

## Calculus 1 Worksheet 92 Implicit Differentiation

1)

Consider the curve given by  $x^2 + 4y^2 = 7 + 3xy$ .

- (a) Show that  $\frac{dy}{dx} = \frac{3y - 2x}{8y - 3x}$ .
- (b) Show that there is a point  $P$  with  $x$ -coordinate 3 at which the line tangent to the curve at  $P$  is horizontal. Find the  $y$ -coordinate of  $P$ .
- (c) Find the value of  $\frac{d^2y}{dx^2}$  at the point  $P$  found in part (b). Does the curve have a local maximum, a local minimum, or neither at the point  $P$ ? Justify your answer.

2)

Consider the curve given by  $y^2 = 2 + xy$ .

- (a) Show that  $\frac{dy}{dx} = \frac{y}{2y - x}$ .
- (b) Find all points  $(x, y)$  on the curve where the line tangent to the curve has slope  $\frac{1}{2}$ .
- (c) Show that there are no points  $(x, y)$  on the curve where the line tangent to the curve is horizontal.
- (d) Let  $x$  and  $y$  be functions of time  $t$  that are related by the equation  $y^2 = 2 + xy$ . At time  $t = 5$ , the value of  $y$  is 3 and  $\frac{dy}{dt} = 6$ . Find the value of  $\frac{dx}{dt}$  at time  $t = 5$ .

3)

Consider the closed curve in the  $xy$ -plane given by

$$x^2 + 2x + y^4 + 4y = 5.$$

- (a) Show that  $\frac{dy}{dx} = \frac{-(x + 1)}{2(y^3 + 1)}$ .
- (b) Write an equation for the line tangent to the curve at the point  $(-2, 1)$ .
- (c) Find the coordinates of the two points on the curve where the line tangent to the curve is vertical.
- (d) Is it possible for this curve to have a horizontal tangent at points where it intersects the  $x$ -axis? Explain your reasoning.