $-\frac{5}{18} \approx -0.278$

(c) By the Alternating Series Estimation Theorem, the error is bounded by the size of the next term, which is $\frac{32}{243}$, or about 0.132.

60. (a) $1 - (x-3) + (x-3)^2 - (x-3)^3 + \cdots + (-1)^n(x-3)^n + \cdots$

(b) $(x-3) - \frac{(x-3)^2}{2} + \frac{(x-3)^3}{3} - \frac{(x-3)^4}{4} + \cdots + (-1)^n\frac{(x-3)^{n+1}}{n+1} + \cdots$

(c) Evaluate at $x = 3.5$. This is an alternating series. By the Alternating Series Estimation Theorem, since the size of the third term is $\frac{1}{24} < 0.05$, the first two terms will suffice. The estimate for $\ln\left(\frac{3}{2}\right)$ is 0.375.

61. (a) $1 - 2x^2 + 2x^4 - \frac{4x^6}{3} + \cdots + (-1)^n\frac{2^n x^{2n}}{n!} + \cdots$

(b) All real numbers. Use the Ratio Test.

(c) This is an alternating series. The difference will be bounded by the magnitude of the fifth term, which is $\frac{(2x^2)^4}{4!} = \frac{2x^8}{3}$.
Since $-0.6 \le x \le 0.6$, this term is less than $\frac{2(0.6)^8}{3}$ which is less than 0.02.

62. (a) $x^2 - x^3 + x^4 - x^5 + \cdots + (-1)^n x^{n+2} + \cdots$

(b) No. The partial sums form the sequence 1, 0, 1, 0, 1, 0, … which has no limit.

63. (a) $\frac{x^3}{3} - \frac{x^7}{7(3!)} + \frac{x^{11}}{11(5!)} + \cdots + \frac{(-1)^n x^{4n+3}}{(4n+3)(2n+1)!} + \cdots$

(b) The first two nonzero terms suffice (through degree 7).

(c) 0.31026830 **(d)** Within 1.5×10^{-7}

64. (a) 0.88566

(b) $\frac{41}{60} \approx 0.68333$

(c) Since f is concave up, the trapezoids used to estimate the area lie above the curve, and the estimate is too large.

(d) Since all the derivatives are positive (and $x > 0$), the remainder, $R_n(x)$, must be positive. This means that $P_n(x)$ is smaller than $f(x)$.

(e) $e - 2 \approx 0.71828$

65. (a) Because $[\$1000(1.08)^{-n}](1.08)^n = \$1000$ will be available after n years.

(b) Assume that the first payment goes to the charity at the end of the first year.
$1000(1.08)^{-1} + 1000(1.08)^{-2} + 1000(1.08)^{-3} + \cdots$

(c) This is a geometric series with sum equal to $12,500. This represents the amount which must be invested today in order to completely fund the perpetuity forever.

66. $16,666.67 [Again, assuming first payment at end of year.]

67. (a) $0\left(\frac{1}{2}\right) + 1\left(\frac{1}{2}\right)^2 + 2\left(\frac{1}{2}\right)^3 + 3\left(\frac{1}{2}\right)^4 + \cdots$

(b) $1 + 2x + 3x^2 + 4x^3 + \cdots$

(c) $x^2 + 2x^3 + 3x^4 + 4x^5 + \cdots$

(d) The expected payoff of the game is $1.

68. (a) $\frac{b^2\sqrt{3}}{4} + \frac{3b^2\sqrt{3}}{4^2} + \frac{3^2b^2\sqrt{3}}{4^3} + \cdots$

(b) $b^2\sqrt{3}$

(c) No, not every point is removed. But the remaining points are "isolated" enough that there are no regions and hence no area remaining.

69. $\frac{1}{(1-x)^2} = 1 + 2x + 3x^2 + 4x^3 + 5x^4 + \cdots$
Substitute $x = \frac{1}{2}$ to get the desired result.

70. (b) Solve $x = \frac{2x^2}{(x-1)^3}$. $x \approx 2.769$.

Chapter 10
Parametric, Vector, and Polar Functions

10.1 Parametric Functions (pp. 513–520)

Quick Review 10.1

1. (1, 0) **2.** (0, −1)
3. $x^2 + y^2 = 1$
4. The portion in the first three quadrants
5. $x = t, y = t^2 + 1, -1 \le t \le 3$
6. $x = 2\cos t + 2, y = 2\sin t + 3, 0 \le t \le 2\pi$
7. $\frac{3}{2}$ **8.** $y = \frac{3}{2}x + 3\sqrt{2}$
9. $y = -\frac{2}{3}x + \frac{5\sqrt{2}}{6}$ **10.** $\frac{31^{3/2} - 8}{27}$

Section 10.1 Exercises

1. (a) $-\frac{1}{2}\tan t$ **(b)** $-\frac{1}{8}\sec^3 t$
2. (a) $\sqrt{3}$ **(b)** 0
3. (a) $-\sqrt{3 + \frac{3}{t}}$ **(b)** $-\frac{\sqrt{3}}{t^{3/2}}$
4. (a) $-t$ **(b)** t^2
5. (a) $\frac{3t^2}{2t - 3}$ **(b)** $\frac{6t^2 - 18t}{(2t - 3)^3}$
6. (a) $\frac{2t - 1}{2t + 1}$ **(b)** $\frac{4}{(2t + 1)^3}$
7. (a) (2, 0) and (2, −2)
(b) (1, −1) and (3, −1)
8. (a) Nowhere
(b) (1, 0) and (−1, 0)
9. (a) At $t = \pm\frac{2}{\sqrt{3}}$, or ≈ (0.845, −3.079) and (3.155, 3.079)
(b) Nowhere
10. (a) (−2, 4) and (−2, −2)
(b) (1, 1) and (−5, 1)
11. 4 **12.** $\frac{21}{2}$
13. $\frac{2\sqrt{2} - 1}{3} \approx 0.609$ **14.** π^2
15. ln 2 **16.** ≈4.497
17. $8\pi^2$ **18.** ≈14.214
19. ≈178.561 **20.** π
21. (a) $x(t) = 2t, y(t) = t + 1, 0 \le t \le 1$
(b) $3\pi\sqrt{5}$
(c) $3\pi\sqrt{5}$
22. (a) Because these values for $x(t)$ and $y(t)$ satisfy $y = \frac{r}{h}x$, which is the equation for the line through the origin and (h, r), and this range of t-values gives the correct initial and terminal points.
(b) $\pi r\sqrt{r^2 + h^2}$ **(c)** $\pi r\sqrt{r^2 + h^2}$
23. (a) π **(b)** π
24. ≈22.103
25. Just substitute x for t and note that $\frac{dx}{dx} = 1$.
26. Use the parametrization $x = g(y), y = y, c \le y \le d$, substitute y for t and note $\frac{dy}{dy} = 1$.
27. At $t = \sqrt{13} - 1$, or ≈(3.394, 5.160)
28. ≈159.485 **29.** ≈144.513
30. $\frac{64\pi a^2}{3}$
31. $3\pi a^2$
32. $5\pi^2 a^3$
33. (a) $x = \cos t + t\sin t, y = \sin t - t\cos t$
(b) $2\pi^2$
34. (a) $x = a(\cos t + t\sin t), y = a(\sin t - t\cos t)$
(b) $2a\pi^2$
35. (a) ≈461.749 ft **(b)** ≈41.125 ft
36. (a) ≈641.236 ft
(b) $\frac{5625}{64} \approx 87.891$ ft
37. (a) ≈840.421 ft
(b) $\frac{16{,}875}{64} \approx 263.672$ ft
38. (a) 703.125 ft
(b) $\frac{5625}{16} = 351.5625$ ft
39. Just substitute x for t and note that $\frac{dx}{dx} = 1$.
40. ≈1273.371
41. ≈9.417
42. ≈116.687

10.2 Vectors in the Plane (pp. 520–529)

Quick Review 10.2

1. $\sqrt{17}$ **2.** $\frac{1}{4}$
3. $b = 11$ **4.** $a = 4$
5. $b = 6$
6. (a) 120° **(b)** $\frac{2\pi}{3}$
7. (a) −30° **(b)** $-\frac{\pi}{6}$

8. (a) $-45°$ **(b)** $-\frac{\pi}{4}$

9. $c \approx 2.832$

10. $\theta \approx 1.046$ radians or $59.935°$

Section 10.2 Exercises

1. (a) $\langle 9, -6 \rangle$ **(b)** $3\sqrt{13}$

2. (a) $\langle 4, -10 \rangle$ **(b)** $2\sqrt{29}$

3. (a) $\langle 1, 3 \rangle$ **(b)** $\sqrt{10}$

4. (a) $\langle 5, -7 \rangle$ **(b)** $\sqrt{74}$

5. (a) $\langle 12, -19 \rangle$ **(b)** $\sqrt{505}$

6. (a) $\langle -16, 29 \rangle$ **(b)** $\sqrt{1097}$

7. (a) $\left\langle \frac{1}{5}, \frac{14}{5} \right\rangle$ **(b)** $\frac{\sqrt{197}}{5}$

8. (a) $\left\langle -3, \frac{70}{13} \right\rangle$ **(b)** $\frac{\sqrt{6421}}{13}$

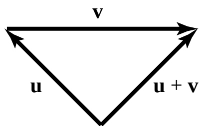
9. $\langle 1, -4 \rangle$ **10.** $\langle -1, 1 \rangle$

11. $\langle -2, -3 \rangle$ **12.** $\langle 0, 0 \rangle$

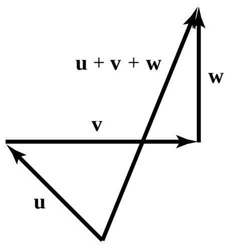
13. $\left\langle -\frac{1}{2}, \frac{\sqrt{3}}{2} \right\rangle$ **14.** $\left\langle -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}} \right\rangle$

15. $\left\langle -\frac{\sqrt{3}}{2}, -\frac{1}{2} \right\rangle$ **16.** $\left\langle -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle$

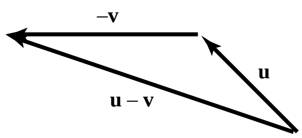
17. (a)



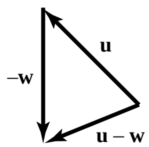
(b)



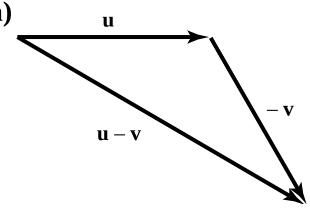
(c)



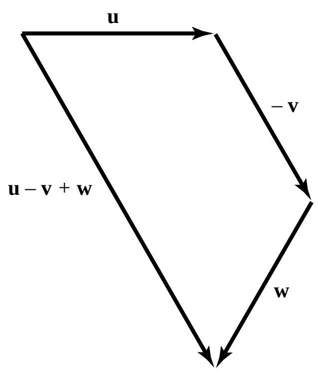
(d)



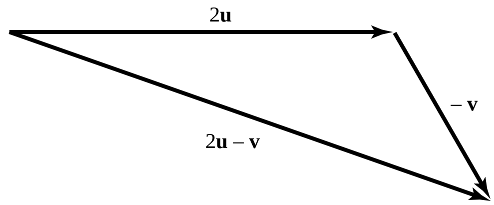
18. (a)



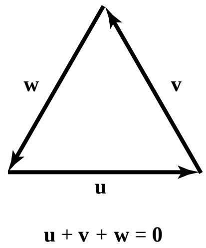
(b)



(c)



(d)



19. $\left\langle \frac{3}{5}, \frac{4}{5} \right\rangle$ **20.** $\left\langle \frac{4}{5}, -\frac{3}{5} \right\rangle$

21. $\left\langle -\frac{15}{17}, \frac{8}{17} \right\rangle$ **22.** $\left\langle -\frac{5}{\sqrt{29}}, -\frac{2}{\sqrt{29}} \right\rangle$

23. Tangent: $\pm\left\langle \frac{1}{\sqrt{17}}, \frac{4}{\sqrt{17}} \right\rangle$

Normal: $\pm\left\langle \frac{4}{\sqrt{17}}, -\frac{1}{\sqrt{17}} \right\rangle$

24. Tangent: $\pm\left\langle \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}} \right\rangle$

Normal: $\pm\left\langle \frac{2}{\sqrt{5}}, -\frac{1}{\sqrt{5}} \right\rangle$

25. Tangent: $\pm\left\langle -\frac{12}{\sqrt{219}}, \frac{5}{\sqrt{73}} \right\rangle \approx \pm\langle -0.811, 0.585 \rangle$

Normal: $\pm\left\langle \frac{5}{\sqrt{73}}, \frac{12}{\sqrt{219}} \right\rangle \approx \langle 0.585, 0.811 \rangle$

26. Tangent: $\pm\left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle$

Normal: $\pm\left\langle \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}} \right\rangle$

27. Angle at $A = \cos^{-1}\left(\frac{1}{\sqrt{5}}\right) \approx 63.435°$

Angle at $B = \cos^{-1}\left(\frac{3}{5}\right) \approx 53.130°$

Angle at $C = \cos^{-1}\left(\frac{1}{\sqrt{5}}\right) \approx 63.435°$

28. $90°$

29. (a) Both equal $u_1(v_1 + w_1) + u_2(v_2 + w_2)$.

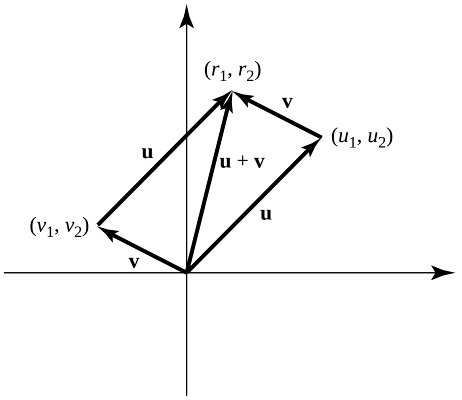
(b) Both equal $(u_1 + v_1)w_1 + (u_2 + v_2)w_2$.

30. Both equal $u_1^2 + u_2^2$.

31. $(\mathbf{u} + \mathbf{v}) \cdot (\mathbf{u} - \mathbf{v})$
$= (u_1 + v_1)(u_1 - v_1) + (u_2 + v_2)(u_2 - v_2)$
$= u_1^2 - v_1^2 + u_2^2 - v_2^2$
$= (u_1^2 + u_2^2) - (v_1^2 + v_2^2)$
$= |\mathbf{u}|^2 - |\mathbf{v}|^2$

32. This comes immediately from $\cos\left(\frac{\pi}{2}\right) = 0$.

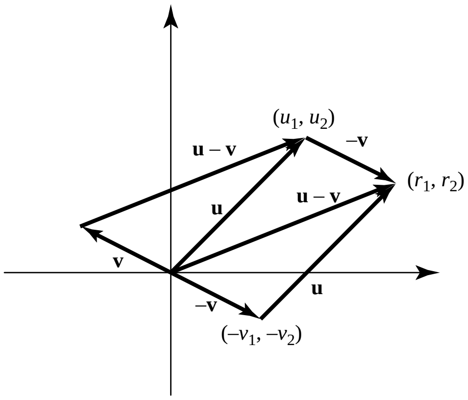
33.



$r_1 - v_1 = u_1$ so $r_1 = u_1 + v_1$
$r_2 - v_2 = u_2$ so $r_2 = u_2 + v_2$

34. (a) To find $\mathbf{u} - \mathbf{v}$, place both vectors with their initial points at the origin. The vector drawn from the terminal point of $\mathbf{v}$ to the terminal point of $\mathbf{u}$ is $\mathbf{u} - \mathbf{v}$. Or, add $\mathbf{u}$ and $-\mathbf{v}$ according to the parallelogram law.

(b)



$r_1 - (-v_1) = u_1$ so $r_1 = u_1 - v_1$
$r_2 - (-v_2) = u_2$ so $r_2 = u_2 - v_2$

35. (a) Let $P = (a, b)$ and $Q = (c, d)$. Then

$$\left(\frac{1}{2}\right)\overrightarrow{OP} + \left(\frac{1}{2}\right)\overrightarrow{OQ} = \left(\frac{1}{2}\right)\langle a, b\rangle + \left(\frac{1}{2}\right)\langle c, d\rangle = \left\langle \frac{(a+c)}{2}, \frac{(b+d)}{2} \right\rangle = \overrightarrow{OM}$$

(b) $\overrightarrow{OM} = \left(\frac{2}{3}\right)\overrightarrow{OP} + \left(\frac{1}{3}\right)\overrightarrow{OQ}$

(c) $\overrightarrow{OM} = \left(\frac{1}{3}\right)\overrightarrow{OP} + \left(\frac{2}{3}\right)\overrightarrow{OQ}$

(d) Possible answer:
M is a fraction of the way from P to Q. Let d be this fraction. Then

$$\overrightarrow{OM} = d\overrightarrow{OQ} + (1 - d)\overrightarrow{OP}.$$

36. $\overrightarrow{CA} = -\mathbf{u} - \mathbf{v}$ and $\overrightarrow{CB} = \mathbf{u} - \mathbf{v}$. Since $|\mathbf{v}| = |\mathbf{u}|$, these vectors are orthogonal, as $(-\mathbf{u} - \mathbf{v}) \cdot (\mathbf{u} - \mathbf{v}) = |\mathbf{v}|^2 - |\mathbf{u}|^2 = 0$.

37. Two adjacent sides of the rhombus can be given by two vectors of the same length, **u** and **v**.
Then the diagonals of the rhombus are $(\mathbf{u} + \mathbf{v})$ and $(\mathbf{u} - \mathbf{v})$. These two vectors are orthogonal since $(\mathbf{u} + \mathbf{v}) \cdot (\mathbf{u} - \mathbf{v}) = |\mathbf{u}|^2 - |\mathbf{v}|^2 = 0$.

38. Two adjacent sides of a rectangle can be given by two vectors **u** and **v**. The diagonals are then $(\mathbf{u} + \mathbf{v})$ and $(\mathbf{u} - \mathbf{v})$. These two vectors will be orthogonal if and only if **u** and **v** are the same length, since
$(\mathbf{u} + \mathbf{v}) \cdot (\mathbf{u} - \mathbf{v}) = |\mathbf{u}|^2 - |\mathbf{v}|^2$.

39. Let two adjacent sides of the parallelogram be given by two vectors **u** and **v**. The diagonals are then $(\mathbf{u} + \mathbf{v})$ and $(\mathbf{u} - \mathbf{v})$. So the lengths of the diagonals satisfy
$|\mathbf{u} + \mathbf{v}|^2 = (\mathbf{u} + \mathbf{v}) \cdot (\mathbf{u} + \mathbf{v})$
$= |\mathbf{u}|^2 + 2\mathbf{u} \cdot \mathbf{v} + |\mathbf{v}|^2$
and $|\mathbf{u} - \mathbf{v}|^2 = (\mathbf{u} - \mathbf{v}) \cdot (\mathbf{u} - \mathbf{v})$
$= |\mathbf{u}|^2 - 2\mathbf{u} \cdot \mathbf{v} + |\mathbf{v}|^2$.
The two lengths will be the same if and only if $\mathbf{u} \cdot \mathbf{v} = 0$, which means that **u** and **v** are perpendicular and the parallelogram is a rectangle.

40. The indicated diagonal is $(\mathbf{u}+\mathbf{v})$. The cosine of the angle between the diagonal and $\mathbf{u}$ is
$\dfrac{(\mathbf{u}+\mathbf{v})\cdot\mathbf{u}}{|\mathbf{u}+\mathbf{v}|\,|\mathbf{u}|}=\dfrac{|\mathbf{u}|^2+\mathbf{v}\cdot\mathbf{u}}{|\mathbf{u}+\mathbf{v}|\,|\mathbf{u}|}$.
But the cosine of the angle between the diagonal and $\mathbf{v}$ is
$\dfrac{[(\mathbf{u}+\mathbf{v})\cdot\mathbf{v}]}{[|\mathbf{u}+\mathbf{v}|\,|\mathbf{v}|]}=\dfrac{[\mathbf{u}\cdot\mathbf{v}+|\mathbf{v}|^2]}{[|\mathbf{u}+\mathbf{v}|\,|\mathbf{v}|]}$.
If $\mathbf{u}$ and $\mathbf{v}$ are the same length then these two quantities are equal, and the diagonal makes the same angle with both sides.

41. The slopes are the same.
42. $\mathbf{v}=\mathbf{0}$
43. $\approx\langle -338.095, 725.046\rangle$
44. $\approx\langle 104.189, -590.885\rangle$
45. Speed $\approx$ 346.735 mph
direction $\approx 14.266°$ east of north
46. $\mathbf{w}\approx\langle 2.205, 1.432\rangle$
47. $\approx$39.337 lb
48. **(a)** $\approx(4.950, 4.950)$
(b) $\approx(-1.978, 0.950)$
49. $\overrightarrow{AB}=\langle -3, 4\rangle=\overrightarrow{CD}$
50. $\overrightarrow{AB}=\langle 2, -5\rangle=\overrightarrow{CD}$
51. $\mathbf{u}=\langle u_1, u_2\rangle$, $\mathbf{v}=\langle v_1, v_2\rangle$, $\mathbf{w}=\langle w_1, w_2\rangle$
(i) $\mathbf{u}+\mathbf{v}=\langle u_1+v_1, u_2+v_2\rangle$
$=\langle v_1+u_1, v_2+u_2\rangle=\mathbf{v}+\mathbf{u}$
(ii) $(\mathbf{u}+\mathbf{v})+\mathbf{w}$
$=\langle u_1+v_1, u_2+v_2\rangle+\langle w_1, w_2\rangle$
$=\langle (u_1+v_1)+w_1, (u_2+v_2)+w_2\rangle$
$=\langle u_1+(v_1+w_1), u_2+(v_2+w_2)\rangle$
$=\mathbf{u}+(\mathbf{v}+\mathbf{w})$
(iii) $\mathbf{u}+\mathbf{0}=\langle u_1, u_2\rangle+\langle 0, 0\rangle=\langle u_1+0, u_2+0\rangle$
$=\langle u_1, u_2\rangle=\mathbf{u}$
(iv) $\mathbf{u}+(-\mathbf{u})=\langle u_1, u_2\rangle+\langle -u_1, -u_2\rangle$
$=\langle u_1-u_1, u_2-u_2\rangle=\langle 0, 0\rangle=\mathbf{0}$
(v) $0\mathbf{u}=0\langle u_1, u_2\rangle=\langle 0u_1, 0u_2\rangle=\langle 0, 0\rangle=\mathbf{0}$
(vi) $1\mathbf{u}=1\langle u_1, u_2\rangle=\langle 1u_1, 1u_2\rangle=\langle u_1, u_2\rangle=\mathbf{u}$
(vii) $a(b\mathbf{u})=a(b\langle u_1, u_2\rangle)=a\langle bu_1, bu_2\rangle$
$=\langle abu_1, abu_2\rangle=ab\langle u_1, u_2\rangle=(ab)\mathbf{u}$
(viii) $a(\mathbf{u}+\mathbf{v})=a\langle u_1+v_1, u_2+v_2\rangle$
$=\langle au_1+av_1, au_2+av_2\rangle$
$=\langle au_1, au_2\rangle+\langle av_1, av_2\rangle$
$=a\langle u_1, u_2\rangle+a\langle v_1, v_2\rangle$
$=a\mathbf{u}+a\mathbf{v}$
(ix) $(a+b)\mathbf{u}=(a+b)\langle u_1, u_2\rangle$
$=\langle (a+b)u_1, (a+b)u_2\rangle$
$=\langle au_1+bu_1, au_2+bu_2\rangle$
$=\langle au_1, au_2\rangle+\langle bu_1, bu_2\rangle=a\mathbf{u}+b\mathbf{u}$

52. $\langle 3, 4\rangle=\left\langle \frac{7}{2}, \frac{7}{2}\right\rangle+\left\langle -\frac{1}{2}, \frac{1}{2}\right\rangle$
53. **(a)** $y=-x-1$
(b) $y=x+3$
54. $\cos^{-1}\left(\dfrac{7\sqrt{2}}{10}\right)\approx 8.130°$

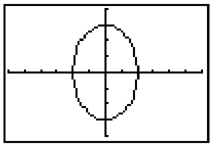
10.3 Vector-valued Functions (pp. 529–539)

Quick Review 10.3

1. $y=\left(-\dfrac{1}{\sqrt{3}}\right)x+\dfrac{4}{\sqrt{3}}$, or approximately
$y=-0.577x+2.309$
2. $y=\sqrt{3}x$
3. 0
4. Undefined; vertical tangent
5. $y=\left(-\dfrac{5\sqrt{3}}{4}\right)x+10$
6. $y=\left(\dfrac{4\sqrt{3}}{15}\right)x+\dfrac{9}{10}$
7. $\dfrac{1}{4}$
8. $\approx$3.400
9. $\approx$2.958
10. $y=xe^x-e^x+3$

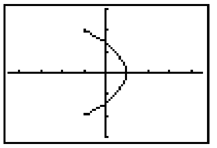
Section 10.3 Exercises

1. $6\mathbf{i}-3\mathbf{j}$
2. $-3\mathbf{i}+4\mathbf{j}$
3. **(a)** $-\mathbf{i}-\mathbf{j}$ **(b)** $7\mathbf{i}+5\mathbf{j}$
4. **(a)** $8\mathbf{i}+2\mathbf{j}$ **(b)** $2\mathbf{i}-6\mathbf{j}$
(c) $15\mathbf{i}-6\mathbf{j}$ **(d)** $\mathbf{i}-16\mathbf{j}$
5. **(a)**



[−6, 6] by [−4, 4]

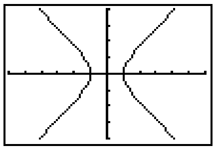
(b) $\mathbf{v}(t)=(-2\sin t)\mathbf{i}+(3\cos t)\mathbf{j}$
$\mathbf{a}(t)=(-2\cos t)\mathbf{i}+(-3\sin t)\mathbf{j}$
(c) Speed = 2; direction = $\langle -1, 0\rangle$
(d) Velocity = $2\langle -1, 0\rangle$
6. **(a)**



[−4.5, 4.5] by [−3, 3]

(b) $\mathbf{v}(t)=(-2\sin 2t)\mathbf{i}+(2\cos t)\mathbf{j}$
$\mathbf{a}(t)=(-4\cos 2t)\mathbf{i}-(2\sin t)\mathbf{j}$
(c) Speed = 2; direction = $\langle 0, 1\rangle$
(d) Velocity = $2\langle 0, 1\rangle$

7. (a)



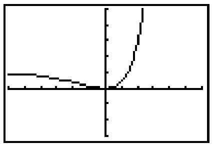
[−6, 6] by [−4, 4]

(b) $\mathbf{v}(t) = (\sec t \tan t)\mathbf{i} + (\sec^2 t)\mathbf{j}$
$\mathbf{a}(t) = (\sec t \tan^2 t + \sec^3 t)\mathbf{i} + (2 \sec^2 t \tan t)\mathbf{j}$

(c) Speed $= \dfrac{2\sqrt{5}}{3}$; direction $= \left\langle \dfrac{1}{\sqrt{5}}, \dfrac{2}{\sqrt{5}} \right\rangle$

(d) Velocity $= \dfrac{2\sqrt{5}}{3}\left\langle \dfrac{1}{\sqrt{5}}, \dfrac{2}{\sqrt{5}} \right\rangle$

8. (a)



[−6, 6] by [−3, 5]

(b) $\mathbf{v}(t) = \left(\dfrac{2}{t+1}\right)\mathbf{i} + (2t)\mathbf{j}$

$\mathbf{a}(t) = \left(-\dfrac{2}{(t+1)^2}\right)\mathbf{i} + 2\mathbf{j}$

(c) Speed $= \sqrt{5}$; direction $= \left\langle \dfrac{1}{\sqrt{5}}, \dfrac{2}{\sqrt{5}} \right\rangle$

(d) Velocity $= \sqrt{5}\left\langle \dfrac{1}{\sqrt{5}}, \dfrac{2}{\sqrt{5}} \right\rangle$

9. (a) $y = -1$ **(b)** $x = 0$

10. (a) $y = -\dfrac{3}{2}x + \dfrac{6\sqrt{2} - 7}{2}$

(b) $y = \dfrac{2}{3}x + \dfrac{5\sqrt{2} + 18}{6}$

11. $-3\mathbf{i} + (4\sqrt{2} - 2)\mathbf{j}$

12. $\left(\sqrt{2} + \dfrac{\pi}{2}\right)\mathbf{j}$

13. $(\sec t)\mathbf{i} + (\ln|\sec t|)\,\mathbf{j} + \mathbf{C}$

14. $(\ln|t|)\mathbf{i} - (\ln|5 - t|)\mathbf{j} + \mathbf{C}$

15. $\mathbf{r}(t) = ((t+1)^{3/2} - 1)\mathbf{i} - (e^{-t} - 1)\mathbf{j}$

16. $\mathbf{r}(t) = \left(\dfrac{t^4}{4} + 2t^2 + 1\right)\mathbf{i} + \left(\dfrac{t^2}{2} + 1\right)\mathbf{j}$

17. $\mathbf{r}(t) = (8t + 100)\mathbf{i} + (-16t^2 + 8t)\mathbf{j}$

18. $\mathbf{r}(t) = \left(-\dfrac{t^2}{2} + 10\right)\mathbf{i} + \left(-\dfrac{t^2}{2} + 10\right)\mathbf{j}$

19. $t = 0, \pi, 2\pi$

20. $t = \dfrac{k\pi}{2}$, k any nonnegative integer

21. $t = \dfrac{k\pi}{2}$, k any nonnegative integer

22. For all values of t

23. $\cos^{-1}\left(\dfrac{3}{5}\right) \approx 53.130°$

24. 90°

25. (a) $3\mathbf{i}$ **(b)** $t \neq 0, -3$
(c) $t = 0, -3$

26. (a) $2\mathbf{i}$ **(b)** $(-1, 0) \cup (0, \infty)$
(c) $(-\infty, -1] \cup \{0\}$

27. 2

28. (a) Initial $= \left(\dfrac{1}{4}, 1\right)$; terminal $= \left(\dfrac{e^8}{4} - 2, e^4\right)$

(b) $\dfrac{e^8 + 7}{4} \approx 746.989$

(c) $\pi\left(\dfrac{e^{16} - 12e^8 - 69}{16}\right) \approx 1{,}737{,}746.456$

29. (a) $\mathbf{v}(t) = (\cos t)\mathbf{i} - (2 \sin 2t)\mathbf{j}$

(b) $t = \dfrac{\pi}{2}, \dfrac{3\pi}{2}$

(c) $y = 1 - 2x^2$, $-1 \le x \le 1$. The particle starts at (0, 1), goes to (1, −1), then goes to (−1, −1), and then goes to (0, 1), tracing the curve twice.

30. (a) $\dfrac{t^2 - 4}{2t^2 - 2t}$

(b) $t = -2$: horizontal tangent at (−28, 16)
$t = 0$: vertical tangent at (0, 0)
$t = 1$: vertical tangent at (−1, −11)
$t = 2$: horizontal tangent (4, −16)

31. $\mathbf{r}(t) = \left(\dfrac{3}{2}t^2 + \dfrac{3\sqrt{10}}{5}t + 1\right)\mathbf{i} + \left(-\dfrac{1}{2}t^2 - \dfrac{\sqrt{10}}{5}t + 2\right)\mathbf{j}$

32. (a) $(2\sqrt{2}, 6)$ **(b)** -6
(c) -12

33. (a) 160 seconds **(b)** 225 meters

(c) $\dfrac{15}{4}$ meters per second

(d) At $t = 80$ seconds

34. (a) $t = 2$

(b) First particle: $\left\langle \dfrac{1}{\sqrt{5}}, -\dfrac{2}{\sqrt{5}} \right\rangle$

Second particle: $\left\langle \dfrac{1}{\sqrt{2}}, \dfrac{1}{\sqrt{2}} \right\rangle$

35. (a) Referring to the figure, look at the circular arc from the point where $t = 0$ to the point "m". On one hand, this arc has length given by $r_0\theta$, but it also has length given by vt. Setting those two quantities equal gives the result.

(b) $\mathbf{a}(t) = -\dfrac{v^2}{r_0}\left[\left(\cos \dfrac{vt}{r_0}\right)\mathbf{i} + \left(\sin \dfrac{vt}{r_0}\right)\mathbf{j}\right]$

(c) From part (b) above, $\mathbf{a}(t) = -\left(\dfrac{v}{r_0}\right)^2 \mathbf{r}(t)$.

So, by Newton's second law, $\mathbf{F} = -m\left(\dfrac{v}{r_0}\right)^2 \mathbf{r}$.

Substituting for $\mathbf{F}$ in the law of gravitation

gives the result.

(d) Set $\dfrac{vT}{r_0} = 2\pi$ and solve for vT.

(e) Substitute $\dfrac{2\pi r_0}{T}$ for v in $v^2 = \dfrac{GM}{r_0}$ and solve for T^2.

36. $y = (e^x - 1)^2 - 1$ or $y = e^{2x} - 2e^x$, for $x \ge 0$

37. **(a)** Apply Corollary 3 to each component separately.

(b) Follows immediately from (a) since any two anti-derivatives of $\mathbf{r}(t)$ must have identical derivatives, namely $\mathbf{r}(t)$.

38. $\frac{d}{dt}|\mathbf{v}|^2 = \frac{d}{dt}(\mathbf{v}\cdot\mathbf{v}) = \mathbf{v}'\cdot\mathbf{v} + \mathbf{v}\cdot\mathbf{v}' = 2\mathbf{v}\cdot\mathbf{v}' = 0.$
Therefore, $|\mathbf{v}|$ is constant.

39. Let $\mathbf{C} = \langle C_1, C_2\rangle$. $\frac{d\mathbf{C}}{dt} = \left\langle \frac{dC_1}{dt}, \frac{dC_2}{dt}\right\rangle = \langle 0, 0\rangle$.

40. **(a)** Suppose $\mathbf{u} = \langle u_1(t), u_2(t)\rangle$.

$$\begin{aligned}\frac{d}{dt}(c\mathbf{u}) &= \frac{d}{dt}\langle cu_1(t), cu_2(t)\rangle\\ &= \left\langle \frac{d}{dt}(cu_1(t)), \frac{d}{dt}(cu_2(t))\right\rangle\\ &= \left\langle c\frac{du_1}{dt}, c\frac{du_2}{dt}\right\rangle = c\left\langle \frac{du_1}{dt}, \frac{du_2}{dt}\right\rangle = c\frac{d\mathbf{u}}{dt}\end{aligned}$$

(b)
$$\begin{aligned}\frac{d}{dt}(f\mathbf{u}) &= \frac{d}{dt}\langle fu_1, fu_2\rangle\\ &= \langle fu_1' + f'u_1, fu_2' + f'u_2\rangle\\ &= \langle fu_1', fu_2'\rangle + \langle f'u_1, f'u_2\rangle\\ &= f\mathbf{u}' + f'\mathbf{u}\end{aligned}$$

41. $\mathbf{u} = \langle u_1, u_2\rangle$, $\mathbf{v} = \langle v_1, v_2\rangle$

(a)
$$\begin{aligned}\frac{d}{dt}(\mathbf{u}+\mathbf{v}) &= \frac{d}{dt}(\langle u_1 + v_1, u_2 + v_2\rangle)\\ &= \left\langle \frac{d}{dt}(u_1+v_1), \frac{d}{dt}(u_2+v_2)\right\rangle\\ &= \langle u_1' + v_1', u_2' + v_2'\rangle\\ &= \langle u_1', u_2'\rangle + \langle v_1', v_2'\rangle = \frac{d\mathbf{u}}{dt} + \frac{d\mathbf{v}}{dt}\end{aligned}$$

(b)
$$\begin{aligned}\frac{d}{dt}(\mathbf{u}-\mathbf{v}) &= \frac{d}{dt}(\langle u_1 - v_1, u_2 - v_2\rangle)\\ &= \left\langle \frac{d}{dt}(u_1-v_1), \frac{d}{dt}(u_2-v_2)\right\rangle\\ &= \langle u_1' - v_1', u_2' - v_2'\rangle\\ &= \langle u_1', u_2'\rangle - \langle v_1', v_2'\rangle\\ &= \frac{d\mathbf{u}}{dt} - \frac{d\mathbf{v}}{dt}\end{aligned}$$

42.
$$\begin{aligned}\frac{d\mathbf{r}}{dt} &= \frac{df}{dt}\mathbf{i} + \frac{dg}{dt}\mathbf{j}\\ \left(\frac{d\mathbf{r}}{dt}\right)\left(\frac{dt}{ds}\right) &= \left(\frac{df}{dt}\mathbf{i} + \frac{dg}{dt}\mathbf{j}\right)\left(\frac{dt}{ds}\right)\\ &= \left(\frac{df}{dt}\cdot\frac{dt}{ds}\right)\mathbf{i} + \left(\frac{dg}{dt}\cdot\frac{dt}{ds}\right)\mathbf{j}\\ &= \frac{df}{ds}\mathbf{i} + \frac{dg}{ds}\mathbf{j}\\ &= \frac{d\mathbf{r}}{ds}\end{aligned}$$

43. $f(t)$ and $g(t)$ differentiable at $c \Rightarrow f(t)$ and $g(t)$ continuous at $c \Rightarrow \mathbf{r}(t) = f(t)\mathbf{i} + g(t)\mathbf{j}$ is continuous at c.

44. **(a)** Let $\mathbf{r}(t) = \langle x(t), y(t)\rangle$.

$$\begin{aligned}\int_a^b k\mathbf{r}(t)\,dt &= \int_a^b \langle kx(t), ky(t)\rangle\,dt\\ &= \left\langle \int_a^b kx(t)\,dt, \int_a^b ky(t)\,dt\right\rangle\\ &= \left\langle k\int_a^b x(t)\,dt, k\int_a^b y(t)\,dt\right\rangle\\ &= k\left\langle \int_a^b x(t)\,dt, \int_a^b y(t)\,dt\right\rangle = k\int_a^b \langle x(t), y(t)\rangle\,dt\\ &= k\int_a^b \mathbf{r}(t)\,dt\end{aligned}$$

(b) Let $\mathbf{r}_1(t) = \langle x_1(t), y_1(t)\rangle$ and $\mathbf{r}_2(t) = \langle x_2(t), y_2(t)\rangle$.

$$\begin{aligned}&\int_a^b (\mathbf{r}_1(t) \pm \mathbf{r}_2(t))\,dt\\ &= \int_a^b (\langle x_1(t), y_1(t)\rangle \pm \langle x_2(t), y_2(t)\rangle)\,dt\\ &= \int_a^b \langle x_1(t) \pm x_2(t), y_1(t) \pm y_2(t)\rangle\,dt\\ &= \left\langle \int_a^b (x_1(t) \pm x_2(t))\,dt, \int_a^b (y_1(t) \pm y_2(t))\,dt\right\rangle\\ &= \left\langle \int_a^b x_1(t)\,dt \pm \int_a^b x_2(t)\,dt, \int_a^b y_1(t)\,dt \pm \int_a^b y_2(t)\,dt\right\rangle\\ &= \left\langle \int_a^b x_1(t)\,dt, \int_a^b y_1(t)\,dt\right\rangle \pm \left\langle \int_a^b x_2(t)\,dt, \int_a^b y_2(t)\,dt\right\rangle\\ &= \int_a^b \mathbf{r}_1(t)\,dt \pm \int_a^b \mathbf{r}_2(t)\,dt\end{aligned}$$

(c) Let $\mathbf{C} = \langle C_1, C_2\rangle$, $\mathbf{r}(t) = \langle x(t), y(t)\rangle$.

$$\begin{aligned}\int_a^b \mathbf{C}\cdot\mathbf{r}(t)\,dt &= \int_a^b (C_1xt + C_2yt)\,dt\\ &= C_1\int_a^b xt\,dt + C_2\int_a^b yt\,dt\\ &= \langle C_1, C_2\rangle\cdot\left\langle \int_a^b xt\,dt, \int_a^b yt\,dt\right\rangle\\ &= \mathbf{C}\cdot\int_a^b \mathbf{r}(t)\,dt\end{aligned}$$

45. **(a)** Let $\mathbf{r}(t) = f(t)\mathbf{i} + g(t)\mathbf{j}$. Then

$$\begin{aligned}\frac{d}{dt}\int_a^t \mathbf{r}(q)\,dq &= \frac{d}{dt}\int_a^t [f(q)\mathbf{i} + g(q)\mathbf{j}]\,dq\\ &= \frac{d}{dt}\left[\left(\int_a^t f(q)\,dq\right)\mathbf{i} + \left(\int_a^t g(q)\,dq\right)\mathbf{j}\right]\\ &= \left(\frac{d}{dt}\int_a^t f(q)\,dq\right)\mathbf{i} + \left(\frac{d}{dt}\int_a^t g(q)\,dq\right)\mathbf{j}\\ &= f(t)\mathbf{i} + g(t)\mathbf{j} = \mathbf{r}(t).\end{aligned}$$

(b) Let $\mathbf{S}(t) = \int_a^t \mathbf{r}(q)\,dq$. Then part (a) shows that $\mathbf{S}(t)$ is an antiderivative of $\mathbf{r}(t)$. Let $\mathbf{R}(t)$ be any antiderivative of $\mathbf{r}(t)$. Then according to 37(b), $\mathbf{S}(t) = \mathbf{R}(t) + \mathbf{C}$. Letting $t = a$, we have $\mathbf{0} = \mathbf{S}(a) = \mathbf{R}(a) + \mathbf{C}$. Therefore, $\mathbf{C} = -\mathbf{R}(a)$ and $\mathbf{S}(t) = \mathbf{R}(t) - \mathbf{R}(a)$. The result follows by letting $t = b$.

10.4 Modeling Projectile Motion (pp. 539–552)

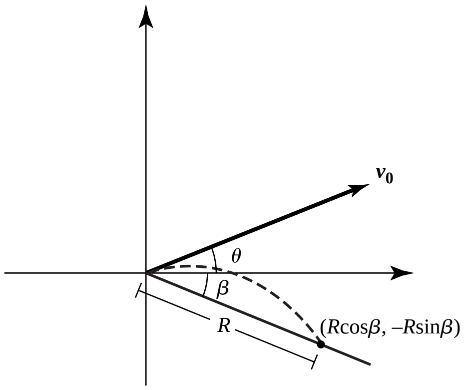
Quick Review 10.4

1. $\langle 50 \cos 25°, 50 \sin 25° \rangle \approx \langle 45.315, 21.131 \rangle$
2. $\langle -40, 40\sqrt{3} \rangle$
3. x-intercepts: $\left(\frac{5}{2}, 0\right)$ and $(-8, 0)$
y-intercept: $(0, -40)$
4. $\left(-\frac{11}{4}, -\frac{441}{8}\right)$
5. x-intercepts: $(0, 0)$ and $(20, 0)$
y-intercept: $(0, 0)$
6. $(10, 100)$
7. $y = -\cos x + 2$
8. $y = \frac{t^3}{3} + 3t + \frac{25}{3}$
9. $y = 16 + 4e^{-t}$
10. $y = 2 - e^{-x^2}$

Section 10.4 Exercises

1. 50 seconds
2. 490 m/sec
3. **(a)** After ≈72.154 seconds, ≈25.510 km downrange
(b) 4020 m
(c) ≈6377.55 m
4. After 2 seconds, $32\sqrt{3} \approx 55.426$ feet away (horizontally)
5. After ≈2.135 seconds, ≈66.421 feet from the stopboard
6. ≈1.184 inches
7. **(a)** $7\sqrt{2} \approx 9.899$ m/sec
(b) ≈18.435° or 71.565°
8. $\approx 3.136 \times 10^{-14}$ meters, or $\approx 3.136 \times 10^{-12}$ cm
9. ≈278.016 ft/sec, or ≈189.556 mph
10. Yes, ≈32.079°
11. No. When it has travelled 135 ft in the horizontal direction, it is only about 29.942 feet above the ground.
12. ≈0.255 feet beyond the pin
13. **(a)** ≈149.307 ft/sec
(b) ≈2.245 seconds
14. In the formula for range, $\sin 2\alpha = \sin 2(90 - \alpha)$.
15. ≈39.261° and 50.739°
16. **(a)** Substitute $2v_0$ for v_0 in the formula for range.
(b) 41%
17. ≈46.597 ft/sec
18. $y(t) = v_0(\sin \alpha)t - \frac{1}{2}gt^2$, and we know the maximum height is $\frac{(v_0 \sin \alpha)^2}{2g}$ and it occurs when $t = \frac{v_0 \sin \alpha}{g}$. Substituting $t = \frac{v_0 \sin \alpha}{2g}$ into the equation for $y(t)$ gives a height of $\frac{3(v_0 \sin \alpha)^2}{8g}$, which is three-fourths of the maximum height.
19. Integrating, $\frac{d}{dt}\mathbf{r}(t) = c_1\mathbf{i} + (-gt + c_2)\mathbf{j}$. The initial condition on the velocity gives $c_1 = v_0 \cos \alpha$ and $c_2 = v_0 \sin \alpha$. Integrating again,
$$\mathbf{r}(t) = ((v_0 \cos \alpha)t + c_3)\mathbf{i} + \left(-\frac{gt^2}{2} + (v_0 \sin \alpha)t + c_4\right)\mathbf{j}.$$
The initial condition on the position gives $c_3 = x_0$ and $c_4 = y_0$.
20. ≈78.7 ft/sec
21. It takes about 1.924 seconds. The arrow passes about 3.698 feet above the rim. (It's 73.698 feet above the ground.)
22. The projectile rises straight up and then falls straight down, returning to the firing point.
23. Angle ≈ 62°
Maximum height = 4 feet (independent of the measured angle)
Speed of engine ≈ 8.507 ft/sec (changes with the angle)
24. The height of A is given by $y_A = (v \sin \alpha)t - \frac{1}{2}gt^2$ and the height of B is given by $y_B = R \tan \alpha - \frac{1}{2}gt^2$. The second terms in y_A and y_B $\left(-\frac{gt^2}{2}\right)$ are equal for any value of t. But A moves R units horizontally to B's line of fall in $\frac{R}{v \cos \alpha}$ time units, and the first terms in y_A and y_B are also equal at that time: $(v \sin \alpha)\left(\frac{R}{v \cos \alpha}\right) = R \tan \alpha$. Therefore, A and B will always be at the same height when A reaches B's line of fall.

25. (a)



$x = (v_0 \cos \theta)t$

$y = (v_0 \sin \theta)t - \frac{1}{2}gt^2$

$x = R\cos\beta \Rightarrow R\cos\beta = (v_0 \cos\theta)t$

$\Rightarrow t = \frac{R\cos\beta}{v_0\cos\theta}$. Then $y = -R\sin\beta$

$\Rightarrow -R\sin\beta = \frac{(v_0\sin\theta)\,R\cos\beta}{v_0\cos\theta} - \frac{g}{2}\,\frac{R^2\cos^2\beta}{v_0^2\cos^2\theta}$

$\Rightarrow R = \frac{2v_0^2}{g\cos^2\beta}\cos\theta\sin(\theta+\beta)$.

Let $f(\theta) = \cos\theta\sin(\theta+\beta)$.

$f'(\theta) = \cos\theta\cos(\theta+\beta) - \sin\theta\sin(\theta+\beta)$

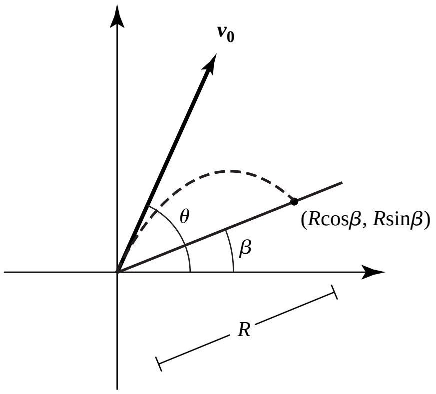
$f'(\theta) = 0 \Rightarrow \tan\theta\tan(\theta+\beta) = 1$

$\Rightarrow \tan\theta = \cot(\theta+\beta)$

$\Rightarrow \theta+\beta = 90° - \theta$. Note that $f''(\theta) < 0$, so R

is maximum when $\theta + \beta = 90° - \theta$.

(b)



$R = \frac{2v_0^2}{g\cos^2\beta}\cos\theta\sin(\theta-\beta)$ is maximum when $\tan\theta = \cot(\theta-\beta)$,

so $\theta - \beta = 90° - \theta$.

The initial velocity vector bisects the angle between the hill and the vertical for max range.

26. (a) $\mathbf{r}(t) = (x(t))\mathbf{i} + (y(t))\mathbf{j}$, where
$x(t) = (145\cos 23° - 14)t$ and
$y(t) = 2.5 + (145\sin 23°)t - 16t^2$.

(b) At $t \approx 1.771$ seconds, it reaches a maximum height of about 52.655 feet.

(c) Range $\approx$ 428.262 feet
flight time $\approx$ 3.585 seconds

(d) At $t \approx 0.342$ and $t \approx 3.199$ seconds, and it is 40.847 and 382.208 feet from home plate at those times.

(e) Yes. According to part (d), the ball is still 20 feet above the ground when it is 382 feet from home plate.

27. (a) (Assuming that "x" is zero at the point of impact.)
$\mathbf{r}(t) = (x(t))\mathbf{i} + (y(t))\mathbf{j}$, where
$x(t) = (35\cos 27°)t$ and
$y(t) = 4 + (35\sin 27°)t - 16t^2$.

(b) At $t \approx 0.497$ seconds, it reaches its maximum height of about 7.945 feet.

(c) Range $\approx$ 37.460 feet
flight time $\approx$ 1.201 seconds

(d) At $t \approx 0.254$ and $t \approx 0.740$ seconds, when it is $\approx$29.554 and $\approx$14.396 feet from where it will land.

(e) Yes. It changes things because the ball won't clear the net.

28. (a) $\mathbf{r}(t) = (x(t))\mathbf{i} + (y(t))\mathbf{j}$, where

$x(t) = \left(\frac{152}{0.12}\right)(1 - e^{-0.12t})(\cos 20°)$ and

$y(t) = 3 + \left(\frac{152}{0.12}\right)(1 - e^{-0.12t})(\sin 20°) + \left(\frac{32}{0.12^2}\right)(1 - 0.12t - e^{-0.12t})$

(b) At $t \approx 1.484$ seconds it reaches its maximum height of about 40.435 feet.

(c) Range $\approx$ 372.323 feet
flight time $\approx$ 3.126 seconds

(d) At $t \approx 0.689$ and $t \approx 2.305$ seconds, when it is about 94.513 and 287.628 feet from home plate.

(e) Yes, the batter has hit a home run since the ball is more than 15 feet above the ground when it passes over the fence.

29. (a) $\mathbf{r}(t) = (x(t))\mathbf{i} + (y(t))\mathbf{j}$, where

$x(t) = \left(\frac{1}{0.08}\right)(1 - e^{-0.08t})(152\cos 20° - 17.6)$

and

$y(t) = 3 + \left(\frac{152}{0.08}\right)(1 - e^{-0.08t})(\sin 20°) + \left(\frac{32}{0.08^2}\right)(1 - 0.08t - e^{-0.08t})$

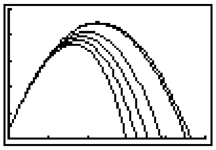
(b) At $t \approx 1.527$ seconds it reaches its maximum height of about 41.893 feet.

(c) Range ≈ 351.734 feet
Flight time ≈ 3.181 seconds

(d) At $t \approx 0.877$ and $t \approx 2.190$ seconds, when it is about 106.028 and 251.530 feet from home plate.

(e) No. The wind gust would need to be greater than 12.846 ft/sec in the direction of the hit in order for the ball to clear the fence for a home run.

30. (a)



[0, 500] by [0, 50]

(b)

drag coeff	time at max ht	max ht
$k = 0.01$	$t \approx 1.612$	44.777
$k = 0.02$	$t \approx 1.599$	44.336
$k = 0.10$	$t \approx 1.505$	41.149
$k = 0.15$	$t \approx 1.454$	39.419
$k = 0.20$	$t \approx 1.407$	37.854
$k = 0.25$	$t \approx 1.363$	36.431

(c)

drag coeff	flight time	range
$k = 0.01$	$t \approx 3.289$	462.152
$k = 0.02$	$t \approx 3.273$	452.478
$k = 0.10$	$t \approx 3.153$	386.274
$k = 0.15$	$t \approx 3.088$	352.983
$k = 0.20$	$t \approx 3.028$	324.410
$k = 0.25$	$t \approx 2.974$	299.661

(d) This follows from the following two limits (as $k \to 0$):

$$\lim_{k\to 0} \frac{1 - e^{-kt}}{k} = t, \text{ and}$$
$$\lim_{k\to 0} \frac{1 - kt - e^{-kt}}{k^2} = -\frac{t^2}{2}.$$

As $k \to 0$, the air resistance approaches 0.

31. The points in question are $(x, y) = \left(\frac{R}{2}, y_{\max}\right)$. So, $x = \frac{{v_0}^2 \sin\alpha \cos\alpha}{g}$, and $y = \frac{(v_0 \sin\alpha)^2}{2g}$. Substituting these into the given equation for the ellipse yields an identity.

32. From Equation (10), find $\left(\frac{d\mathbf{r}}{dt}\right)$ and $\left(\frac{d^2\mathbf{r}}{dt^2}\right)$, and then show that they satisfy both the equation and the initial conditions.

10.5 Polar Coordinates and Polar Graphs (pp. 552–559)

Quick Review 10.5

1. $y = -x + 2$ **2.** $x^2 + y^2 = 9$

3. $(x + 2)^2 + (y - 4)^2 = 4$

4. (a) No
(b) No
(c) Yes

5. (a) No
(b) No
(c) No

6. (a) No
(b) Yes
(c) No

7. (a) Yes
(b) Yes
(c) Yes

8. Graph $y = (x - 2)^{1/2}$ and $y = -(x - 2)^{1/2}$

9. Graph $y = \left(\frac{4 - x^2}{3}\right)^{1/2}$ and $y = -\left(\frac{4 - x^2}{3}\right)^{1/2}$

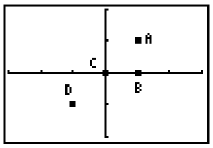
10. $(x - 2)^2 + (y + 3)^2 = 4$, center $= (2, -3)$, radius $= 2$

Section 10.5 Exercises

1. (a) and (e) are the same.
(b) and (g) are the same.
(c) and (h) are the same.
(d) and (f) are the same.

2. (a) and (f) are the same.
(b) and (h) are the same.
(c) and (g) are the same.
(d) and (e) are the same.

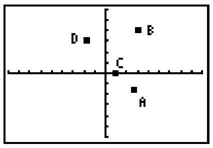
3.



[−3, 3] by [−2, 2]

(a) (1, 1)
(b) (1, 0)
(c) (0, 0)
(d) (−1, −1)

4.



[−9, 9] by [−6, 6]

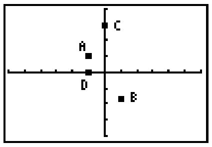
(a) $\left(\frac{3\sqrt{3}}{2}, -\frac{3}{2}\right)$

(b) (3, 4)

(c) (1, 0)

(d) $(-\sqrt{3}, 3)$

5.



[−6, 6] by [−4, 4]

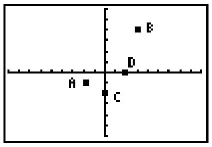
(a) $\left(\sqrt{2}, \frac{3\pi}{4}\right)$ or $\left(\sqrt{2}, -\frac{5\pi}{4}\right)$

(b) $\left(2, -\frac{\pi}{3}\right)$ or $\left(-2, \frac{2\pi}{3}\right)$

(c) $\left(3, \frac{\pi}{2}\right)$ or $\left(3, \frac{5\pi}{2}\right)$

(d) $(1, \pi)$ or $(-1, 0)$

6.



[−9, 9] by [−6, 6]

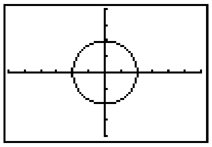
(a) $\left(2, \frac{7\pi}{6}\right)$ or $\left(-2, \frac{\pi}{6}\right)$

(b) $\left(5, \tan^{-1}\frac{4}{3}\right)$ or $\left(-5, \pi + \tan^{-1}\frac{4}{3}\right)$

(c) $\left(2, \frac{3\pi}{2}\right)$ or $\left(2, -\frac{\pi}{2}\right)$

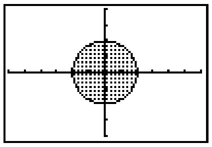
(d) $(2, 0)$ or $(2, 2\pi)$

7.



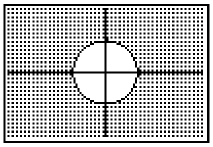
[−6, 6] by [−4, 4]

8.



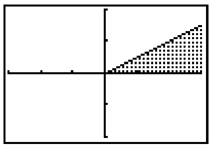
[−6, 6] by [−4, 4]

9.



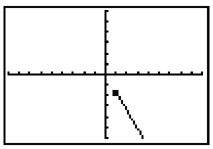
[−3, 3] by [−2, 2]

10.



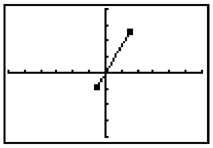
[−3, 3] by [−2, 2]

11.



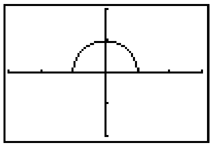
[−9, 9] by [−6, 6]

12.



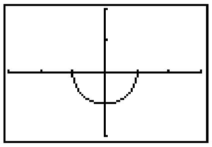
[−6, 6] by [−4, 4]

13.



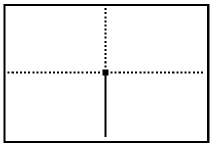
[−3, 3] by [−2, 2]

14.



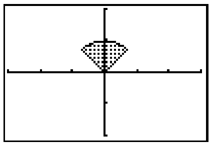
[−3, 3] by [−2, 2]

15.



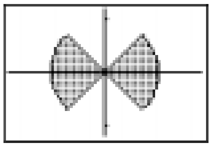
[−3, 3] by [−2, 2]

16.



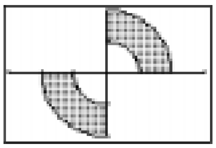
[−3, 3] by [−2, 2]

17.



[−1.8, 1.8] by [−1.2, 1.2]

18.



[−3, 3] by [−2, 2]

19. $y = 0$, the x-axis

20. $x = 0$, the y-axis

21. $y = 4$, a horizontal line

22. $x = -3$, a vertical line

23. $x + y = 1$, a line (slope $= -1$, y-intercept $= 1$)

24. $x^2 + y^2 = 1$, a circle (center $= (0, 0)$, radius $= 1$)

25. $x^2 + y^2 = 4y$, a circle (center $= (0, 2)$, radius $= 2$)

26. $y - 2x = 5$, a line (slope $= 2$, y-intercept $= 5$)

27. $xy = 1$, a hyperbola

28. $y^2 = x$, a parabola

29. $y = e^x$, the exponential curve

30. $x^2 = y^2$, the union of two lines: $y = \pm x$

31. $y = \ln x$, the logarithmic curve

32. $(x + y)^2 = 1$,
the union of two lines: $x + y = \pm 1$

33. $(x + 2)^2 + y^2 = 4$, a circle (center = $(-2, 0)$, radius = 2)

34. $x^2 + (y - 4)^2 = 16$, a circle (center = $(0, 4)$, radius = 4)

35. $(x - 1)^2 + (y - 1)^2 = 2$, a circle (center = $(1, 1)$, radius = $\sqrt{2}$)

36. $x + \sqrt{3}y = 4$, a line $\left(\text{slope} = -\frac{1}{\sqrt{3}}, y\text{-intercept} = \frac{4}{\sqrt{3}}\right)$

37. $r \cos \theta = 7$

38. $r \sin \theta = 1$

39. $\theta = \frac{\pi}{4}$

40. $r \cos \theta - r \sin \theta = 3$

41. $r^2 = 4$ or $r = 2$

42. $r^2(\cos^2 \theta - \sin^2 \theta) = 1$

43. $r^2(4 \cos^2 \theta + 9 \sin^2 \theta) = 36$

44. $r^2 \cos \theta \sin \theta = 2$ or $r^2 \sin 2\theta = 4$

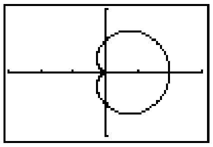
45. $r \sin^2 \theta = 4 \cos \theta$

46. $r^2(1 + \cos \theta \sin \theta) = 1$

47. $r = 4 \sin \theta$

48. $r^2 - 6r \cos \theta + 2r \sin \theta + 6 = 0$

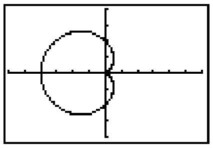
49. (a)



$[-3, 3]$ by $[-2, 2]$

(b) Length of interval = 2π

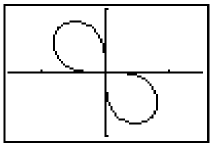
50. (a)



$[-6, 6]$ by $[-4, 4]$

(b) Length of interval = 2π

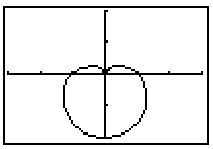
51. (a)



$[-1.5, 1.5]$ by $[-1, 1]$

(b) Length of interval = $\frac{\pi}{2}$

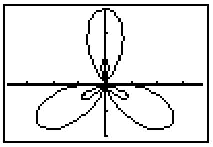
52. (a)



$[-3, 3]$ by $[-2, 2]$

(b) Length of interval = 2π

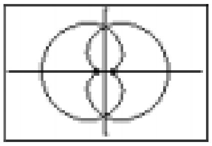
53. (a)



$[-3.75, 3.75]$ by $[-2, 3]$

(b) Length of interval = 2π

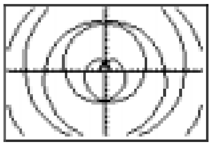
54. (a)



$[-1.5, 1.5]$ by $[-1, 1]$

(b) Length of interval = 4π

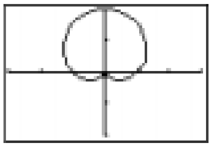
55. (a)



$[-15, 15]$ by $[-10, 10]$

(b) Required interval = $(-\infty, \infty)$

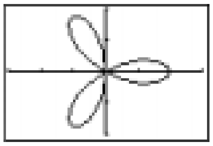
56. (a)



$[-3, 3]$ by $[-2, 2]$

(b) Length of interval = 2π

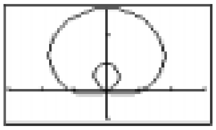
57. (a)



$[-3, 3]$ by $[-2, 2]$

(b) Length of interval = π

58. (a)



$[-3, 3]$ by $[-1, 3]$

(b) Length of interval = 2π

59. x-axis, y-axis, origin

60. Origin

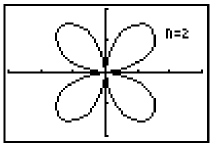
61. y-axis

62. x-axis, y-axis, origin

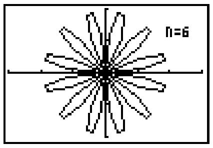
63. (a) Because $r = a \sec \theta$ is equivalent to $r \cos \theta = a$, which is equivalent to the Cartesian equation $x = a$.

(b) $r = a \csc \theta$ is equivalent to $y = a$.

64. (a) The graph is the same for $n = 2$ and $n = -2$, and in general, it's the same for $n = 2k$ and $n = -2k$. The graphs for $n = 2$, 4, and 6 are roses with 4, 8, and 12 "petals" respectively. The graphs for $n = \pm 2$ and $n = \pm 6$ are shown below.



$[-3, 3]$ by $[-2, 2]$

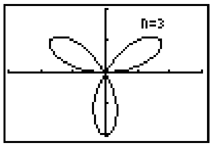


$[-3, 3]$ by $[-2, 2]$

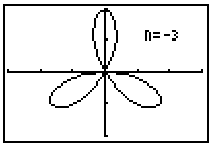
(b) 2π

(c) The graph is a rose with $2|n|$ "petals".

(d) The graphs are roses with 3, 5, and 7 "petals" respectively. The "center leaf" points upward if $n = -3$, $+5$, or -7. The graphs for $n = 3$ and $n = -3$ are shown below.



$[-3, 3]$ by $[-2, 2]$



$[-3, 3]$ by $[-2, 2]$

(e) π

(f) The graph is a rose with $|n|$ "petals".

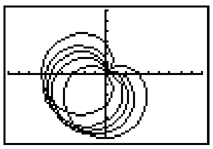
65. (a) We have $x = r\cos\theta$ and $y = r\sin\theta$. By taking $t = \theta$, we have $r = f(t)$, so $x = f(t)\cos t$ and $y = f(t)\sin t$.

(b) $x = 3\cos t$, $y = 3\sin t$

(c) $x = (1 - \cos t)\cos t$, $y = (1 - \cos t)\sin t$

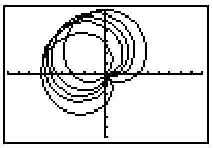
(d) $x = (3\sin 2t)\cos t$, $y = (3\sin 2t)\sin t$

66. (a)



$[-9, 9]$ by $[-6, 6]$

(b)

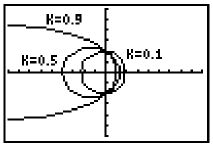


$[-9, 9]$ by $[-6, 6]$

(c) The graph of r_2 is the graph of r_1 rotated counterclockwise about the origin by the angle α.

67. $d = [(x_2 - x_1)^2 + (y_2 + y_1)^2]^{1/2}$
$= [(r_2\cos\theta_2 - r_1\cos\theta_1)^2 + (r_2\sin\theta_2 - r_1\sin\theta_1)^2]^{1/2}$
and then simplify using trigonometric identities.

68. (a)

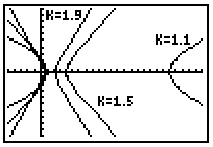


$[-9, 9]$ by $[-6, 6]$

The graphs are ellipses.

(b) As $k \to 0^+$, the graph approaches the circle of radius 2 centered at the origin.

69. (a)

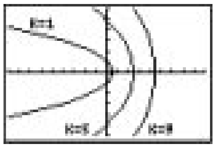


$[-5, 25]$ by $[-10, 10]$

The graphs are hyperbolas.

(b) As $k \to 1^+$, the right branch of the hyperbola goes to infinity and "disappears". The left branch approaches the parabola $y^2 = 4 - 4x$.

70. (a)



$[-9, 9]$ by $[-6, 6]$

The graphs are parabolas.

(b) As $k \to 0^+$, the limit of the graph is the negative x-axis.

10.6 Calculus of Polar Curves (pp. 559–568)

Quick Review 10.6

1. $-\frac{5}{3}\cot t$ **2.** $-\frac{5}{3}\cot 2 \approx 0.763$

3. (0, 5) and (0, −5) **4.** (3, 0) and (−3, 0)

5. ≈ 12.763

6. The upper half of the outer loop

7. The inner loop

8. The lower half of the outer loop

9. 36 **10.** ≈ 2.403

Section 10.6 Exercises

1. At $\theta = 0$: -1
At $\theta = \pi$: 1

2. At $\theta = 0$: undefined
At $\theta = -\frac{\pi}{2}$: 0
At $\theta = \frac{\pi}{2}$: 0
At $\theta = \pi$: undefined

3. At $(2, 0)$: $-\frac{2}{3}$

At $\left(-1, \frac{\pi}{2}\right)$: 0

At $(2, \pi)$: $\frac{2}{3}$

At $\left(5, \frac{3\pi}{2}\right)$: 0

4. At $\left(1.5, \frac{\pi}{3}\right)$: undefined

At $\left(4.5, \frac{2\pi}{3}\right)$: 0

At $(6, \pi)$: undefined

At $\left(3, \frac{3\pi}{2}\right)$: 1

5. $\theta = \frac{\pi}{2}$ $\quad [x = 0]$

6. $\theta = \frac{\pi}{6}$ $\quad \left[y = \frac{1}{\sqrt{3}}x\right]$

$\theta = \frac{\pi}{2}$ $\quad [x = 0]$

$\theta = \frac{5\pi}{6}$ $\quad \left[y = -\frac{1}{\sqrt{3}}x\right]$

7. $\theta = 0$ $\quad [y = 0]$

$\theta = \frac{\pi}{5}$ $\quad \left[y = \left(\tan\frac{\pi}{5}\right)x\right]$

$\theta = \frac{2\pi}{5}$ $\quad \left[y = \left(\tan\frac{2\pi}{5}\right)x\right]$

$\theta = \frac{3\pi}{5}$ $\quad \left[y = \left(\tan\frac{3\pi}{5}\right)x\right]$

$\theta = \frac{4\pi}{5}$ $\quad \left[y = \left(\tan\frac{4\pi}{5}\right)x\right]$

8. $\theta = 0$ $\quad [y = 0]$

$\theta = \frac{\pi}{2}$ $\quad [x = 0]$

9. Horizontal at: $\left(-\frac{1}{2}, \frac{\pi}{6}\right)$ $\left[y = -\frac{1}{4}\right]$,

$\left(-\frac{1}{2}, \frac{5\pi}{6}\right)$ $\left[y = -\frac{1}{4}\right]$,

$\left(-2, \frac{3\pi}{2}\right)$ $[y = 2]$

Vertical at: $\left(0, \frac{\pi}{2}\right)$ $[x = 0]$,

$\left(-\frac{3}{2}, \frac{7\pi}{6}\right)$ $\left[x = \frac{3\sqrt{3}}{4}\right]$,

$\left(-\frac{3}{2}, \frac{11\pi}{6}\right)$ $\left[x = -\frac{3\sqrt{3}}{4}\right]$

10. Horizontal at: $\left(\frac{3}{2}, \frac{\pi}{3}\right)$ $\left[y = \frac{3\sqrt{3}}{4}\right]$,

$(0, \pi)$ $[y = 0]$,

$\left(\frac{3}{2}, \frac{5\pi}{3}\right)$ $\left[y = -\frac{3\sqrt{3}}{4}\right]$

Vertical at: $(2, 0)$ $[x = 2]$,

$\left(\frac{1}{2}, \frac{2\pi}{3}\right)$ $\left[x = -\frac{1}{4}\right]$,

$\left(\frac{1}{2}, \frac{4\pi}{3}\right)$ $\left[x = -\frac{1}{4}\right]$,

$(2, 2\pi)$ $[x = 2]$

11. Horizontal at: $(0, 0)$ $[y = 0]$,

$\left(2, \frac{\pi}{2}\right)$ $[y = 2]$,

$(0, \pi)$ $[y = 0]$

Vertical at: $\left(\sqrt{2}, \frac{\pi}{4}\right)$ $[x = 1]$,

$\left(\sqrt{2}, \frac{3\pi}{4}\right)$ $[x = -1]$

12. Horizontal at: $(-0.676, 0.405)$ $[y \approx -0.267]$,

$(5.176, 2.146)$ $[y \approx 4.343]$,

$(5.176, 4.137)$ $[y \approx -4.343]$,

$(-0.676, 5.878)$ $[y \approx 0.267]$

Vertical at: $(-1, 0)$ $[x = -1]$,

$(1.5, 1.186)$ $\left[x = \frac{9}{16}\right]$,

$(7, \pi)$ $[x = -7]$,

$(1.5, 5.097)$ $\left[x = \frac{9}{16}\right]$,

$(-1, 2\pi)$ $[x = -1]$

13. 18π

14. $\frac{3}{2}\pi a^2$

15. $2a^2$

16. $\frac{\pi}{8}$

17. 2

18. 4

19. $\frac{\pi}{2} - 1$

20. $\frac{2\pi}{3} - \frac{\sqrt{3}}{2}$

21. $5\pi - 8$

22. $6\pi - 16$

23. $a^2\pi$

24. $3\sqrt{3} - \pi$

25. $8 - \pi$

26. **(a)** $\frac{3\sqrt{3}}{2} + 2\pi$ **(b)** $3\sqrt{3} + \pi$

27. $12\pi - 9\sqrt{3}$

28. $\frac{3\sqrt{3}}{4}$

29. **(a)** $\frac{3}{2} - \frac{\pi}{4}$

(b) Yes. $x = \tan\theta\cos\theta \Rightarrow x = \sin\theta$

$y = \tan\theta\sin\theta \Rightarrow y = \frac{\sin^2\theta}{\cos\theta}$

$\lim_{\theta\to-\pi/2^+} x = -1,\ \lim_{\theta\to-\pi/2^+} y = \infty$

$\lim_{\theta\to\pi/2^-} x = 1,\ \lim_{\theta\to\pi/2^-} y = \infty$

30. The integral given is incorrect because $r = \cos\theta$ sweeps out the circle twice as θ goes from 0 to 2π. You can't use equation (2) from the text on the interval $[0, 2\pi]$ because $r = \cos\theta$ is negative for $\frac{\pi}{2} < \theta < \frac{3\pi}{2}$. The correct area is $\frac{5\pi}{4}$, which can be found by computing the areas of the cardioid $\frac{3\pi}{2}$ and the circle $\frac{\pi}{4}$ separately and subtracting.

31. $\frac{19}{3}$

32. $e^{\pi} - 1$

33. 8

34. $2a$

35. ≈ 6.887

36. ≈ 2.296

37. $\frac{\pi + 3}{8}$

38. 2π

39. $\pi\sqrt{2} \approx 4.443$

40. $\sqrt{5}\pi(e^{\pi/2} + 1) \approx 40.818$

41. $(4 - 2\sqrt{2})\pi \approx 3.681$

42. $4a^2\pi^2$

43. $\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2$

$= (f'(\theta)\cos\theta - f(\theta)\sin\theta)^2$

$\quad + (f'(\theta)\sin\theta + f(\theta)\cos\theta)^2$

$= (f'(\theta)\cos\theta)^2 + (f(\theta)\sin\theta)^2 + (f'(\theta)\sin\theta)^2$

$\quad + (f(\theta)\cos\theta)^2$

$= (f(\theta))^2(\sin^2\theta + \cos^2\theta)$

$\quad + (f'(\theta))^2(\cos^2\theta + \sin^2\theta)$

$= (f(\theta))^2 + (f'(\theta))^2 = r^2 + \left(\frac{dr}{d\theta}\right)^2$

44. **(a)** a

(b) a

(c) $\frac{2a}{\pi}$

45. If $g(\theta) = 2f(\theta)$, then $\sqrt{g(\theta)^2 + g'(\theta)^2} = 2\sqrt{f(\theta)^2 + f'(\theta)^2}$, so the length of g is 2 times the length of f.

46. If $g(\theta) = 2f(\theta)$, then

$2\pi\, g(\theta)\sin\theta\sqrt{g(\theta)^2 + g'(\theta)^2}$

$= 4[2\pi\, f(\theta)\sin\theta\sqrt{f(\theta)^2 + f'(\theta)^2}]$,

so the area generated by g is 4 times that of f.

47. **(a)** Let $r = 1.75 + \frac{0.06\theta}{2\pi}$.

(b) Since $\frac{dr}{d\theta} = \frac{b}{2\pi}$, this is just Equation 4 for the length of the curve.

(c) ≈ 741.420 cm, or ≈ 7.414 m

(d) $\left(r^2 + \left(\frac{b}{2\pi}\right)^2\right)^{1/2} = r\left(1 + \left(\frac{b}{2\pi r}\right)^2\right)^{1/2} \approx r$ since $\left(\frac{b}{2\pi r}\right)^2$ is a very small quantity squared.

(e) $L \approx 741.420$ cm (from part (c)), $L_a \approx 741.416$ cm

48. **(a)** Use the approximation, L_a, from 47(e). If the reel has made n complete turns, then the angle is $2\pi n$. So from the integral, $L_a = \pi b n^2 + 2\pi r_0 n$. Solving for n gives $n = \left(\frac{r_0}{b}\right)\left(\sqrt{\frac{bL}{r_0^2\pi} + 1} - 1\right)$.

(b) The take up reel slows down as time progresses.

(c) Since L is proportional to time, the formula in part (a) shows that n will grow roughly as the square root of time.

49. $\left(\frac{5a}{6}, 0\right)$

50. $\left(0, \frac{4a}{3\pi}\right)$

Chapter 10 Review Exercises (pp. 569–572)

1. **(a)** $\langle -17, 32\rangle$

(b) $\sqrt{1313}$

2. **(a)** $\langle -1, -1\rangle$

(b) $\sqrt{2}$

3. **(a)** $\langle 6, -8\rangle$

(b) 10

4. **(a)** $\langle 10, -25\rangle$

(b) $\sqrt{725} = 5\sqrt{29}$

5. $\left\langle -\frac{\sqrt{3}}{2}, -\frac{1}{2}\right\rangle$ [assuming counterclockwise]

6. $\left\langle \frac{\sqrt{3}}{2}, \frac{1}{2}\right\rangle$

7. $\left\langle \frac{8}{\sqrt{17}}, -\frac{2}{\sqrt{17}}\right\rangle$

8. $\langle -3, -4\rangle$

9. **(a)** $y = \frac{\sqrt{3}}{2}x + \frac{1}{4}$

(b) $\frac{1}{4}$

10. (a) $y = -3x + \frac{13}{4}$

(b) 6

11. (a) $\left(0, \frac{1}{2}\right)$ and $\left(0, -\frac{1}{2}\right)$

(b) Nowhere

12. (a) $(0, 2)$ and $(0, -2)$

(b) $(-2, 0)$ and $(2, 0)$

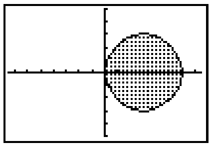
13. (a) $(0, 0)$

(b) Nowhere

14. (a) $(0, 9)$ and $(0, -9)$

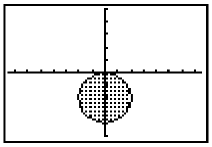
(b) $(-4, 0)$ and $(4, 0)$

15.



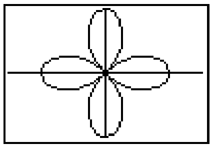
$[-7.5, 7.5]$ by $[-5, 5]$

16.



$[-7.5, 7.5]$ by $[-5, 5]$

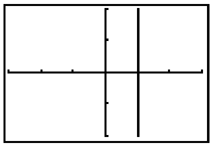
17. (a)



$[-1.5, 1.5]$ by $[-1, 1]$

(b) 2π

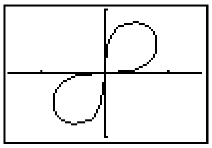
18. (a)



$[-3, 3]$ by $[-2, 2]$

(b) π

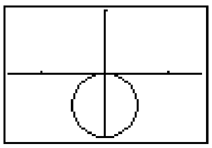
19. (a)



$[-1.5, 1.5]$ by $[-1, 1]$

(b) $\frac{\pi}{2}$

20. (a)



$[-1.5, 1.5]$ by $[-1, 1]$

(b) π

21. Tangent lines at $\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}$, and $\frac{7\pi}{4}$
Cartesian equations are $y = \pm x$.

22. Tangent lines at $\theta = \frac{\pi}{2}$ and $\frac{3\pi}{2}$
Cartesian equation is $x = 0$.

23. Horizontal: $y = 0, y \approx \pm 0.443, y \approx \pm 1.739$
Vertical: $x = 2, x \approx 0.067, x \approx -1.104$

24. Horizontal: $y = \frac{1}{2}, y = -4$
Vertical: $x = 0, x \approx \pm 2.598$

25. $y = \pm x + \sqrt{2}$ and $y = \pm x - \sqrt{2}$

26. $y = x - 1$ and $y = -x - 1$

27. $x = y$, a line

28. $x^2 + y^2 = 3x$,
a circle $\left(\text{center} = \left(\frac{3}{2}, 0\right), \text{radius} = \frac{3}{2}\right)$

29. $x^2 = 4y$, a parabola

30. $x - \sqrt{3}y = 4\sqrt{3}$ or $y = \frac{x}{\sqrt{3}} - 4$, a line

31. $r = -5 \sin \theta$

32. $r = 2 \sin \theta$

33. $r^2 \cos^2 \theta + 4r^2 \sin^2 \theta = 16$, or
$r^2 = \frac{16}{\cos^2 \theta + 4 \sin^2 \theta}$

34. $(r \cos \theta + 2)^2 + (r \sin \theta - 5)^2 = 16$

35. $\frac{\ln 2 + 24}{8} \approx 3.087$

36. $4\sqrt{3}$

37. 8

38. $\pi\sqrt{2}$

39. $\pi - 3$

40. $\pi\sqrt{2}$

41. ≈ 3.183

42. ≈ 12.363

43. $\frac{9\pi}{2}$

44. $\frac{\pi}{12}$

45. $\frac{\pi}{4} + 2$

46. 5π

47. $\frac{76\pi}{3}$

48. ≈ 10.110

49. $\pi(2 - \sqrt{2}) \approx 1.840$

50. $4\pi \approx 12.566$

51. (a) $\mathbf{v}(t) = (-4 \sin t)\mathbf{i} + (\sqrt{2} \cos t)\mathbf{j}$
$\mathbf{a}(t) = (-4 \cos t)\mathbf{i} + (-\sqrt{2} \sin t)\mathbf{j}$

(b) 3

(c) $\cos^{-1} \frac{7}{9} \approx 38.942°$

52. (a) $\mathbf{v}(t) = (\sqrt{3} \sec t \tan t)\mathbf{i} + (\sqrt{3} \sec^2 t)\mathbf{j}$
$\mathbf{a}(t) = \sqrt{3}\,(\sec t \tan^2 t + \sec^3 t)\mathbf{i}$
$+ (2\sqrt{3} \sec^2 t \tan t)\mathbf{j}$

(b) $\sqrt{3}$

(c) $90°$

53. 1

54. $\mathbf{a}(t) = (-2\, e^t \sin t)\mathbf{i} + (2\, e^t \cos t)\mathbf{j}$, and
$\mathbf{r}(t) \cdot \mathbf{a}(t) = 0$ for all t. The angle between $\mathbf{r}$ and $\mathbf{a}$ is always $90°$.

55. $6\mathbf{i}$

56. $3\mathbf{i} + (\ln 2)\mathbf{j}$

57. $\mathbf{r}(t) = (\cos t - 1)\mathbf{i} + (\sin t + 1)\mathbf{j}$

58. $\mathbf{r}(t) = (\tan^{-1} t + 1)\mathbf{i} + \sqrt{t^2 + 1}\mathbf{j}$

59. $\mathbf{r}(t) = \mathbf{i} + t^2\mathbf{j}$

60. $\mathbf{r}(t) = (-t^2 + 6t - 2)\mathbf{i} + (-t^2 + 2t + 2)\mathbf{j}$

61. (a) $\frac{\pi\sqrt{17}}{4} \approx 3.238$

(b) x-component: $\frac{3\pi^2}{16\sqrt{2}}$

y-component: $-\frac{5\pi^2}{16\sqrt{2}}$

(c) $\frac{x^2}{9} + \frac{y^2}{25} = 1$

62. (a) ≈ 25.874

(b) $\frac{1250\pi}{3}$

(c) ≈ 1040.728

63. (a) 1

(b) $e^3\sqrt{2}$

(c) $(e^3 - 1)\sqrt{2}$

64. (a) $\frac{104}{5}$

(b) $\frac{4144}{135}$

(c) $\frac{dy}{dx} = \frac{3}{5}\sqrt{x + 2}$

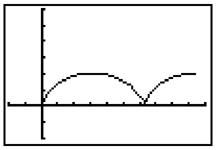
65. Speed $\approx$ 591.982 mph
Direction $\approx$ 8.179° north of east

66. Direction $\approx$ 2.073°
Length $\approx$ 411.891 lbs

67. It hits the ground $\approx$ 2.135 seconds later, approximately 66.421 horizontal feet from where it left the thrower's hand. Assuming it doesn't bounce or roll, it will still be there 3 seconds after it was thrown.

68. 57 feet

69. (a)



[−2, 10] by [−2, 6]

(b) $\mathbf{v}(0) = \langle 0, 0\rangle$ $\quad\mathbf{v}(1) = \langle 2\pi, 0\rangle$
$\mathbf{a}(0) = \langle 0, \pi^2\rangle$ $\quad\mathbf{a}(1) = \langle 0, -\pi^2\rangle$
$\mathbf{v}(2) = \langle 0, 0\rangle$ $\quad\mathbf{v}(3) = \langle 2\pi, 0\rangle$
$\mathbf{a}(2) = \langle 0, \pi^2\rangle$ $\quad\mathbf{a}(3) = \langle 0, -\pi^2\rangle$

(c) Topmost point: 2π ft/sec
center of wheel: π ft/sec
Reasons: Since the wheel rolls half a circumference, or π feet every second, the center of the wheel will move π feet every second. Since the rim of the wheel is turning at a rate of π ft/sec about the center, the velocity of the topmost point relative to the center is π ft/sec, giving it a total velocity of 2π ft/sec.

70. For 4325 yds: $v_0 \approx$ 644.360 ft/sec
For 4752 yds: $v_0 \approx$ 675.420 ft/sec

71. (a) $\approx$59.195 ft/sec

(b) $\approx$74.584 ft/sec

72. (a) $\approx$91.08 ft/sec

(b) 59.97 ft

73. We have $x = (v_0 t)\cos\alpha$ and $y + \frac{gt^2}{2} = (v_0 t)\sin\alpha$. Squaring and adding gives $x^2 + \left(y + \frac{gt^2}{2}\right)^2 = (v_0 t)^2(\cos^2\alpha + \sin^2\alpha) = v_0{}^2 t^2$.

74. (a) $\mathbf{r}(t) = (155\cos 18° - 11.7)t\mathbf{i} + (4 + 155\sin 18° t - 16t^2)\mathbf{j}$

$x(t) = (155\cos 18° - 11.7)t$

$y(t) = 4 + 155\sin 18° t - 16t^2$

(b) At $\approx$1.497 seconds, it reaches a maximum height of $\approx$39.847 ft.

(c) Range $\approx$ 417.307 ft
Flight time $\approx$ 3.075 sec

(d) At times $t \approx 0.534$ and $t \approx 2.460$ seconds, when it is $\approx$72.406 and $\approx$333.867 feet from home plate.

(e) Yes, the batter has hit a home run. When the ball is 380 feet from home plate (at $t \approx 2.800$ seconds), it is approximately 12.673 feet off the ground and therefore clears the fence by at least two feet.

75. (a)

$$\mathbf{r}(t) = \left[(155\cos 18° - 11.7)\left(\frac{1}{0.09}\right)(1 - e^{-0.09t})\right]\mathbf{i} + \left[4 + \left(\frac{155\sin 18°}{0.09}\right)(1 - e^{-0.09t}) + \frac{32}{0.09^2}(1 - 0.09t - e^{-0.09t})\right]\mathbf{j}$$

$$x(t) = (155\cos 18° - 11.7)\left(\frac{1}{0.09}\right)(1 - e^{-0.09t})$$

$$y(t) = 4 + \left(\frac{155\sin 18°}{0.09}\right)(1 - e^{-0.09t}) + \frac{32}{0.09^2}(1 - 0.09t - e^{-0.09t})$$

(b) At $\approx$1.404 seconds, it reaches a maximum height of $\approx$36.921 feet.

(c) Range $\approx$ 352.520 ft
Flight time $\approx$ 2.959 secs

(d) At times $t \approx 0.753$ and $t \approx 2.068$ seconds, when it is $\approx$98.799 and $\approx$256.138 feet from home plate

(e) No, the batter has not hit a home run. If the drag coefficient k is less than $\approx$0.011, the hit will be a home run.

76. (a) $\overrightarrow{BD} = \overrightarrow{AD} - \overrightarrow{AB}$

(b) $\overrightarrow{AP} = \overrightarrow{AB} + \frac{1}{2}\overrightarrow{BD} = \frac{1}{2}\overrightarrow{AB} + \frac{1}{2}\overrightarrow{AD}$

(c) $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{AD}$, so by part (b), $\overrightarrow{AP} = \frac{1}{2}\overrightarrow{AC}$.

77. The widths between the successive turns are constant and are given by $2\pi a$.

Cumulative Review Exercises (pp. 573–576)

1. 0 **2.** $\frac{3}{4}$

3. -1 **4.** -1

5. 3 **6.** 1

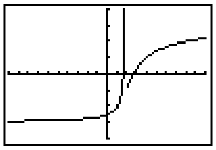
7. e^2 **8.** 3

9. **(a)** 1 **(b)** 1
(c) 1 **(d)** Yes
(e) No

10. All $x \le -2$ and $x \ge 2$

11. Horizontal: $y = 0$
vertical: $x = 0$, $x = \frac{1}{2}$

12. One possible answer:



$[-10, 10]$ by $[-4, 4]$

13. $\frac{1}{5}$ **14.** $-\frac{3}{(x-2)^2}$

15. $\frac{3\sin(\sqrt{1-3x})}{2\sqrt{1-3x}}$

16. $\sin x \sec^2 x + \tan x \cos x = \frac{(\sin x)(1+\cos^2 x)}{\cos^2 x}$

17. $\frac{2x}{x^2+1}$ **18.** $(2x-1)e^{x^2-x}$

19. $2x\tan^{-1} x + \frac{x^2}{1+x^2}$ **20.** $(x^{-3} - 3x^{-4})e^x$

21. $\frac{3\csc^2 x}{(1+\cos x)^4}(1 - \csc x \cot x - \cos x \csc x \cot x)$
$= \frac{3(1-2\cos x)}{(\sin^4 x)(1+\cos x)^3}$

22. $-\frac{1}{\sqrt{1-x^2}} + \frac{1}{1+x^2}$ **23.** $\frac{1 + xy\sin(xy)}{2xy - x^2\sin(xy)}$

24. $\frac{|x|}{2x\sqrt{|x|}}$ **25.** $\cot t = \frac{x-1}{1-y}$

26. $(\cos x)^{x-1}[\cos x \ln(\cos x) - x\sin x]$

27. $\sqrt{1+x^3}$

28. $2x\sin(x^2) - 2\sin(2x)$

29. $\frac{(2y+2)^2(\sec^3 x + \sec x \tan^2 x) - 2\sec^2 x \tan^2 x}{(2y+2)^3}$

30. -1

31. **(a)** $v = 3t^2 - 12t + 9$, $a = 6t - 12$
(b) $t = 1$ or $t = 3$
(c) Right: $0 \le t < 1$, $3 < t \le 5$
Left: $1 < t < 3$
(d) -3 m/sec

32. **(a)** $y = -2x + 1$ **(b)** $y = \frac{1}{2}x - \frac{3}{2}$

33. **(a)** $y = \left(\frac{3 - \pi\sqrt{3}}{6}\right)\left(x - \frac{\pi}{3}\right) + \frac{\pi}{6}$
$\approx -0.407x + 0.950$

(b) $y = \left(\frac{6}{\pi\sqrt{3} - 3}\right)\left(x - \frac{\pi}{3}\right) + \frac{\pi}{6}$
$\approx 2.458x - 2.050$

34. **(a)** $y = -\frac{\sqrt{3}}{2}x + 2\sqrt{3} \approx -0.866x + 3.464$

(b) $y = \frac{2}{\sqrt{3}}x + \frac{5}{2\sqrt{3}} \approx 1.155x + 1.443$

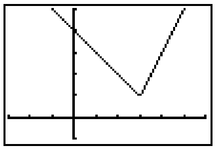
35. **(a)** $y = -\frac{\sqrt{3}}{2}x + 2\sqrt{3} \approx -0.866x + 3.464$

(b) $y = \frac{2}{\sqrt{3}}x + \frac{5}{2\sqrt{3}} \approx 1.155x + 1.443$

36. **(a)** $y = \sqrt{2}x - 1 \approx 1.414x - 1$

(b) $y = -\frac{1}{\sqrt{2}}x + 2 \approx -0.707x + 2$

37. $f(x) = \begin{cases} -x + 4, & x \le 3 \\ 2x - 5, & x > 3 \end{cases}$

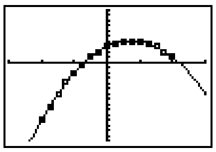


$[-3, 6]$ by $[-1, 5]$

38. **(a)** $x \ne 0, 2$ **(b)** $x = 0$
(c) $x = 2$
(d) Absolute maximum of 2 at $x = 0$
Absolute minimum of 0 at $x = -2, 2, 3$

39. According to the Mean Value Theorem the driver's speed at some time was $\frac{111}{1.5} = 74$ mph.

40. **(a)** Increasing in $[-0.7, 2]$, decreasing in $[-2, -0.7]$, and has a local minimum at $x \approx -0.7$.
(b) $y \approx -2x^2 + 3x + 3$



$[-3, 3]$ by $[-15, 10]$

(c) $f(x) = -\frac{2}{3}x^3 + \frac{3}{2}x^2 + 3x + 1$

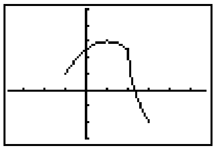
41. $f(x) = x^2 - 3x - \cos x - 1$

42. **(a)** $\left[-2, -\frac{2\sqrt{6}}{3}\right], \left[0, \frac{2\sqrt{6}}{3}\right]$

(b) $\left[-\frac{2\sqrt{6}}{3}, 0\right], \left[\frac{2\sqrt{6}}{3}, 2\right]$

(c) Approximately $(-1.042, 1.042)$
(d) Approximately $(-2, -1.042)$, $(1.042, 2)$
(e) Local max of approximately 3.079 at $x = -\frac{2\sqrt{6}}{3}$ and $x = \frac{2\sqrt{6}}{3}$; local min of 0 at $x = 0$
(f) $\approx(\pm 1.042, 1.853)$

43. **(a)** f has an absolute maximum at $x = 1$ and an absolute minimum at $x = 3$.
(b) f has a point of inflection at $x = 2$.
(c) One possible answer:



$[-3.7, 5.7]$ by $[-3,5]$

44. Dimensions: $4\sqrt{2}$ by $\sqrt{2}$, area: 8

45. $y = \sqrt{2}\left(x - \frac{\pi}{4}\right) + \sqrt{2} \approx 1.414x + 0.303$

46. 3%

47. **(a)** About 1.9 ft/sec
(b) About 0.15 rad/sec

48. **(a)** $\frac{9}{16\pi} \approx 0.179$ in./min
(b) $\frac{36}{25\pi} \approx 0.458$ in./min

49. **(a)** 165 in. **(b)** 165 in.

50. 2.5

51. 2π

52. $\ln 3 + \frac{26}{3} \approx 9.765$

53. 1

54. 7

55. $\frac{\ln 2}{1 + \ln 2} \approx 0.409$

56. $-2\mathbf{i} + (\ln 3)\mathbf{j}$

57. $-\cot(e^x + 1) - e^x + C$

58. $\frac{1}{2}\tan^{-1}\left(\frac{s}{2}\right) + C$

59. $\frac{1}{2\cos^2(x - 3)} + C$

60. $\frac{e^{-x}}{5}(2\sin 2x - \cos 2x) + C$

61. $\frac{8}{7}\ln|x - 6| - \frac{1}{7}\ln|x + 1| + C$
$= \frac{1}{7}\ln\frac{(x - 6)^8}{|x + 1|} + C$

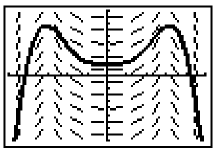
62. 8975 ft^3

63. $y = -\frac{1}{t + 1} - \frac{1}{2}e^{-2t} + \frac{7}{2}$

64. $y = -\frac{1}{4}\sin 2\theta + \cos\theta + \frac{1}{2}\theta - \frac{\pi}{4}$

65. $\int x^2 \sin x\, dx = (2 - x^2)\cos x + 2x\sin x + C$

The graph of the slope field of the differential equation $\frac{dy}{dx} = x^2 \sin x$ and the antiderivative $y = (2 - x^2)\cos x + 2x\sin x$ is shown below.



$[-5, 5]$ by $[-10, 10]$

66. $e^x(x - 1) + C$

67. **(a)** $y = 4268e^{kt}$, $k = \frac{\ln(5/3)}{3} \approx 0.170$
(b) About 4268

68. **(a)** About 6.13°C
(b) About 2 hours and 27 minutes after it was 65°C above room temperature, or about 2 hours and 12 minutes after it was 50°C above room temperature.

69. $y = \frac{500}{1 + Ce^{-0.08x}}$

70. $y = Ce^{x^2/2 + 3x} + 4$

71.

x	y
0	0
0.1	0.1
0.2	0.2095
0.3	0.3285
0.4	0.4568
0.5	0.5946
0.6	0.7418
0.7	0.8986
0.8	1.0649
0.9	1.2411
1.0	1.4273

72. 4

73. $\frac{64}{3}$

74. ≈ 16.039

75. $\frac{27\pi}{2} \approx 42.412$

76. $\frac{\pi}{14} \approx 0.224$

77. $\frac{128\pi}{3} \approx 134.041$

78. ≈ 0.763

79. ≈ 2.556

80. 4

81. ≈ 6.110

82. ≈ 8.423

83. ≈ 3.470

84. ≈ 32.683

85. $\frac{9\pi}{280} \approx 0.101$

86. $8\pi - 2\pi^2 \approx 5.394$

87. **(a)** 1.2 m **(b)** 180 J

88. **(a)** 70,686 ft-lb **(b)** 4 min, 17 sec

89. ≈ 12.166 ft^3

90. $f(x) = \ln x$ grows slower than $g(x) = \sqrt{x}$.

91. Converges

92. Diverges

93. Converges

94. Converges

95. Diverges

96. Converges

97. $1 - 2x + 4x^2 - 8x^3 + \cdots + (-1)^n 2^n x^n + \cdots$;
$-\frac{1}{2} < x < \frac{1}{2}$

98. **(a)** $x - \frac{x^5}{5 \cdot 2!} + \frac{x^9}{9 \cdot 4!} - \frac{x^{13}}{13 \cdot 6!} + \cdots + (-1)^n \frac{x^{4n+1}}{(4n + 1) \cdot (2n)!} + \cdots$
(b) $-\infty < x < \infty$; Since the cosine series converges for all real numbers, so does the integrated series, by the term-by-term integration theorem (Section 9.1, Theorem 2).

99. $\ln 2 + x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots + (-1)^{n-1}\frac{x^n}{n} + \cdots; -1 < x \le 1.$

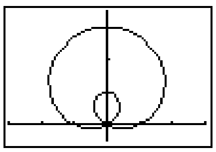
100. $(x - 2\pi) - \frac{(x-2\pi)^3}{3!} + \frac{(x-2\pi)^5}{5!} - \cdots + (-1)^n\frac{(x-2\pi)^{2n+1}}{(2n+1)!} + \cdots$

101. $P_6(x) = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - \frac{x^5}{5!} + \frac{x^6}{6!}$
By the Alternating Series Estimation Theorem, $|\text{error}| \le \left|\frac{x^7}{7!}\right| \le \frac{1}{7!} < 0.001.$

102. $1 + \frac{1}{3}x - \frac{2}{2! \cdot 3^2}x^2 + \frac{2 \cdot 5}{3! \cdot 3^3}x^3 - \frac{2 \cdot 5 \cdot 8}{4! \cdot 3^4}x^4 + \cdots + (-1)^{n-1}\frac{2 \cdot 5 \cdot \cdots \cdot (3n-4)}{n! \cdot 3^n}x^n + \cdots$

$R = 1$

103. Converges
104. Diverges
105. Converges
106. Converges
107. **(a)** $R = 1; -3 < x \le -1$
(b) $-3 < x < -1$
(c) At $x = -1$
108. **(a)** $R = 1; -1 \le x \le 1$
(b) $-1 \le x \le 1$
(c) Nowhere
109. $\left\langle \frac{2}{\sqrt{13}}, -\frac{3}{\sqrt{13}} \right\rangle$
110. $\left\langle \frac{1}{2}, \frac{\sqrt{3}}{2} \right\rangle$
111. Tangent: $\left\langle -\frac{4}{5}, -\frac{3}{5} \right\rangle, \left\langle \frac{4}{5}, \frac{3}{5} \right\rangle$
normal: $\left\langle \frac{3}{5}, -\frac{4}{5} \right\rangle, \left\langle -\frac{3}{5}, \frac{4}{5} \right\rangle$
112. **(a)** $\mathbf{v}(t) = (-\cos t)\mathbf{i} + (1 + \sin t)\mathbf{j}$
$\mathbf{a}(t) = (\sin t)\mathbf{i} + (\cos t)\mathbf{j}$
(b) 4
113. Yes. When $x = 130$ ft, $t = \frac{130}{100}\cos 45° \approx 1.838$ sec and $y \approx 75.9$ ft, high enough to easily clear the 35-ft tree.
114. $x - y = 2$; line with slope 1 and y-intercept -2
115.



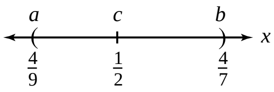
$[-3, 3]$ by $[-0.5, 3.5]$

2π

116. Horizontal tangent lines: $y = 0, y = \pm\frac{3\sqrt{3}}{4}$
vertical tangent lines: $x = -2, x = \frac{1}{4}$

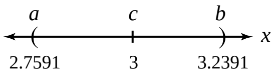
Appendix A3 (pp. 584–592)

1.



$\delta = \frac{1}{18}$

2.

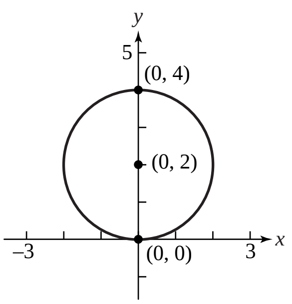


$\delta = 0.2391$

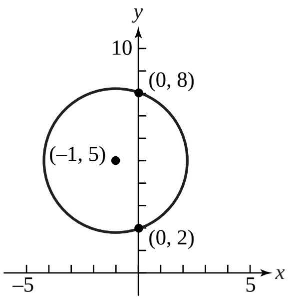
3. $\delta = 0.39$
4. $\delta = 0.36$
5. $(-2.01, -1.99); \delta = 0.01$
6. $(-0.19, 0.21); \delta = 0.19$
7. $(3, 15); \delta = 5$
8. $(-\sqrt{4.5}, -\sqrt{3.5}); \delta = \sqrt{4.5} - 2 \approx 0.121$
9. **(a)** -4
(b) $\delta = 0.05$
10. **(a)** 2
(b) $\delta = \frac{1}{12}$
11. **(a)** $\sin 1 \approx 0.841$
(b) $\delta = 0.018$
12. **(a)** $\frac{1}{3}$
(b) $\delta = 0.155$
13. $\delta = \min\{1 - \sqrt{1 - \epsilon}, \sqrt{1 + \epsilon} - 1\}$
14. $\delta = \min\left\{\sqrt{3} - \sqrt{\frac{3}{1 + 3\epsilon}}, \sqrt{\frac{3}{1 - 3\epsilon}} - \sqrt{3}\right\}$
15. **(a)** $I = (5, 5 + \epsilon^2)$
(b) $\lim_{x \to 5^+} \sqrt{x - 5} = 0$
16. **(a)** $I = (4 - \epsilon^2, 4)$
(b) $\lim_{x \to 4^-} \sqrt{4 - x} = 0$

Appendix A5.1 (pp. 593–606)

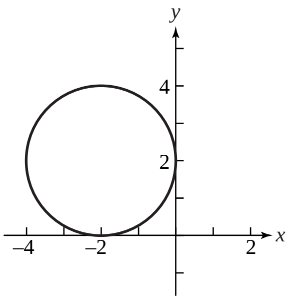
1. $x^2 + (y - 2)^2 = 4$



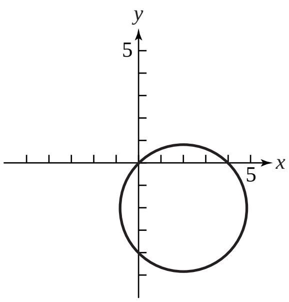
2. $(x + 1)^2 + (y - 5)^2 = 10$



3. Center $= (-2, 2)$; radius $= 2$



4. Center $= (2, -2)$; radius $= 2\sqrt{2}$



5. The circle with center at (1, 0) and radius 2 plus its interior.

6. The region exterior to the unit circle and interior to the circle with center at (0, 0) and radius 2.

7. $y^2 = 8x$; focus is (2, 0); directrix is $x = -2$

8. $y^2 = -4x$; focus is $(-1, 0)$; directrix is $x = 1$

9. $x^2 = -6y$; focus is $\left(0, -\frac{3}{2}\right)$; directrix is $y = \frac{3}{2}$

10. $x^2 = 2y$; focus is $\left(0, \frac{1}{2}\right)$; directrix is $y = -\frac{1}{2}$

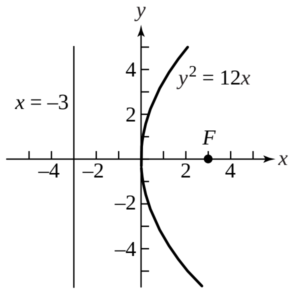
11. $\frac{x^2}{4} - \frac{y^2}{9} = 1$; foci are $(\pm\sqrt{13}, 0)$; vertices are $(\pm 2, 0)$; asymptotes are $y = \pm\frac{3}{2}x$

12. $\frac{x^2}{4} + \frac{y^2}{9} = 1$; foci are $(0, \pm\sqrt{5})$; vertices are $(0, \pm 3)$

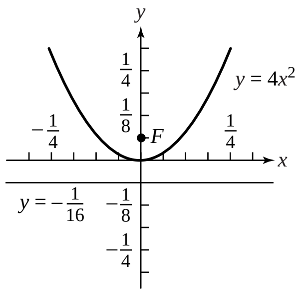
13. $\frac{x^2}{2} + y^2 = 1$; foci are $(\pm 1, 0)$; vertices are $(\pm\sqrt{2}, 0)$

14. $\frac{y^2}{4} - x^2 = 1$; foci are $(0, \pm\sqrt{5})$; vertices are $(0, \pm 2)$; asymptotes are $y = \pm 2x$

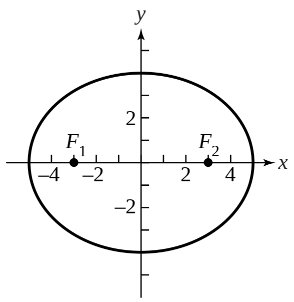
15. Focus is (3, 0); directrix is $x = -3$



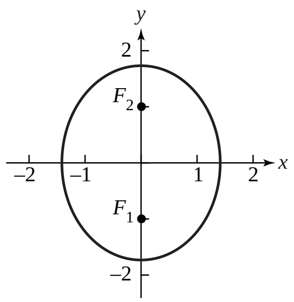
16. Focus is $\left(0, \frac{1}{16}\right)$; directrix is $y = -\frac{1}{16}$



17. $\frac{x^2}{25} + \frac{y^2}{16} = 1$; foci are $(\pm 3, 0)$



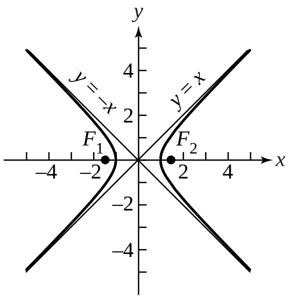
18. $\frac{x^2}{2} + \frac{y^2}{3} = 1$; foci are $(0, \pm 1)$



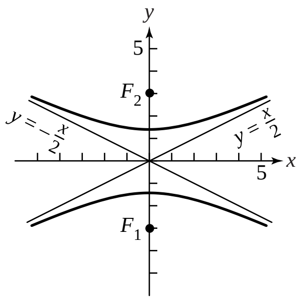
19. $\frac{x^2}{4} + \frac{y^2}{2} = 1$

20. $\frac{x^2}{9} + \frac{y^2}{25} = 1$

21. $x^2 - y^2 = 1$; asymptotes are $y = \pm x$;
foci are $(\pm\sqrt{2}, 0)$



22. $\dfrac{y^2}{2} - \dfrac{x^2}{8} = 1$; asymptotes are $y = \pm\dfrac{x}{2}$;
foci are $(0, \pm\sqrt{10})$

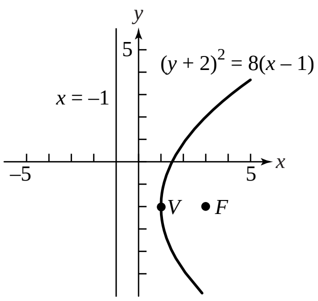


23. $y^2 - x^2 = 1$

24. $\dfrac{x^2}{9} - \dfrac{y^2}{16} = 1$

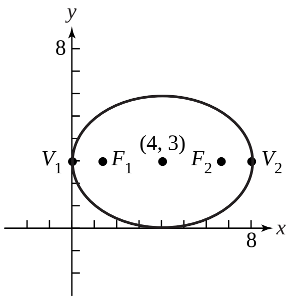
25. (a) Vertex is $(1, -2)$; focus is $(3, -2)$;
directrix is $x = -1$

(b)



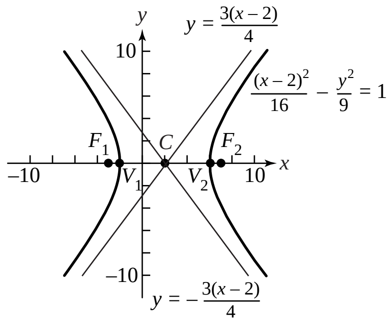
26. (a) Foci are $(4 \pm \sqrt{7}, 3)$; vertices are $(0, 3)$ and $(8, 3)$; center is $(4, 3)$

(b)



27. (a) Center is $(2, 0)$; foci are $(-3, 0)$ and $(7, 0)$;
asymptotes are $y = \pm\dfrac{3(x-2)}{4}$;
vertices are $(-2, 0)$ and $(6, 0)$

(b)



28. $(y + 3)^2 = 4(x + 2)$; vertex is $(-2, -3)$;
focus is $(-1, -3)$; directrix is $x = -3$

29. $\dfrac{(x+2)^2}{6} + \dfrac{(y+1)^2}{9} = 1$; vertices are $(-2, 2)$ and $(-2, -4)$; foci are $(-2, -1 \pm \sqrt{3})$;
center is $(-2, -1)$

30. $\dfrac{(x-2)^2}{4} - \dfrac{(y-2)^2}{5} = 1$; vertices are $(4, 2)$ and $(0, 2)$; foci are $(5, 2)$ and $(-1, 2)$;
center is $(2, 2)$;
asymptotes are $y = \pm\dfrac{\sqrt{5}}{2}(x - 2) + 2$

31. $(y - 1)^2 - (x + 1)^2 = 1$; vertices are $(-1, 2)$ and $(-1, 0)$; foci are $(-1, 1 \pm \sqrt{2})$;
center is $(-1, 1)$;
asymptotes are $y = \pm(x + 1) + 1$

32. Circle; center is $(-2, 0)$; radius is 4

33. Circle; center is $(7, -3)$, radius is 1

34. Parabola; focus is $(-1, 0)$; vertex is $(-1, 1)$

35. Ellipse; center is $(-2, 0)$;
foci are $(-4, 0)$ and $(0, 0)$;
vertices are $(-2 \pm \sqrt{5}, 0)$

36. Hyperbola; center is $(1, 2)$; foci are $(1 \pm \sqrt{2}, 2)$;
vertices are $(2, 2)$ and $(0, 2)$;
asymptotes are $y = \pm(x - 1) + 2$

37. Volume of the parabolic solid is $V_1 = \dfrac{\pi hb^2}{8}$;
volume of the cone is $V_2 = \dfrac{\pi hb^2}{12}$; $\dfrac{V_1}{V_2} = \dfrac{3}{2}$

38. (a) Volume of the solid formed by revolving A about the y-axis is $V_1 = \dfrac{\pi x^2\sqrt{kx}}{5}$; volume of the solid formed by revolving B about the y-axis is $V_3 = V_2 - V_1 = \dfrac{4\pi x^2\sqrt{kx}}{5}$; $\dfrac{V_2}{V_1} = \dfrac{4}{1} = 4$

(b) 1:1 $\left(\text{both equal to } \dfrac{\pi k x^2}{2}.\right)$

39. The slopes of the two tangents to $y^2 = 4px$ from the point $(-p, a)$ are $m_1 = \dfrac{2p}{a + \sqrt{a^2 + 4p^2}}$ and $m_2 = \dfrac{2p}{a - \sqrt{a^2 + 4p^2}}$, and $m_1 m_2 = -1$.

40. $2\sqrt{2}$ by $\sqrt{2}$; area $= 4$

41. **(a)** 24π
(b) 16π

42. 24π

43. 24π

44. $y = \dfrac{wx^2}{2H}$

45. $\dfrac{dr_A}{dt} = \dfrac{dr_B}{dt} \Rightarrow \dfrac{d}{dt}(r_A - r_B) = 0$
$\Rightarrow r_A - r_B =$ a constant

46. PF will always equal PB because the string has constant length $AB = FP + PA = AP + PB$.

Appendix A5.2 (pp. 606–611)

1. $e = \dfrac{3}{5}$; foci are $(\pm 3, 0)$; directrices are $x = \pm\dfrac{25}{3}$

2. $e = \dfrac{1}{\sqrt{2}}$; foci are $(0, \pm 1)$; directrices are $y = \pm 2$

3. $e = \dfrac{1}{\sqrt{3}}$; foci are $(0, \pm 1)$; directrices are $y = \pm 3$

4. $e = \dfrac{1}{\sqrt{3}}$; foci are $(\pm\sqrt{3}, 0)$; directrices are $x = \pm 3\sqrt{3}$

5. $\dfrac{x^2}{27} + \dfrac{y^2}{36} = 1$

6. $\dfrac{x^2}{1600} + \dfrac{y^2}{1536} = 1$

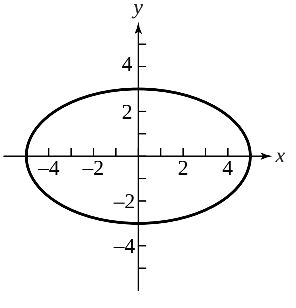
7. $\dfrac{x^2}{100} + \dfrac{y^2}{94.24} = 1$

8. $\dfrac{x^2}{4851} + \dfrac{y^2}{4900} = 1$

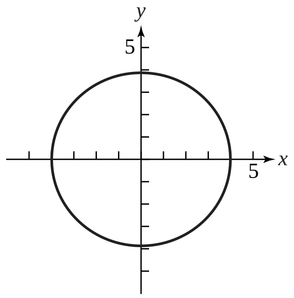
9. $e = \dfrac{\sqrt{5}}{3}$; $\dfrac{x^2}{9} + \dfrac{y^2}{4} = 1$

10. $e = \dfrac{1}{2}$; $\dfrac{x^2}{64} + \dfrac{y^2}{48} = 1$

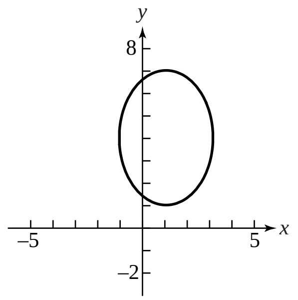
11. Take $c = 4$ and $a = 5$, then $e = \dfrac{c}{a} = \dfrac{4}{5}$ and $b = 3$. The equation is $\dfrac{x^2}{25} + \dfrac{y^2}{9} = 1$.



12. Take $c = 1$ and $a = 4$, then $e = \dfrac{c}{a} = \dfrac{1}{4}$ and $b = \sqrt{15}$. Therefore, $\dfrac{x^2}{16} + \dfrac{y^2}{15} = 1$ is a model of Pluto's orbit.



13. $\dfrac{(x-1)^2}{4} + \dfrac{(y-4)^2}{9} = 1$; foci are $(1, 4 \pm \sqrt{5})$; $e = \dfrac{\sqrt{5}}{3}$; directrices are $y = 4 \pm \dfrac{9\sqrt{5}}{5}$.



14. $\dfrac{x^2}{36} + \dfrac{y^2}{20} = 1$

15. $e = \dfrac{5}{4}$; foci are $(\pm 5, 0)$; directrices are $x = \pm\dfrac{16}{5}$

16. $e = \sqrt{2}$; foci are $(0, \pm 4)$; directrices are $y = \pm 2$

17. $e = \sqrt{5}$; foci are $(\pm\sqrt{10}, 0)$; directrices are $x = \pm\dfrac{\sqrt{10}}{5}$

18. $e = \sqrt{5}$; foci are $(0, \pm\sqrt{10})$; directrices are $y = \pm\dfrac{2}{\sqrt{10}}$

19. $y^2 - \dfrac{x^2}{8} = 1$

20. $x^2 - \dfrac{y^2}{8} = 1$

21. $e = \sqrt{2}$; $\dfrac{x^2}{8} - \dfrac{y^2}{8} = 1$

22. $e = 2$; $x^2 - \dfrac{y^2}{3} = 1$

23. $\dfrac{(y-6)^2}{36} - \dfrac{(x-1)^2}{45} = 1$

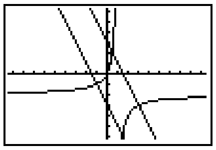
24. $\dfrac{x^2}{a^2} - \dfrac{y^2}{a^2(e^2 - 1)} = 1$

25. $a = 0$, $b = -4$, $c = 0$; $e = \dfrac{\sqrt{3}}{2}$

Appendix A5.3 (pp. 612–618)

1. Hyperbola **2.** Parabola
3. Ellipse **4.** Ellipse
5. Parabola **6.** Hyperbola
7. Parabola **8.** Ellipse (circle)
9. Hyperbola **10.** Parabola
11. Hyperbola **12.** Hyperbola
13. Ellipse **14.** Hyperbola
15. Ellipse **16.** Parabola
17. $(x')^2 - (y')^2 = 4$; hyperbola
18. $3(x')^2 + (y')^2 = 2$; ellipse
19. $4(x')^2 + 16y' = 0$; parabola
20. $(x')^2 + 5(y')^2 = 2$; ellipse
21. $(y')^2 = 1$; parallel horizontal lines
22. $4(y')^2 = 1$; parallel horizontal lines
23. $(x')^2 + 4y' = 0$; parabola
24. $(x')^2 - (y')^2 - 2\sqrt{2}x' + 2 = 0$; hyperbola
25. $4(x')^2 + 2(y')^2 = 19$; ellipse
26. $5(x')^2 - 3(y')^2 = 7$; hyperbola

27. $\sin\alpha = \frac{1}{\sqrt{5}}$, $\cos\alpha = \frac{2}{\sqrt{5}}$; or $\sin\alpha = -\frac{2}{\sqrt{5}}$, $\cos\alpha = \frac{1}{\sqrt{5}}$
28. $\sin\alpha = \frac{2}{\sqrt{5}}$, $\cos\alpha = \frac{1}{\sqrt{5}}$; or $\sin\alpha = -\frac{1}{\sqrt{5}}$, $\cos\alpha = \frac{2}{\sqrt{5}}$
29. $\sin\alpha \approx 0.23$, $\cos\alpha \approx 0.97$; $A' \approx 0.88$, $B' \approx 0.00$, $C' \approx 3.12$, $D' \approx 0.74$, $E' \approx -1.20$, $F' = -3$; $0.88(x')^2 + 3.12(y')^2 + 0.74x' - 1.20y' - 3 = 0$; ellipse
30. $\sin\alpha \approx 0.10$, $\cos\alpha \approx 0.995$; $A' \approx 2.05$, $B' \approx 0.00$, $C' \approx -3.05$, $D' \approx 2.99$, $E' \approx -0.30$, $F' = -7$; $2.05(x')^2 - 3.05(y')^2 + 2.99x' - 0.30y' - 7 = 0$; hyperbola
31. $\sin\alpha \approx 0.45$, $\cos\alpha \approx 0.89$; $A' \approx 0.00$, $B' \approx 0.00$, $C' \approx 5.00$, $D' = 0$, $E' = 0$, $F' = -5$; $5.00(y')^2 - 5 = 0$ or $y' = \pm 1.00$; parallel lines
32. $\sin\alpha \approx 0.32$, $\cos\alpha \approx 0.95$; $A' \approx 0.00$, $B' \approx 0.00$, $C' \approx 20.00$, $D' = 0$, $E' = 0$, $F' = -49$; $20.00(y')^2 - 49 = 0$; parallel lines
33. $\sin\alpha \approx 0.63$, $\cos\alpha \approx 0.77$; $A' \approx 5.05$, $B' \approx 0.00$, $C' \approx -0.05$, $D' \approx -5.07$, $E' \approx -6.19$, $F' = -1$; $5.05(x')^2 - 0.05(y')^2 - 5.07x' - 6.19y' - 1 = 0$; hyperbola
34. $\sin\alpha \approx -0.38$, $\cos\alpha \approx 0.92$; $A' \approx 0.55$, $B' \approx 0.00$, $C' \approx 10.45$, $D' \approx 18.48$, $E' \approx -7.65$, $F' = -86$; $0.55(x')^2 + 10.45(y')^2 + 18.48x' - 7.65y' - 86 = 0$; ellipse
35. **(a)** $(x')^2 - (y')^2 = 2$
(b) $(x')^2 - (y')^2 = 2a$
36. Yes, the graph is a hyperbola: with $AC < 0$ we have $-4AC > 0$ and $B^2 - 4AC > 0$.
37. Yes, $x^2 + 4xy + 5y^2 - 1 = 0$
38. $B' = B\cos 2\alpha = 0$
39. **(a)** $\frac{(x')^2}{b^2} + \frac{(y')^2}{a^2} = 1$
(b) $\frac{(y')^2}{a^2} - \frac{(x')^2}{b^2} = 1$
(c) $(x')^2 + (y')^2 = a^2$
(d) $y' = -\frac{1}{m}x'$
(e) $y' = -\frac{1}{m}x' + \frac{b}{m}$
40. **(a)** $\frac{(x')^2}{a^2} + \frac{(y')^2}{b^2} = 1$
(b) $\frac{(x')^2}{a^2} - \frac{(y')^2}{b^2} = 1$
(c) $(x')^2 + (y')^2 = a^2$
(d) $y' = mx'$
(e) $y' = mx' + b$
41. **(a)** Hyperbola
(b) $y = -\frac{2x}{x-1}$
(c) At $(3, -3)$: $y = -2x + 3$;
At $(-1, -1)$: $y = -2x - 3$



$[-9.4, 9.4]$ by $[-6.2, 6.2]$

42. **(a)** False. If $A = C = 1$, $B = 2$, the graph is a parabola.
(b) False. If $A = C = 1$, $B = 2$, the graph is a parabola.
(c) True. $B^2 - 4AC > 0$, the graph is a hyperbola.
43. **(a)** Parabola
(b) The equation can be written in the form $(x + 2y + 3)^2 = 0$.
44. **(a)** Parabola
(b) The equation can be written in the form $(3x + y - 2)^2 = 0$.

Appendix A6 (pp. 618–627)

1. $\cosh x = \frac{5}{4}$; $\tanh x = -\frac{3}{5}$; $\coth x = -\frac{5}{3}$; $\operatorname{sech} x = \frac{4}{5}$; $\operatorname{csch} x = -\frac{4}{3}$
2. $\cosh x = \frac{5}{3}$; $\tanh x = \frac{4}{5}$; $\coth x = \frac{5}{4}$; $\operatorname{sech} x = \frac{3}{5}$; $\operatorname{csch} x = \frac{3}{4}$

3. $\sinh x = \frac{8}{15}$; $\tanh x = \frac{8}{17}$; $\coth x\ \frac{17}{8}$; $\operatorname{sech} x = \frac{15}{17}$; $\operatorname{csch} x = \frac{15}{8}$

4. $\sinh x = \frac{12}{5}$; $\tanh x = \frac{12}{13}$; $\coth x\ \frac{13}{12}$; $\operatorname{sech} x = \frac{5}{13}$; $\operatorname{csch} x = \frac{5}{12}$

5. $x + \frac{1}{x}$

6. $\frac{x^4 - 1}{2x^2}$

7. e^{5x}

8. e^{-3x}

9. e^{4x}

10. 0

13. $\frac{dy}{dx} = 2\cosh\frac{x}{3}$

14. $\frac{dy}{dx} = \cosh(2x + 1)$

15. $\frac{dy}{dt} = \operatorname{sech}^2\sqrt{t} + \frac{\tanh\sqrt{t}}{\sqrt{t}}$

16. $\frac{dy}{dt} = -\operatorname{sech}^2\frac{1}{t} + 2t\tanh\frac{1}{t}$

17. $\frac{dy}{dz} = \coth z$

18. $\frac{dy}{dz} = \tanh z$

19. $\frac{dy}{d\theta} = (\operatorname{sech}\theta\tanh\theta)(\ln\operatorname{sech}\theta)$

20. $\frac{dy}{d\theta} = (\operatorname{csch}\theta\coth\theta)(\ln\operatorname{csch}\theta)$

21. $\frac{dy}{dx} = \tanh^3 x$

22. $\frac{dy}{dx} = \coth^3 x$

23. $y = 2x$; $\frac{dy}{dx} = 2$

24. $y = 4x$; $\frac{dy}{dx} = 4$

25. $\frac{dy}{dx} = \frac{1}{2\sqrt{x(1 + x)}}$

26. $\frac{dy}{dx} = \frac{1}{\sqrt{4x^2 + 7x + 3}}$

27. $\frac{dy}{d\theta} = \frac{1}{1 + \theta} - \tanh^{-1}\theta$

28. $\frac{dy}{d\theta} = (2\theta + 2)\tanh^{-1}(\theta + 1) - 1$

29. $\frac{dy}{dt} = \frac{1}{2\sqrt{t}} - \coth^{-1}\sqrt{t}$

30. $\frac{dy}{dt} = 1 - 2t\coth^{-1} t$

31. $\frac{dy}{dx} = -\operatorname{sech}^{-1} x$

32. $\frac{dy}{dx} = -\frac{x}{\sqrt{1 - x^2}}\operatorname{sech}^{-1} x$

33. $\frac{dy}{d\theta} = \frac{\ln 2}{\sqrt{1 + \left(\frac{1}{2}\right)^{2\theta}}}$

34. $\frac{dy}{d\theta} = -\frac{\ln 2}{\sqrt{1 + 2^{2\theta}}}$

35. $\frac{dy}{dx} = |\sec x|$

36. $\frac{dy}{dx} = \sec x,\ 0 < x < \frac{\pi}{2}$

37. (a) $\frac{d}{dx}(\tan^{-1}(\sinh x) + C) = \operatorname{sech} x$

(b) $\frac{d}{dx}(\sin^{-1}(\tanh x) + C) = \operatorname{sech} x$

38. $\frac{d}{dx}\left(\frac{x^2}{2}\operatorname{sech}^{-1} x - \frac{1}{2}\sqrt{1 - x^2} + C\right) = x\operatorname{sech}^{-1} x$

39. $\frac{d}{dx}\left(\frac{x^2 - 1}{2}\coth^{-1} x + \frac{x}{2} + C\right) = x\coth^{-1} x$

40. $\frac{d}{dx}\left(x\tanh^{-1}x + \frac{1}{2}\ln(1 - x^2) + C\right) = \tanh^{-1} x$

41. $\frac{\cosh 2x}{2} + C$

42. $5\cosh\frac{x}{5} + C$

43. $12\sinh\left(\frac{x}{2} - \ln 3\right) + C$

44. $\frac{4}{3}\sinh(3x - \ln 2) + C$

45. $7\ln\left|\cosh\frac{x}{7}\right| + C$

46. $\sqrt{3}\ln\left|\sinh\frac{\theta}{\sqrt{3}}\right| + C$

47. $\tanh\left(x - \frac{1}{2}\right) + C$

48. $\coth(5 - x) + C$

49. $-2\operatorname{sech}\sqrt{t} + C$

50. $-\operatorname{csch}(\ln t) + C$

51. $\ln\left(\frac{5}{2}\right) \approx 0.916$

52. $\frac{1}{2}\ln\left(\frac{17}{8}\right) \approx 0.377$

53. $\frac{3}{32} + \ln 2 \approx 0.787$

54. $\ln 4 - \frac{3}{4} \approx 0.636$

55. $e - e^{-1} \approx 2.350$

56. $e - e^{-1} - 2 \approx 1.086$

57. $\frac{3}{4}$

58. $8(e^2 - e^{-2} - e + e^{-1}) \approx 39.227$

59. $\frac{3}{8} + \ln\sqrt{2} \approx 0.722$

60. $\frac{99}{10} - 2\ln 10 \approx 5.295$

61. 2π

62. π

63. $\left(2\ln\frac{199}{100} - \frac{99}{100}\right)\pi \approx 1.214$

64. **(a)** $\frac{6}{5}$

(b) $\frac{\sinh ab}{a}$

65. **(a)** If $g(x) = \frac{f(x) + f(-x)}{2}$, then $g(-x) = \frac{f(-x) + f(x)}{2} = g(x)$. Thus, $\frac{f(x) + f(-x)}{2}$ is even. If $h(x) = \frac{f(x) - f(-x)}{2}$, then $h(-x) = \frac{f(-x) - f(x)}{2} = -\frac{f(x) - f(-x)}{2} = -h(x)$. Thus $\frac{f(x) - f(-x)}{2}$ is odd.

(b) Even part: $\frac{e^x + e^{-x}}{2} = \cosh x$

Odd part: $\frac{e^x - e^{-x}}{2} = \sinh x$

66. **(a)** If f is even, then

$$\frac{f(x) + f(-x)}{2} + \frac{f(x) - f(-x)}{2} = \frac{2f(x)}{2} + \frac{f(x) - f(x)}{2} = f(x) + 0$$

(b) If f is odd, then

$$\frac{f(x) + f(-x)}{2} + \frac{f(x) - f(-x)}{2} = \frac{f(x) - f(x)}{2} + \frac{f(x) + f(x)}{2} = 0 + f(x)$$

68. **(a)** $a = \frac{d^2s}{dt^2} = -k^2s$ is directed toward the origin.

(b) $a = \frac{d^2s}{dt^2} = k^2s$ is directed away from the origin.

69. $y = \operatorname{sech}^{-1}(x) - \sqrt{1 - x^2}$

70. $16\pi \ln 6 + \frac{455\pi}{9} \approx 248.889$

72. **(c)** $A(0) = 0$, $C = 0$, $u = 2A(u)$