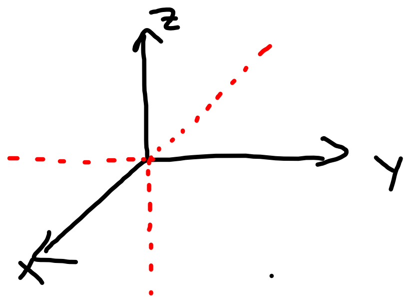


(x, y, z)

xy trace set $z=0$
xz trace set $y=0$
yz trace set $x=0$

distance formula in 3D:

$$\boxed{d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}}$$

Ex: $P_1=(0,0,0)$ and $P_2=(1,-2,3)$
Ex: $P_1=(2,-3,-3)$ and $P_2=(4,1,-1)$

Find direction angles of $v=-6i+12j+4k$ (α, β, r)

$\|v\|=\sqrt{(-6)^2+(12)^2+(4)^2}=\sqrt{196}=14$

$\cos\alpha=\frac{-6}{14}=-\frac{3}{7}$, $\alpha \doteq 115.4^\circ$

$\cos\beta=\frac{12}{14}=\frac{6}{7}$, $\beta \doteq 31^\circ$

$\cos r=\frac{4}{14}=\frac{2}{7}$, $r=73.4^\circ$

$v=\|v\|(\cos\alpha\, i+\cos\beta\, j+\cos r\, k)$
$=14(\cos 115.4^\circ + \cos 31^\circ + \cos 73.4^\circ)$

---

64. p.624

$r=3$, $P_0(-1,1,2)$

$\sqrt{(x+1)^2+(y-1)^2+(z-2)^2}=3$

$(x+1)^2+(y-1)^2+(z-2)^2=9$

---

66 p.624 $x^2+y^2+z^2+2x-2z=-1$

$x^2+2x+y^2+z^2-2z=-1$

$x^2+2x+1+y^2+z^2-2z+1=-1+1+1$

$(x+1)^2+y^2+(z-1)^2=1$

c: $(-1,0,1)$ $r=1$

Work $\Rightarrow \|F\| \cdot \overrightarrow{AB}$

① If an angle btwn two nonzero vectors $= 0$ or π, the vectors are $\parallel$.

$v = \alpha w$

② If an angle btwn two nonzero vectors $= \frac{\pi}{2}$, the vectors are $\perp$.

$(v \cdot w = 0)$

③ Decomposing a vector into 2 $\perp$ vectors:

$$\text{proj}_w v = \frac{v \cdot w}{\|w\|^2} w$$

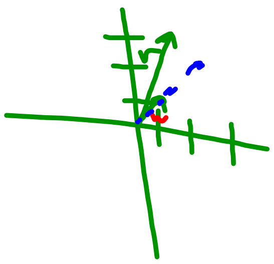
$$\Rightarrow v_1 = \text{proj}_w v = \frac{v \cdot w}{\|w\|^2} w$$

$$v_2 = v - v_1$$

Ex: Find vector projection of $v = i + 3j$ onto $w = i + j$. Decompose v into two vectors v_1 and v_2 where v_1 is $\parallel$ to w and $v_2 \perp w$.

① Graph it

② Rewrite into formulas: v_1 and v_2.



$$v_1 = \text{proj}_w v = \frac{v \cdot w}{\|w\|^2} w$$

$$= \frac{1+3}{(\sqrt{1^2+1^2})^2} w$$

$$= \frac{4}{2} w = 2w$$

$$= 2(i + j)$$

$$v_2 = (i + 3j) - (2i + 2j)$$

$$= i + 3j - 2i - 2j$$

$$= -i + j$$

72. p. 624

$F = 1$ newton

$2i + 2j + k$ moving 3M

$\cos\alpha = \frac{2}{\sqrt{2^2+2^2+1^2}} = \frac{2}{\sqrt{9}} = \frac{2}{3}$

$\cos\beta = \frac{2}{3}$

$\cos\gamma = \frac{1}{3}$

$F = 1\left(\frac{2}{3}i + \frac{2}{3}j + \frac{1}{3}k\right)$

$= \left(\frac{2}{3}, \frac{2}{3}, \frac{1}{3}\right) \cdot (1,2,2)$ where $(1,2,2)$ is the position vector from $(0,0,0)$ to $(1,2,2)$.

$W = F(1i + 2j + 2k)$

$= 1\left(1 \cdot \frac{2}{3} + \frac{2}{3} \cdot 2 + \frac{1}{3} \cdot 2\right)$

$= \frac{8}{3}$ joules

~~8~~ 9. 7.

$$\begin{vmatrix} \overset{+}{i} & \overset{-}{j} & \overset{+}{k} \\ 2 & 3 & 5 \\ 1 & 2 & 3 \end{vmatrix}$$

$$D = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$\det D = ad - bc$

$v = 2i + 3j + 5k$

$w = i + 2j + 3k$

$v \times w = (9-10)i - (6-5)j$

$+ (4-3)k$

$= -i - j + k$

§9.7

$$\begin{array}{ccc} i & j & k \\ \end{array}$$
$$\begin{vmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{vmatrix}$$

$$(6-6)i - (3-3)j + (2-2)k$$

$$\langle 0, 0, 0 \rangle$$

31. $v\cdot(u \times w)$ $\quad v = \langle -3, 3, 2\rangle$

$$v\cdot \begin{array}{ccc} i & j & k \\ \hline 2 & -3 & 1 \\ 1 & 1 & 3 \end{array}$$

$v\cdot\left(+(-10)i - (5)j + (5)k\right)$

$v\cdot(-10i - 5j + 5k)$

$-3(-10)\quad 3(-5)\quad 2(5)$

$30 - 15 + 10$

25

40. $u = P_1P_2 = \langle 2, 3, 1 \rangle$

$v = P_1P_3 = \langle -2, 4, 1 \rangle$

$$u \times v = \begin{vmatrix} i & j & k \\ 2 & 3 & 1 \\ -2 & 4 & 1 \end{vmatrix}$$

$= -i - 4j + 14k$

$\text{Area} = \|u \times v\| = \sqrt{(-1)^2 + (-4)^2 + (14)^2} \approx 14.6$

37. $u = \langle 2, -3, 1 \rangle$

$i + j = \langle 1, 1, 0 \rangle$

$$u \times (i+j) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 1 \\ 1 & 1 & 0 \end{vmatrix}$$

$$= \boxed{-1i + j + 5k}$$

Review Ch.9 #98.

98) $V_a = 500j$

$V_w = 60\left(\frac{\sqrt{2}}{2}i - \frac{\sqrt{2}}{2}j\right) = 30\sqrt{2}i - 30\sqrt{2}j$

$V_g = V_a + V_w = 500j + 30\sqrt{2}i - 30\sqrt{2}j$

$= 30\sqrt{2}i + (500 - 30\sqrt{2})j$

$\|V_g\| = \sqrt{(30\sqrt{2})^2 + (500 - 30\sqrt{2})^2} \approx 459.5 \text{ m/h}$

$\Rightarrow \cos\theta = \frac{V_g \cdot j}{\|V_g\| \|j\|} = \frac{(30\sqrt{2})(0) + (500 - 30\sqrt{2})(1)}{459.5\left(\sqrt{0^2 + 1^2}\right)} \approx .9958$

$\theta = \cos^{-1}(.9958) \approx 5.3^\circ$

N 5.3° E.

$V_1 = \overrightarrow{AB} = \langle 1, 2, 0 \rangle$

$V_2 = \overrightarrow{AC} = \langle -2, 4, -1 \rangle$

$$V_1 \times V_2 = \begin{array}{ccc} + & - & + \\ \hat{i} & \hat{j} & \hat{k} \end{array} \begin{vmatrix} 1 & 2 & 0 \\ -2 & 4 & -1 \end{vmatrix}$$

$= -2i + j + 8k$
(normal to plane)
($\perp$)

$P_0 = (0, 0, 4)$

$n_\perp = \begin{array}{ccc} a & b & c \\ (-2, & 1, & 8) \end{array}$

$\Rightarrow a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$

$-2(x - 0) + 1(y - 0) + 8(z - 4) = 0$

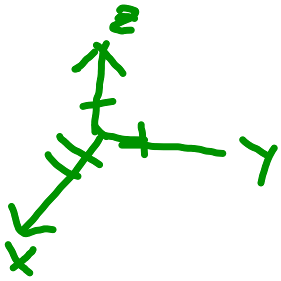
$-2x + y + 8z - 32 = 0$

4. $w = (2, 1, 1)$ $\quad n = (0, 4, 1)$

$$W \times H = \begin{array}{ccc} i & j & k \\ \end{array}$$
$$\begin{bmatrix} 2 & 1 & 1 \\ 0 & 4 & 1 \end{bmatrix}$$

$-3i - 2j + 8k$

$\langle -3, -2, 8 \rangle$



$a(x - x_0) + b(y - y_0) + c(z - z_0)$

$-3(x - 2)$